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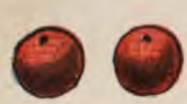
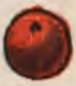
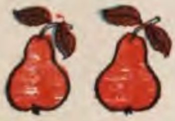
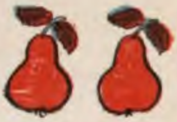
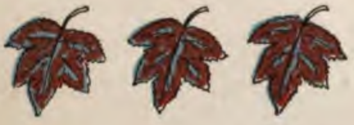
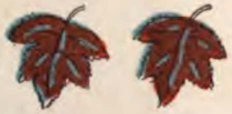
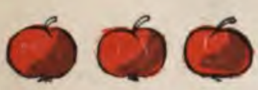
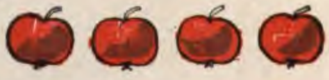
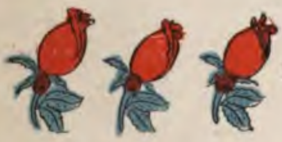
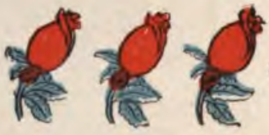
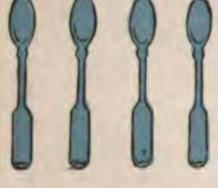
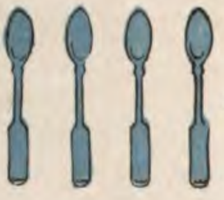
 and  are — ———

 from  are — ———

 and  are — ———

 and  are — ———

 from  are — ———

 and  are — ———

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FOR
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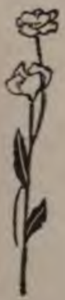

BY

Alvord D. Robinson

Author of "The Workers' Arithmetic," "Modern Methods in Arithmetic," "Night School Helps"

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PREFACE.

For Whom —

PRIMARILY this HELP is for teachers and intelligent parents. They will find in it much explanatory matter which is in accord with the best methods of to-day. The supplementary work is full, and will enable them to explain the actual business forms and transactions, as well as the elementary work, in a clear and concise manner. Older pupils can gain much practical knowledge from it along the lines of the work of the accountant, practical estimates, land surveying, insurance, stocks, bonds, etc. Most of these are illustrated from actual life, and these illustrations will do more to clear these matters up in the minds of pupils than will hours of explanation.

A Place of Its Own —

The number of text-books written on arithmetic in this country is legion. There has been a remarkable increase in the last few years. Each new author has felt that his book would be more practical and more satisfactory than those before. Of them all, however, there are but three or four that have become especially prominent, and that because of real merit. The author makes no attempt to compete with them, or to present any new plan of teaching the subject. The HELP has a field of its own, as the most casual perusal will disclose.

Why Written —

Some years since, Col. Parker said in effect that the arithmetic of the future would contain neither rules, definitions nor

arithmetic work, will especially appreciate this HELP. The articles and illustrations combine to give the phases of arithmetic that belong to actual life a clearness and virility that otherwise can only be obtained from actually sharing in the various business transactions treated.

IN THE HOME —

A few of its uses as a home book are here enumerated: —

1. For the pupil who is unavoidably absent for a few days the present spiral method affords no chance to learn what the teacher has given the class in his absence. To be sure, if he is unusually bright he will pick up in various ways what he missed. If he is not bright it is probable that he becomes a stumbling-block to the class for a time. With this HELP in his home he can, in most cases, make up the deficiency.

2. In the teaching of denominate numbers, practical estimates, percentage, exchange, discount, stocks, bonds, notes, insurance, and other business matters, the pupil cannot always follow the teacher's explanation. If he has a simple and truthful treatment of these subjects, well illustrated, at home he may easily inform himself, or the intelligent parent, after reading it, can give him a home talk on the subject that is not a "*lesson*" and which he will remember because of the place and manner in which it was given.

3. If his memory fails him in some important point either while in school or later, it is a reference work to which he may refer to renew his knowledge. He cannot always have a teacher, and no pupil can remember all he is taught and sometimes he forgets that which he afterwards needs. The arithmetics of to-day leave a vacancy here which the HELP fills in a way to assist in making him an independent thinker.

PROBLEMS AND SOLUTIONS —

After each subject a series of problems with solutions and other explanatory matter is given. Wherever it seemed wise,

CONTENTS.

PAGE

PART ONE —

In the Kindergarten - - - - - - - - - - - - - - 13
Numbers for Little People - - - - - - - - - - 16
Measures for Little People - - - - - - - - - - 40

PART TWO —

The Common Signs - - - - - - - - - - - - - - - 55
The Fundamental Operations: — Addition, Subtraction, Multiplication and Division - - - - - 58

PART THREE —

Factoring - - - - - - - - - - - - - - - - - - - 131
Greatest Common Divisor - - - - - - - - - - - 135
Least Common Multiple - - - - - - - - - - - - 140
Cancellation - - - - - - - - - - - - - - - - - 144

PART FOUR —

Common Fractions - - - - - - - - - - - - - - - 147

PART FIVE —

United States Money - - - - - - - - - - - - - 179
Decimal Fractions - - - - - - - - - - - - - - - 185
Price Marks - - - - - - - - - - - - - - - - - 217
Bills, Cash Book, Accounts Purchase and Accounts Sales - - - - - - - - - - - - - - - - - 218

PART SIX —

Denominate Numbers - - - - - - - - - - - - - 225
Practical Estimates - - - - - - - - - - - - - - 262

PART SEVEN —

Percentage - - - - - - - - - - - - - - - - - 283
Profit and Loss, Commission, Trade Discount - - - 300
Interest - - - - - - - - - - - - - - - - - - - 309
Bank Discount - - - - - - - - - - - - - - - - 334
Exchange - - - - - - - - - - - - - - - - - - 339
Taxes and Duties - - - - - - - - - - - - - - - 348
Insurance - - - - - - - - - - - - - - - - - - 356
Stocks and Bonds - - - - - - - - - - - - - - - 363

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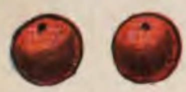
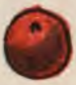
the tent. My **four** cousins saw me but I did not see them.

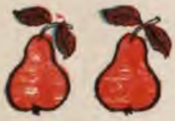
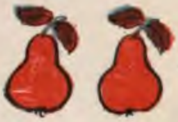
I liked the trained seals best of all. There were **five** of them. The clowns were very funny. They made everybody laugh. One clown counted **one, two, three, four, five, six,** and then jumped off the tight wire. He did not hurt himself.

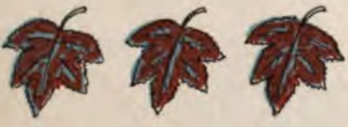
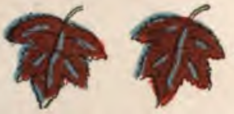
The circus had **seven** elephants this year and **eight** camels. Last year I rode on an elephant's back in a basket with **eight** girls and boys. The elephant must be strong to carry **nine** people like that.

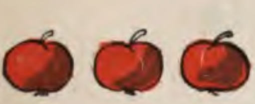
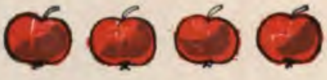
There were **ten** little boys I know who could **not**

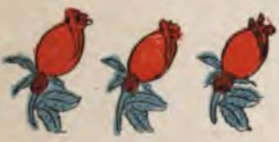
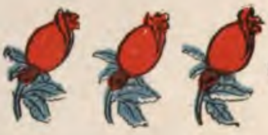
GROUPING AND COLOR EXERCISES.

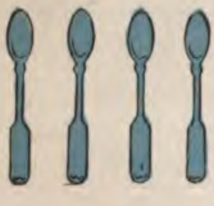
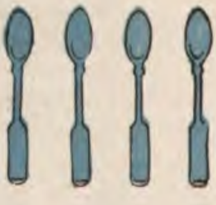
 and  are — ——

 from  are — ——

 and  are — ——

 and  are — ——

 from  are — ——

 and  are — ——

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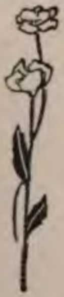

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many we mean. When we wish to say in writing that we have ten of anything we write a **0** *after the* **1** *and have* **10**, *ten.*

This book devotes several pages that may be utilized for counting exercises. One page shows groups of objects in colors; another gives a counting drill in colors; a third gives combinations of **2** *and* **1**, **2** *and* **3**, *etc., to* **10**; *and the addition drills may be used also as counting drills.*

Remember to have the child express himself in complete sentences.

NUMBERS FROM 1 TO 100.

0	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99

100

Things for the Child to do:

1. Cut out 40 squares of paper one inch on a side.

2. Number the squares from 1 to 40.

3. Show the teacher the square that has on it the number she asks for.

To the Teacher:

Figures placed on the blackboard should be written large and in color, yellow preferred.

Be careful that your models of figures are correct.

Use of Colored Pages:

Ask the child questions like these:
How many fish on this page?
How many spools do you see?
How many flags are waving?

Accept only complete statements in answers, as,
There is one fish here.
I see two flags waving, etc.

TEACHING THE COMBINATIONS.

The following plan is suggested in teaching the combinations in Addition and Subtraction:—

1. Add 1 to every number up to **10**; later to **20.**

2. Subtract 1 from every number up to **10**; later to **20.**

3. Add

$$\begin{array}{rrrrrrrrr} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 & +10 \end{array}$$

Here the addends are equal and easily added. The figures should be placed as above and not **2 + 2, 3 + 3,** etc., because the vertical form is the natural one which the child will use all through life. It does not look so formal and represents better what he really does with the objects.

4. Add $\begin{array}{rrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 & +10 \end{array}$

Here one addend exceeds the other by **1.**

5. Subtract $\begin{array}{rrrrrrrrr} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 \end{array}$

Here the minuend is **1** greater in each case.

6. Teach the parts of **10.**

$$\begin{array}{rrrrr} 5 & 2 & 3 & 4 & 9 \\ +5 & +8 & +7 & +6 & +1 \end{array}$$

7. In adding **9** to numbers have the child think of **9** as **10**

thus $\begin{array}{r} 9 \\ +6 \end{array} = \begin{array}{r} 10 \\ +5 \end{array}$ and $\begin{array}{r} 9 \\ +8 \end{array} = \begin{array}{r} 10 \\ +7 \end{array}$

8. Teach the corresponding subtractions.
9. Add **8** to each number up to **10.**
10. Teach the corresponding subtractions.
11. Add **7, 6,** and **5** to each number up to **10.**
12. Teach the corresponding subtractions.
13. Review and give combinations not taught above.

NOTE.—Objects should be grouped by tens and units, in showing numbers above ten.

The numbers written should represent objects counted.

THE DOMINO METHOD FOR DRILL ON COMBINATIONS.

Devices to be Used with the Domino Method:

1. Cut paper dominoes for the child and mark them.

2. A piece of paper, with a figure showing the sum of the dots, may be placed by the child at the right side of each domino.

3. In place of dots on the paper dominoes, little

pictures of common objects may be cut out of papers and magazines and pasted on the domino slips.

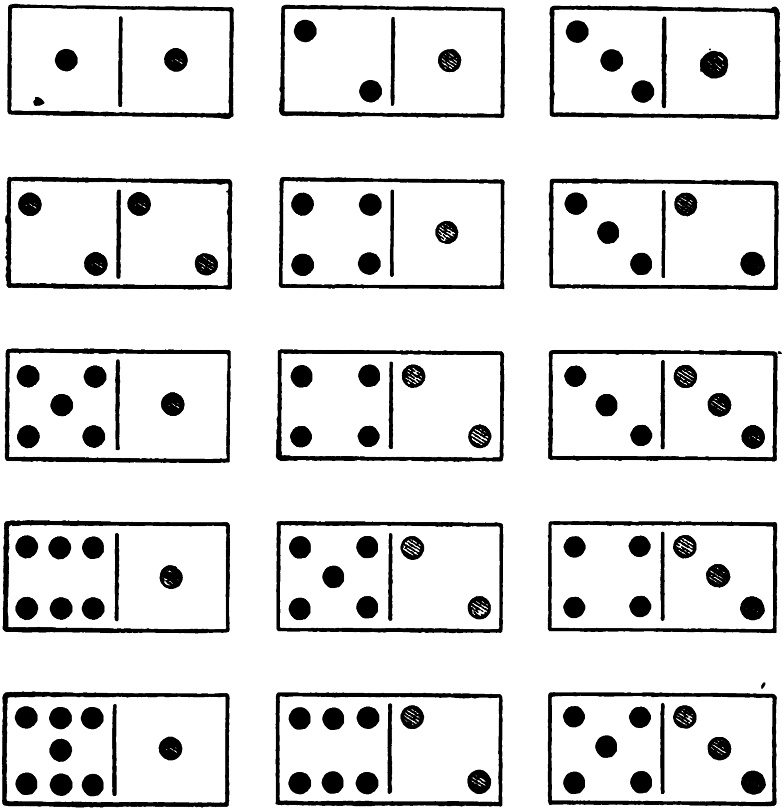
4. Count by **2's** to **20**; by **3's**; by **4's.**

5. Subtract by **2's** to **20**; by **3's**; by **4's.**

6. Try adding rapidly

9	**3**	**6**	**5**	**8**	**9**	**4**
+9	**+8**	**+7**	**+10**	**+6**	**+7**	**+8**

If one can add readily subtraction needs very little attention



With any plan used in teaching the combinations, paper dominoes will be a great help, because they can be adapted to any method, since dominoes may be made to suit the teacher.

If the method advised here with its thirteen subheads is followed, mark dominoes with a different number of dots on one end and have only one dot on the other end. Proceed in the order given in the plan proposed here, marking the dominoes to carry out the ideas suggested.

NUMBER STORIES.

TALK:

All number work between 1 *and* 10 *should be taught by means of objects and this oral work made interesting and practical, the child using objects at every step to show the processes involved.*

Allow no written work until little problems are solved readily in connection with actual handling of objects or counters.

At first the child should simply do as you direct with the counters. Later, for language training and drill in the combinations, he should be led to tell number stories.

The plan is this:—

(1) You tell the story, the child using the objects or counters as called for in the story.

(2) Later the child should tell stories from examples and equations, using objects and counters to illustrate.

(3) Still later equations and examples should be formed from given problems.

To show the process of development the following illustrations are given:—

First Kind of Number Story.

Show me **5** counters. Show me **6** counters. Call the counters oranges. **2** oranges and **3** oranges are how many oranges? I am going to tell you a story about oranges.

STORY.

I had 2 oranges and my sister gave me 4 more and we had 6 oranges in all.

Who can tell me another story about oranges? (The child should be using the counters to show all this.)

THE CHILD'S STORY.

A little boy had 2 oranges and his mother gave him 2 more. Then he had 4 oranges.

A Later Development.

$$\begin{array}{r}5\\+3\end{array}$$ (written by teacher.)

Who can tell a story about this?

THE CHILD'S STORY (counters used constantly).

Five chickens and 3 chickens are 8 chickens.

A Still Later Development.

I had 4 quarts of milk and sold 2. How many quarts have I left?

THE CHILD WRITES:

$$\begin{array}{r}4\\-2\\\hline 2\end{array}$$

COLORED BLOCKS.

TALK:

The use of colored blocks in number story work is very popular with primary teachers.

Provide the children with colored blocks and ask for number stories bringing in all the combinations and processes.

One page of the book shows colored cuts of blocks and suggestive arrangements of them.

TEACHING THE PRIMARY IDEAS OF DIVISION.

In teaching the two ideas of division—division by measurement (division proper) and the fractional idea of division (partition)—proceed very slowly and see that each step is thoroughly understood.

The following suggestions may be useful:—

DIVISION BY MEASUREMENT.

Use colored blocks or any other counters in illustrating the process.

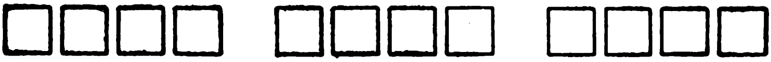
EXAMPLE:

$4\underline{|\,12\,}$

The teacher should ask the child, "How would you count this story?"

Facts Given by Child.

12 = whole number of blocks.
4 = number in each part.
We want to know the number of parts.
We place the blocks so, 4 in each part:—



There are 3 parts.

DIVISION BY PARTITION.

Make use of colored blocks or substitutes to show the process here.

EXAMPLE:

$\frac{1}{4}$ of 12 = 3

The teacher should ask the child, "How would you count this story?"

NUMBER STORIES WITH COLORED BLOCKS.

🟦🟦🟦 and 🟥🟥 are — ——

🟥🟥🟥🟥 and 🟦🟦🟦 are — ——

🟫🟫 from 🟫🟫🟫🟫🟫 are -- ——

🟫🟫🟫 from 🟥🟥🟥🟥🟥🟥 are —

🟦🟦🟦 taken 2 times are — ——

🟥🟥🟥🟥🟥🟥🟥🟥 put in 4 piles mak

— 🟫s in each pile.

🟥🟥🟥🟥🟥🟥 are — dozen 🟦s.

Note.—Talk this way: "Three blocks and two blocks are fi
blocks," etc.

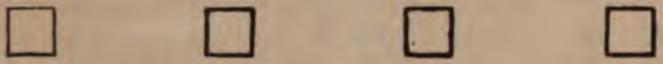
Facts Given by Child.

12 = whole number of blocks.

4 = number of parts.

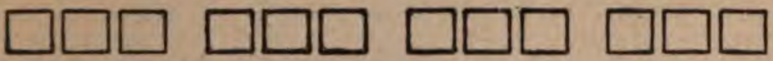
We want to know the number in each part.

We place the blocks so, as we know there are **4** parts:—



We have put one in each part.

Now we will put one in each part until the **12** blocks are gone:—



There are **3** in each part.

NUMBER STORIES IN MULTIPLICATION.

EXAMPLE:

3 × **4** = —

Facts Given by Child.

3 × **4** means **3 4's.**

I count my blocks by fours—I take **1** four, another, another.

I find that **3 4's** are **12.**

3 × **4** = **12.**

To the Teacher:

Now the child is ready to give a number story about **3** × **4.**

NOTE.—If the pupil is in doubt about any part of the work in number stories, he should use objects, and then practice much without them.

MATERIAL FOR NUMBER STORIES.

Let the child tell stories about the following:—

dogs 3 +4	cows 4 +5	eggs 7 −2
books 6 +3	trees ½ of 6	horses 8 +2
pencils 3 +5	chickens ⅓ of 6	squirrels 8 −2
chairs 6 −4	cups 4 +2	ducks 7 −3

These are but a few of the many that will suggest themselves.

PICTURING PROBLEMS.

TALK:

Having little children draw problem pictures for busy work is very pleasant and profitable recreation for them. Show them how to draw and continue to do so through the primary grades.

Encourage the invention of simple illustrations that show what problems tell but remember that too much pains divert children from the true purpose. Fair drawings are all that should be required

Problem picturing is a means, not an end.

DRILL ON COMBINATIONS FROM 1 TO 10.

TALK:

The following manner of teaching the combinations to ten is given to illustrate the nature of primary lessons.

Squares are used because they can be easily made by the child and always look well.

Make little problems, involving the following combinations, and have the child solve them. Make them very simple.

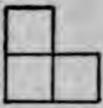
Have the child complete the examples on combinations, and tell a number story about each.

2.

1 + 1 = 2
2 1's = 2

2 − 1 = 1
2 × 1 = 2

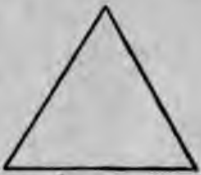
2 − 2 = 0
1 × 2 = 2

3. 

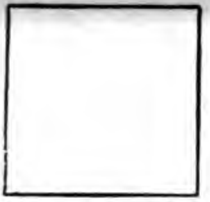
1 + 2 =
2 + 1 =
3 − 1 =
3 − 2 =
$\frac{1}{3}$ of 3 =
$\frac{2}{3}$ of 3 =

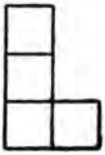
3 ÷ 2 =
3 ÷ 1 =

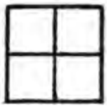
1 + 1 + 1 =
3 × 1 =
3 1's =



A triangle has three sides.

4. 





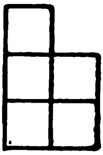
A square has 4 sides.

1 + 3 =
4 − 1 =

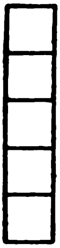
3 × 1 =
4 − 2 =

2 × 2 =
4 ÷ 2 =

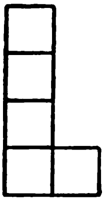
5. A nickel =



3 + 2 = 5



5 1's = 5



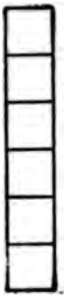
4 + 1 = 5

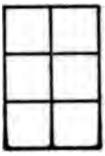
1 + 4 =
4 + 1 =
5 − 4 =

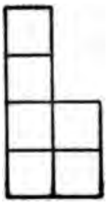
3 + 2 =
2 + 3 =
5 − 2 =

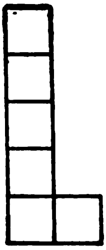
2 + 2 + 1 =
1 + 1 + 1 + 2 =
5 − 2 − 2 =
5 × 1 =
5 ÷ 1 =

6.


6 1's = 6


3 + 3 = 6

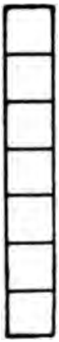

4 + 2 = 6

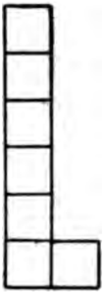

5 + 1 = 6

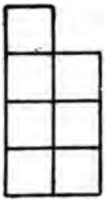
5 + 1 =
1 + 5 =
6 − 1 =
6 − 5 =
2 + 2 + 2 =
6 − 2 − 2 − 2 =
2 + 1 + 3 =
6 ÷ 2 =
A half dozen =
A half foot =

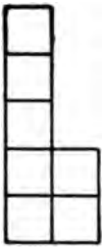
3 + 3 =
6 − 3 =
6 ÷ 3 =
2 + 4 =
½ of 6 =

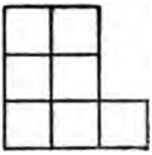
7.


7 1's = 7


6 + 1 = 7


4 + 3 = 7


5 + 2 = 7


3 + 3 + 1 = 7

6 + 1 =
7 − 5 =
7 ÷ 3 =
7 − 2 − 2 − 2 − 1 =

7 − 2 =
7 − 4 =
7 ÷ 2 =

1 + 6 =
7 − 6 =
4 + 3 =

7 − 5 =
2 + 2 + 2 + 1 =
2 × 3 + 1 =

---

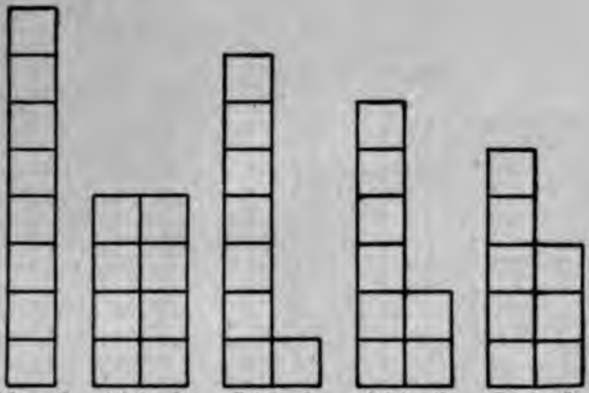
1 + 7 =
7 + 1 =
6 + 2 =
2 + 6 =
8 − 1 =
8 − 7 =

8 − 2 =
8 − 6 =
8 ÷ 2 =
5 + 3 =
3 + 5 =

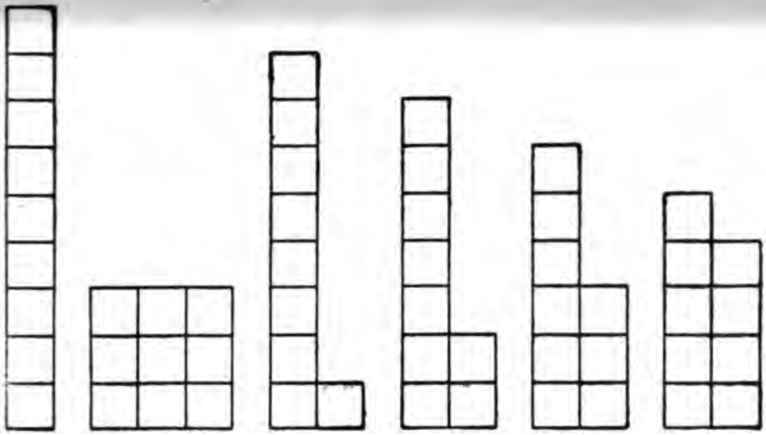
8 − 3 =
8 − 5 =
4 + 4 =
8 − 4 =
½ of 8 =

¼ of 8 =
4 × 2 =
3 + 3 + 2 =
3 × 2 + 2 =
2 + 2 + 2 + 2 =

8.

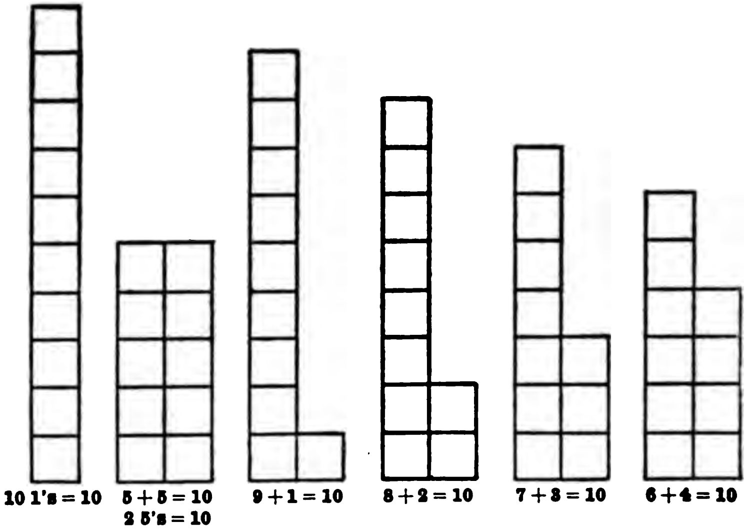

8 1's = 8 4 + 4 = 8 7 + 1 = 8 6 + 2 = 8 5 + 3 = 8

9.


9 1's = 9 3 + 3 + 3 = 9 8 + 1 = 9 7 + 2 = 9 6 + 3 = 9 5 + 4 = 9

1 + 8 = 3 + 6 = 4 + 5 = 8 + 1 =
6 + 3 = 5 + 4 = 2 + 7 = 9 − 3 =
9 − 4 = 7 + 2 = 9 − 6 = 9 − 4 − 5 =
9 − 1 = 9 + 3 = 9 + 4 = 9 − 8 =
2 + 2 + 2 + 2 + 1 = 9 − 2 − 2 − 2 − 2 − 1 =

10.



1 + 9 =
9 + 1 =
2 + 8 =
8 + 2 =

10 − 9 =
10 − 8 =
10 − 7 =
7 + 3 =

6 + 4 =
4 + 6 =
5 + 5 =
5 × 2 =

2 × 5 =
10 ÷ 2 =
3 + 3 + 3 + 1 =
10 − 3 =

1 dime = 2 nickels =

READING AND WRITING NUMBERS.

TALK:

WHEN the child first enters school he should have no written number work. The written work is left until the latter part of the first school term.

That the order may be understood the following general plan of the first few weeks of the child's number work is given:—

NOTE THE ORDER:

Order of Work.

1. Discover the child's knowledge of number.
2. Teach each number to 10 *as a whole.*

3. Teach the child to count to **50,** *later to* **100.**

4. Have the child measure oblongs, triangles, circles, and squares to determine which are largest.

5. Facts about each number from **1** *to* **10** *to be discovered by the child.*

6. Teach figures to **8,** *later to* **15.**

7. Have little problems solved involving facts discovered by the child.

8. Let the child invent problems.

Notice in the above plan that the child has nothing to do with figures for a time. Then he is taught to make them one by one. After practice he should be able to make them easily and accurately.

To learn the child's knowledge of figures, a good plan is to show a certain number of objects and have him write the figure or figures which represents the number.

Reverse this by making figures and having the child show that number of objects.

READING NUMBERS.

When we spell a word, that is, when we read the letters in the word, we read from **left to right.**

To read a number we do the same thing for we read the figures from left to right.

325 is read "Three hundred twenty-five."

The number is read differently if we wish to tell the **place** and **value** of a certain figure, for then we read the figures themselves from **right to left,** thus—

325 means 5 units, 2 tens, 3 hundreds.

Never use "AND" in reading whole numbers.

325 should never be read, "Three hundred and twenty-five," but "Three hundred twenty-five."

NUMBERS ABOVE TEN.

TALK:

Before the child starts with the combinations and separations involved in number work above 10, he should have mastered the following:—

1. He is acquainted with quarts and pints, gallons and quarts, bushels and pecks, pecks and quarts, and yards and feet.

2. He must know halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, and tenths.

3. He must be able to make all combinations up to 12, whether the process is by addition or by multiplication.

4. He must be able to make all separations up to 12, whether the process is by subtraction or division.

Before proceeding to number work above 10, test the child's previous work thoroughly. A good deal depends on this first knowledge.

HOW TO TEST THE CHILD'S EARLY WORK.

1. Show objects rapidly, separating and uniting them.

Ask for answers to bring out the child's knowledge of addition, subtraction, multiplication, and division.

2. Give oral and written work without the use of objects.

3. Give extended practice in problem work.

4. Child to form problems.

5. Child to make drawings to show processes.

TALK:

When the child was taught to write the figures eleven and twelve, he was told to write them in the same way as he wrote the figures two, three, four, etc.

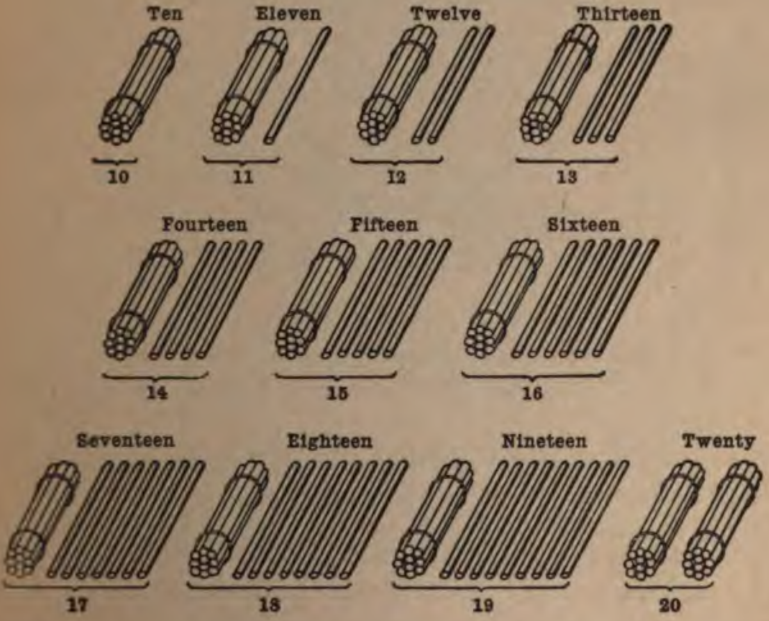
Now he is to be taught that numbers between ten and twenty are made up of tens and units.

Use toothpicks or splints to show that eleven is composed of 1 ten and 1 unit; that twelve is made up of 1 ten and 2 units, etc.

Ask the child to show what fourteen means to him; fifteen; sixteen, etc.

Give him a pile of splints and several rubber bands and have him show the groups of tens and units in each number from ten to twenty.

THE TENS AND UNITS.



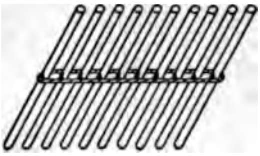
Numbers between ten and twenty are composed of tens and units as shown above.

Eleven means 1 ten and 1 unit; twelve, 1 ten and 2 units, etc.

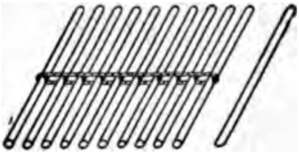
Use the Following Plan Also:

Take splints. Make several rows like these:—

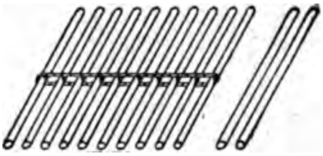
(1)



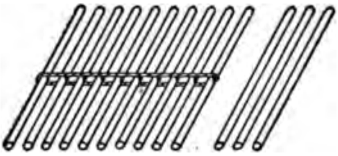
(2)



(3)



(4)



Ask the child to tell the number of splints in the upper row; in the second; in the third; etc.

Ask him to tell of what *eleven* is composed; *twelve; thirteen*, etc.

Tell him the right-hand place is the *unit's place*. the left-hand place is the *ten's place*.

Another Method:

Tens.	Units.
1	0
1	1
1	2
1	3
1	4
1	5
1	6

Things for the Child to Do:

1. Have the child show with objects the tens and units in 12, 13, 14, 15, 16, 17, 18, 19.

2. Drill on many of these:—

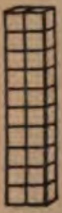
1 ten and 2 units make 12.
1 ten and 3 units make 13.

3. Write the following and have the child write it:—

eleven, 11, 1 ten 1 unit.
twelve, 12, 1 ten 2 units.
thirteen, 13, 1 ten 3 units.
fourteen, 14, 1 ten 4 units.
fifteen, 15, 1 ten 5 units.
sixteen, 16, 1 ten 6 units.
seventeen, 17, 1 ten 7 units.
eighteen, 18, 1 ten 8 units.
nineteen, 19, 1 ten 9 units.

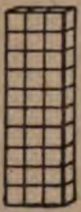
NUMBERS ABOVE TWENTY.

A



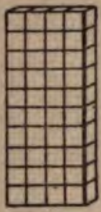
20
Twenty

B



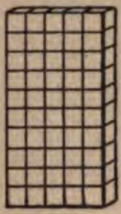
30
Thirty

C



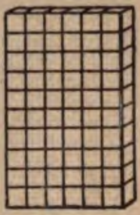
40
Forty

D



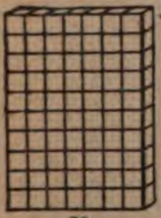
50
Fifty

E



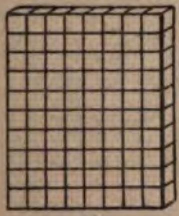
60
Sixty

F



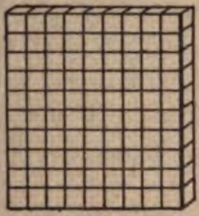
70
Seventy

G



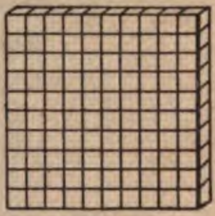
80
Eighty

H



90
Ninety

I



100
One hundred

FROM 10 TO 100.

Up to 10 units we simply name the number of units by the given figure.

Up to 10 tens the tens are named by the second figure from the right.

70 means 0 units and 7 tens.

ten, 10, means 1 ten.
twenty, 20, means 2 tens.
thirty, 30, means 3 tens.
forty, 40, means 4 tens.
fifty, 50, means 5 tens.
sixty, 60, means 6 tens.
seventy, 70, means 7 tens
eighty, 80, means 8 tens.
ninety, 90, means 9 tens.

FROM 100 TO 1000.

If the number has more than 10 tens the groups of tens are arranged into larger groups called hundreds.

one hundred, 100.
two hundred, 200.
three hundred, 300.
four hundred, 400.
five hundred, 500.
six hundred, 600.
seven hundred, 700.
eight hundred, 800.
nine hundred, 900.

FROM 20 TO 100, CONTAINING UNITS.

Show the following by means of toothpicks or splints:—

twenty-one, 21, 2 tens 1 unit.
twenty-two, 22, 2 tens 2 units.
twenty-three, 23, 2 tens 3 units.
seventy-five, 75, 7 tens 5 units.
ninety-two, 92, 9 tens 2 units.
forty-six, 46, 4 tens 6 units.
eighty-nine, 89, 8 tens 9 units.

FROM 100 TO 1000, CONTAINING UNITS.

Numbers between 100 and 1000 are read by naming the hundreds, tens, and units in succession.

Thus—

192—one hundred ninety-two.
652—six hundred fifty-two.
207—two hundred seven.

ABOVE 1000.

Read 1,892,632.

Here the 2 represents 1's, or units of the *first order.*
- the 3 represents 10's, or units of the *second order.*
- the 6 represents 100's, or units of the *third order.*
- the 2 represents 1000's, or units of the *fourth order.*
- the 8 represents 100000's, or units of the *sixth order*, etc., etc.

The order of any figure is the number of its place, starting from the right.

In reading large numbers we arrange the orders into groups of three figures each.

We call each group a *period.*

Commas are used to keep the periods separate.

NAMES OF THE PERIODS.

units,
thousands,
millions,
billions,
trillions,
quadrillions,
quintillions,
sextillions,
septillions,
octillions,
nonillions,
decillions.

trillions	billions	millions	thousands	units
225,	**902,**	**302,**	**472,**	**399.**

ROMAN NOTATION.

TALK:

The child sees a different kind of notation on the dial of a clock or watch. The lessons and chapters of his text-books are also designated by this style of nota-

tion, so he should be taught the meaning of the characters used and how they are connected to form other numbers.

The work will be easy for the child, because daily contact with these characters fixes them in his mind.

Tell him these characters are Roman and the reason why.

At this stage only teach the Roman Numerals to 100.

I = **1**
V = **5**
X = **10**
L = **50**
C = **100**

From these five characters can be formed all the Roman figures from 1 to 100.

Two Things to Remember:

1. When a small Roman Numeral is placed before a larger one—subtract.

2. When a small Roman Numeral is placed after a larger one—add.

Thus—

VI
Here V=5 and I=1.
The larger Roman Numeral is placed before the smaller one, so we add.
5+1=6.
Then VI=6.

XIX
Here X=10.
Then IX=9. Why?
And XIX=19. Why?

Have much practice work, requiring the child to write out results as above.

Interesting Facts about Roman Numerals:

The following facts about the origin and development of Roman Notation can be made very interesting, and will tend to fix the characters in one's mind:—

At first one was written I,
two was written II,
three was written III,
four was written IIII,
five was written IIIII,
six was written IIIIII,
seven was written IIIIIII,
eight was written IIIIIIII,
nine was written IIIIIIIII.

It took too long to make ten marks, and too long to count them after they were made, so one mark with another across it was used to denote 10, thus X. The upper or first half of the X, or V, was used to show 5.

At first twenty was written XX,
thirty was written XXX,
forty was written XXXX,
ninety was written XXXXXXXXX.

It took too long to make the crosses and too long to count them in reading, so [was used to show 100. Half of the [or L was used to show 50. The [gradually became C in trying to make the [rapidly.

Write these where the child can see them plainly:

I = 1
II = 2
III = 3
IV = 4
V = 5
VI = 6
VII = 7
VIII = 8
IX = 9
X = 10

XI = 11
XII = 12
XIII = 13
XIV = 14
XV = 15
XVI = 16
XVII = 17
XVIII = 18
XIX = 19
XX = 20

XXI = 21
XXII = 22
XXIII = 23
XXIV = 24
XXV = 25
XXVI = 26
XXVII = 27
XXVIII = 28
XXIX = 29
XXX = 30

XXXI = 31
XXXII = 32
XXXIII = 33
XXXIV = 34
XXXV = 35
XXXVI = 36
XXXVII = 37
XXXVIII = 38
XXXIX = 39
XL = 40
XLI = 41
XLII = 42
XLIII = 43
XLIV = 44
XLV = 45
XLVI = 46
C = 100.
XLVII = 47
XLVIII = 48
XLIX = 49
L = 50
LX = 60
LXX = 70
LXXX = 80
XC = 90

MEASURES FOR LITTLE PEOPLE.

MEASUREMENT AND COMPARISON IN PRIMARY ARITHMETIC.

TALK:

The child has been taught to count. Now while he is telling you how many objects he is dealing with, teach him to tell how much he is dealing with.

In other words, have the child measure and compare as well as count.

Measurement *and* **Comparison** *of objects and numbers are most important in the study of arithmetic. One author makes this plain when he says that real arithmetic is the measuring of quantity. The successful teacher realizes this and is not satisfied with the answer, "Three feet and three feet make six feet." She wants the child to measure the distance. She wants him to know a foot.*

Repeating rules and tables is of litle practical value if the child does not know what each denomination means. It is a fine thing for him to know his tables; but it means more to know what they mean. What does a peck mean to him? An hour? A yard?

Some parents reproach the teacher by telling her that, "To-day young children don't figure as rapidly as they used to."

The reason for this is plain, if the parents stop to *think. It is because in the best schools mere rapid mechanical work is left until there is a proper understanding of processes.*

Rules and definitions are left until the child can make them for himself. When he is able to give an original rule or definition the teacher may improve the wording, not the thought. The child should furnish that from his experiences.

At school children should have access to foot rules, yard rules, dry and liquid measures, and scales. At home every child should have a foot rule. While he is young he should prove everything by actual measurement and comparison.

Arithmetic cannot be too practical.

METHOD:

Here is a foot measure. Draw a line one foot long. Another.

How many lines have you now?

If you cover up one, how many?

Draw another. Now how many have you? 2 and 1 are how many?

How many ones in 4? How many twos? etc.

For the Child to Learn Objectively:

12 inches make 1 foot.
3 feet make 1 yard.
12 things make 1 dozen.
5 cents make 1 nickel.
10 cents make 1 dime.
2 pints make 1 quart.
8 quarts make 1 peck.
4 pecks make 1 bushel.
7 days make 1 week.*
16 ounces make 1 pound.

*** Objectively so far as pointing it out on a calendar.**

LONG MEASURE.

TALK:

Teach the child to estimate distances and then to verify every estimate by actual measurement.

Teach half inches as well as inches.

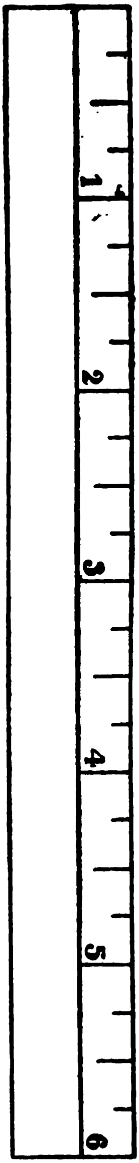
Long Measure Is Used to Measure Length.

12 inches make 1 foot.
3 feet make 1 yard.

in. means inch or inches.
ft. means foot or feet.

For the Child to Do:

1. Cut a strip of paper 12 inches long and 1 inch wide.
2. Mark the inches on it.
3. How many inches long is it?
4. What do you call a measure 12 inches long?
5. Draw a line 2 inches long, as near as you can, without using a ruler.
6. Measure it with a ruler. Did you guess nearly right? Try again.
7. Measure this page. How long is it? How wide?
8. Draw a line on the ground 1 yard long.
9. $\frac{1}{2}$ of a foot is how many inches?
10. 1 yard is how many inches? What is measured by the yard?
11. Ask your mother how many yards of cloth she needs for a dress.
12. What is measured by the foot?
13. How tall are you?



TELLING TIME.

TALK:

The best way to teach a child to tell time is to use the clock itself, and not a picture of one.

A little practice each day and a few moments spent in asking questions will soon overcome all difficulties.

Turn to the page on Roman Numerals and note the method of teaching.

60 minutes make 1 hour.
12 hours make 1 half day.
24 hours make 1 day.

A new day begins at midnight and lasts until the next midnight.

One hour after midnight is 1 o'clock, 2 hours after midnight is 2 o'clock, and so on, until 12 o'clock at noon.

At noon one-half the day has gone and we begin the afternoon, which is the last half of the day.

One hour after noon is 1 o'clock, 2 hours after is 2 o'clock, etc.

If a person says he was at a certain place at 2 o'clock, he must say forenoon or afternoon, so we will know which half of the day he means. If it was 2 o'clock in the morning he would write *2 A.M.*, and if 2 o'clock in the afternoon, *2 P.M.*

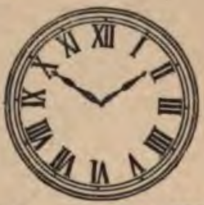
METHOD OF TEACHING A CHILD TO TELL TIME.

First—Teach him to tell the hour hand from the minute hand.

Next—Teach him when he first looks at the dial, to find the hour hand and then notice which Roman Numeral it is nearest. *This will tell about what time it is.*

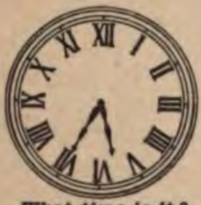
Then—Find the minute hand. *The minute hand will tell exactly what time it is.*

TO ILLUSTRATE:



Take this clock. The hour hand is near the Roman Numeral II, which stands for 2. Tell the child it is somewhere near two o'clock. The minute hand will tell *how near.*

It must always point to the XII before it is exactly the hour. If it is one numeral away from the XII, toward the left, it is 5 minutes of two. If it is two numerals away to the left, it is 10 minutes of two, etc. If it is one numeral away to the right, it is 5 minutes after two, etc. Proceed in this way and keep at it.

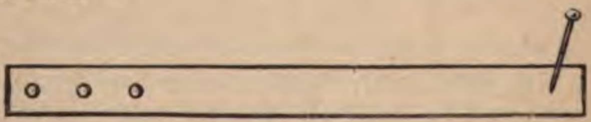


What time is it?

Things to Do:

1. Have the child draw several large clock faces on paper, using a 3-inch strip of cardboard or heavy paper with holes in it, to make the circle.

Like this—



Stick a pin through the end having no holes and use it as an axis in the making of the circle, which will be marked by the pencil point, placed in any one of the holes. The whole makes a compass.

2. Have the child make the clock (just drawn) show the time school opens.

3. Have him show the time school closes.

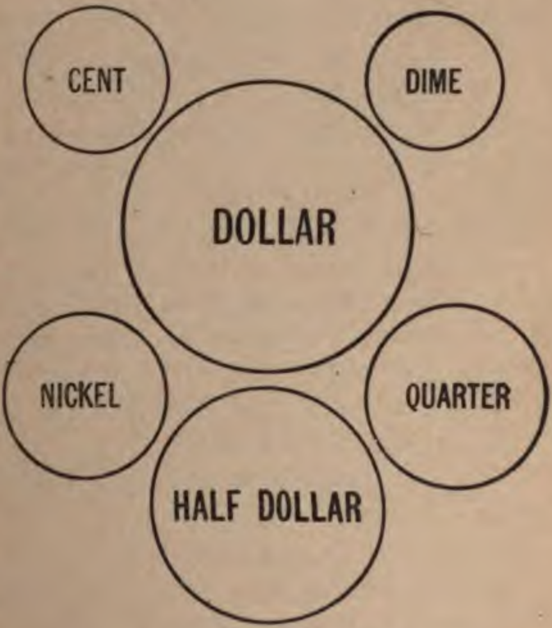
4. Have him show the time he rises in the morning; the time he retires.

MONEY.

TALK:

Use actual coins in teaching money. Lessons on money should be given frequently after the first year of school life.

Begin by teaching the value of the cent and the nickel, then the dime, then the quarter, then the half-dollar, and then the dollar.



Make problems involving change. Develop the ability to make change rapidly. The child may have some money of his own and he should be taught the comparative values of the coins. The correct method of making change is given on the next page.

A recent law forbids printing illustrations of U. S. coins, so the size of each only is given here.

10 cents make 1 dime.
2 five-cent pieces make 1 dime.
100 cents make 1 dollar.
A quarter of a dollar = 25 cents.
A half-dollar = 50 cents.

$ means dollars and ¢ means cents.

A 5-cent piece is called a nickel, because it is made of nickel.

A cent piece is made of copper.

The other coins named are made of silver.

Things for the Child to Do:

1. Name the lowest coin in United States money.
2. Name the coins in their order to one dollar.
3. How many cents equal a dime?
4. Mary has 3 5-cent pieces. How many cents has she?
5. How many cents in a dime and a nickel?
6. How many cents in a quarter-dollar?
7. How many cents in 2 quarter-dollars? in 3?
8. How many cents in 4 half-dollars? in 2?
9. How many cents in a dime and 3 cents?
10. Read—$6, $99, $76, 20¢, 19¢.
11. 1 dime+2 nickels= ———
12. How many quarters in a half-dollar?

How to Make Change.

First—Name the cost of goods.

Then—Add enough money to make even money.

Last—Add the large coin.

TO ILLUSTRATE:

I bought some cloth for 35¢ and gave the clerk 50¢. Count my change.

How to count it—
35¢ for cloth
And 5¢ makes 40¢ (to make even money)
And 10¢ makes 50¢.

TO ILLUSTRATE:

I buy suspenders for 32¢ and give the clerk $1. Count my change.

How to count it—
32¢ for suspenders
And 3¢ makes 35¢ (to make even money)
And 5¢ makes 40¢
And 10¢ makes 50¢
And 50¢ makes $1

NOTE.—Think of it in this way—32¢, 35¢, 40¢, 50¢, $1.

LIQUID MEASURE.

TALK:

If you have quart and pint measures allow the child to empty water from one to the other.

The small family kerosene can generally holds a gallon, so it may be used.

Do not forget that it is very important that the child memorize thoroughly the tables given.

On teaching the table of liquid measure proceed in this manner:—

Here is a gill cup. I am going to fill it to see how many gills make a pint. Now I have poured one gill; now two; now three; now four. Is the pint cup full? How many gills in a pint?

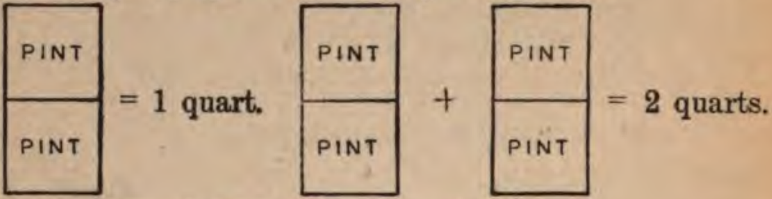
Let us prove what you say by pouring the water back. One gill; two gills; three; four. Were we right then?

Let us write what we have just said and proved,
4 gills = 1 pint.

Proceed in this way with the other measures, only let the child do more of the work for himself.

Things for the Child to Do:

1. Mention 3 liquids sold by your grocer.
2. One pint cup holds 4 gills; 3 pint cups hold —— times —— gills, or —— gills.
3. Does your kerosene come in a gallon can?
4. Mention all the things you know that are sold by the pint; by the quart; by the gallon.



In the Same Way Show by Drawings:

1. The number of quarts in 2 gallons.
2. The number of pints in 2 quarts and a pint.
3. The number of pints in 1 gallon and a quart.

THE CALENDAR.

TALK:

Procure a calendar for the child to own.

A good way to make him familiar with the use of the days of the week and month, as found on a calendar, is to ask him to look up and tell on what day of the week the next Fourth of July will fall; Christmas; New Year; his birthday.

Have him distinguish between the day of the week and the day of the month.

1903 July 1903

Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
First Quar. 1st 30th	Full Moon 9th	Last Quar. 17th	1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	New Moon 23d

7 days make 1 week.
30 days make 1 month.

Days of the Week.

Sunday
Monday
Tuesday
Wednesday
Thursday
Friday
Saturday.

Months of the Year.

January
February
March
April
May
June
July
August
September
October
November
December.

Things for the Child to Do:

1. Name the days of the week and the months of the year.
2. What day of the week does Christmas fall on this year?
3. Is 22 days longer than 3 weeks?
4. Is 5 weeks longer than a month? How much?
5. How many days in 3 weeks?
6. On what day of the week will your next birthday be, etc.?

Remember:

The days of the week in their order.
The months of the year in their order.

DRY MEASURE.

8 quarts make a peck (pk.).
4 pecks make a bushel (bu.).

TALK:

Dry measure is sometimes used to measure dry articles. Formerly grains, vegetables, and fruits of all kinds were bought and sold by dry measure.

Don't teach the child, as most arithmetics say, that grains, fruits, and vegetables are bought and sold by the measured bushel. They are not.

The weight bushel, the case, and the box have come into common use for these articles. More will be found on this under Denominate Numbers.

Teach the names of the measures before you attempt to teach the number of one measure in another, etc.

Things for the Child to Do:

1. How many qts. in a peck? 2 pks.? 3 pks.?
2. How many pks. in a bu.? 2 bu.? 2½ bu.?
3. Name some things that you buy by the quart; by the pk.; by the bu.

SQUARE MEASURE.

How long is each side of the figure on page 51?

How many squares in the figure?

Because each square is one inch on each side, we call it a square inch.

How many square inches in the whole figure?

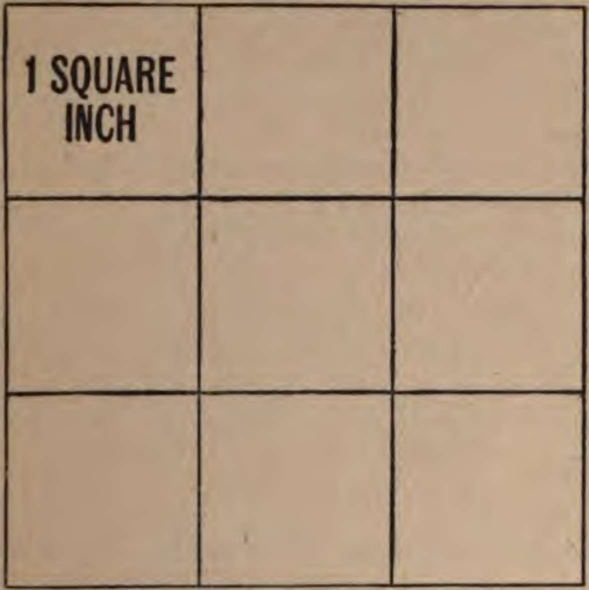
The number of squares is called the **area.**

9 square inches is the *area* of this figure.

Show by a drawing how many square inches in a figure 3 in. wide and 9 in. long.

Its area is —— square inches.

We need not always draw a figure to get the area.



You will notice that the *length times the width will give the number of squares in the figure.*

Find the area of a figure 9 in. long and 5 in. wide, without a drawing. What did you do? *Make a rule.*

Cut out a piece of paper 1 in. wide and 4 in. long. Cut it into square inches. Place the cut squares so as to form a large square. How long is it now? How wide?

TESTS FOR LITTLE PEOPLE.

1. Christmas is in what month?
2. Write the year in which you were born.
3. Draw two triangles making every line 2 inches long.

4. How many legs have 4 horses?

5. Name all the coins you know.

6. What does 8 A.M. mean to you?

7. Copy your calendar for December.

8. What time is it 12 hours after midnight?

9. 2 times nothing is what?

10. Write or tell 7 colors you know by sight.

11. If you were born in 1892, how old would you be now?

12. Name the shortest month. Does it have the same number of days every year?

13. Count backward from 31 to 2.

14. Can you tell how many square inches there are on one page of this book?

15. What number added to 8 will make 22?

16. What will it cost to mail 3 letters?

17. 10 days after your birthday is what day of the month?

18. How many 4's in 2 dozen?

19. What does XXVII stand for?

20. When the hour hand points to X and the minute hand to XII, what time is it?

21. How many days in 5 weeks?

PART TWO.

The Common Signs and the Fundamental Operations—Addition, Subtraction, Multiplication and Division.

"The secret of success is constancy of purpose."

—*Benjamin Disraeli.*

PRACTICAL TEACHING.

WHAT shall be said for the teacher who fears to omit certain problems which are not utilitarian and whose culture value is counterbalanced by the fact that they give a false notion of business, or to omit those traditional puzzles which depend for their difficulty upon their ambiguity of statement? Many a teacher, especially in our country schools, will confess to such a fear of omitting problems, lest he be accused of an inability to solve them.

It would be well for all teachers to assist in creating a sentiment in favor of omitting the unquestionably superfluous or dangerous, and thus to avoid this weak criticism. It should also be understood by timid teachers that it is no disgrace to be unable to solve every puzzle that may be sent in, or even every legitimate problem.

David Eugene Smith,
in Teachers' Professional Library.

THE COMMON SIGNS.

+ This sign is read *and* or *plus*.

When a pupil is first taught how many two apples and two apples are, we say "2 apples and 2 apples are 4 apples." When we write it this way:

2 apples
$\underline{+2}$ apples, the sign + before the last figure indicates that we are to combine the numbers by adding them.

Teach the child to know that wherever he sees the sign + it means that the numbers with which it is used are to be added.

= These two short straight lines are read *equals* or *are*. They show that the numbers joined or connected by the sign = are equal, as, $4+2=6$; $4+2=3+3$. In the first case $4+2$ are 6, so we write it $4+2=6$. In the second case $4+2$ are 6 and $3+3$ are 6. We can show that they are equal by writing it thus:—

$$4+2=3+3.$$

The number or numbers on the left of the sign *equal* those on the right of it.

— This sign is read *minus*, *less*, or *from*, and is the sign of *difference* or *subtraction*. Whenever it is used it means that the number after it is to be taken away from the number before it. $4-2$ means that we are to take 2 from 4, or subtract 2 from 4.

We may read it 4 *less* 2, 4 *minus* 2, or 2 *from* 4. The meaning is the same in all three. The expression 4 *minus* 2 is the best one. When the child needs it teach it in that way.

× This sign means *multiplied by.* 6 × 3 should be read, *6 multiplied by 3.* It may be read 6 *times* 3.

The sign shows that the number before it is to be taken as many times as there are units in the number after the sign.

In 6 × 3 it means that 6 is to be taken 3 times.

When the number after the sign has a name, as, 3 × $5; 4 × 6 cows, etc., read the sign *times* always.

3 × $5 is read *3 times $5.* To say 3 multiplied by $5 is not right.

NOTE.—This sign is sometimes used in certain kinds of business so that it is read *by*. A pane of glass 10 inches wide and 14 inches long is indicated in this way: 10 × 14 and is read "10 by 14."

The size of timbers is also shown in that way, as, a stick 4 × 5 means a stick 4 inches by 5 inches in diameter.

TALK:

Don't try to tell the children all these meanings of the sign at once. Tell them only when they begin to need them, and illustrate by examples. They will not need to know the last use of the sign until they have been in school some years.

÷ This sign is read *divided by.* It has two meanings. Teach the pupil only the first one given below. The last one is too difficult for young children. It is not needed in their early work, and when they get older they will understand it readily.

Teach this:—The sign means *divided by*, and it shows that we are to find how many times the number after the sign is contained in the number before the sign.

Use very small numbers to teach the use of the sign, as, 4 ÷ 2; 6 ÷ 2; 6 ÷ 3; etc.

The sign may mean to find one of the equal parts of a number, as, $8 ÷ 2 may mean to find ½ of $8. Do not try to teach this meaning at first. Leave it till a later time.

, This is an ordinary comma and is put between numbers to show that they are to be called separately, as, 1, 2, 3, 7, 9, which means that the numbers one, two, three, seven, nine, are to be read as if written in words, each by itself.

The comma is also used to make the reading of large numbers easy. To read the number 2719346 would be slow if commas were not used to show the hundreds, thousands, etc. If written in this way—2,719,346—you see at a glance that the 2 is millions, the 719 is thousands, etc.

. This sign is made like an ordinary period, but is called a decimal point. It is of no practical value to know why, but anyone who wishes to know may ask any teacher to explain it.

The most common use of this mark is in writing dollars and cents. The decimal point is always put directly after the dollars' figures, or just before the cents' figures, as in $45.05, which is read, "Forty-five dollars and five cents."

The decimal point is always read AND.

∴ These three dots arranged as shown are read *therefore*. They may be read *hence*. This sign is often used in the last sentence of a solution.

( ) The parenthesis indicates that all the numbers contained therein are to be considered together, as—

$$4\ (6-3+2+4)$$

means that all of that part within the parenthesis are to be treated together; in this case multiplied by 4.

Other signs that are used in the same way are.

Brackets $[4-3]$

The vinculum $\overline{4-2+3}$

Braces $\left\{ 6-4+2 \right\}$

These are called **signs of aggregation.**

ADDITION.

TALK:

It is very important that boys and girls learn early to add quickly and correctly. Slow and faulty adding is the first great trouble with most boys and girls who leave school to work at anything where even the simplest computations are required. Business men complain of them everywhere.

There is no one operation in all arithmetic that is half so valuable to you as a business man or woman, as to be able to add rapidly and accurately.

Practice half an hour each day and in a short while you will be surprised at your own proficiency.

BE CORRECT.

Impress upon the child that one figure wrong in an answer makes a failure. Lead him to realize that unless he gets his results absolutely correct he is worthless as an accountant or bookkeeper.

Illustrate the importance of accuracy by the use of a few problems like this:—

$$\begin{array}{r} \$414 \\ 326 \\ \underline{716} \\ 6 \end{array}$$

Show him that by making a mistake of one in adding the units' column someone may lose a dollar; that a mistake of one in the tens' column means ten dollars; while a mistake of one in the hundreds' column means a hundred dollars. Let him illustrate it to you.

Impress accuracy at the outset.

DEFINITIONS.

In addition and other subjects of arithmetic it is not desirable, in many cases, to pay any attention to definitions at first. Explain processes and lead the child to perform them. When the time comes for definitions let the child give one of his own if he can. If he cannot, help him make one.

His arithmetic will say something like this:—

"ADDITION is the process of finding the sum of two or more like numbers."

When he is to learn it, explain as far as you can what is meant by *process*.

The process is not the sum you get, but the thing done in getting it.

If he has learned numbers to ten he knows all he needs to know of the definition of addition now. It makes little difference to him in actual life, if he never knows more about it than he knows now, if he has been rightly taught thus far.

Sometime in the study of addition the teacher tells the pupil that the numbers in an addition are called **addends.** For instance, if you are to add four, six, and ten, each one of these is an addend.

The SUM is the name of the number he gets when he has finished.

Drill him on these names until he has them memorized. Let him call the *addends* and *sum* by their proper names in the same way that he calls the parts of an animal head, feet, etc.

THE WAY.

Probably when you went to school there was not much made of the "forty-five facts of addition" as given on this page. But if you ever learned to add quickly and correctly it was because you learned these facts in some way, perhaps each by itself from meeting it frequently.

When the teacher is teaching these facts of addition you can help drill the child on them until you know they are thoroughly learned. After that *drill* is the great help to make boys and girls rapid and accurate.

THE FORTY-FIVE FACTS OF ADDITION.

1	2	2	3	3	4	3	4	5
1	1	2	1	2	1	3	2	1
2	3	4	4	5	5	6	6	6
4	5	6	4	5	6	7	5	6
3	2	1	4	3	2	1	4	3
7	7	7	8	8	8	8	9	9
7	8	5	6	7	8	9	6	7
2	1	5	4	3	2	1	5	4
9	9	10	10	10	10	10	11	11
8	9	6	7	8	9	7	8	9
3	2	6	5	4	3	6	5	4
11	11	12	12	12	12	13	13	13
7	8	9	8	9	8	9	9	9
7	6	5	7	6	8	7	8	9
14	14	14	15	15	16	16	17	18

NOTE.—Be sure that the nine facts underscored are thoroughly learned. They give most trouble.

To Add Numbers They Must be of the Same Kind.

Give the pupil, say 6 buttons and 3 pins, and ask him what he has. Teach him to know that he can add only numbers of the same name.

Use many illustrations here, as, "3 cows and 4 sheep make how many ?"—what ? They cannot be added.

Teach also that the sum is always of the same name as the addends.

If the addends are dollars the sum will be dollars; if the addends are books the sum will be books. Think of other illustrations and use them.

DEVICES.

TALK:

Never tire the child by keeping him at work for a long time on the combinations given in the table. Get variety of ways for calling out the practice. Here are a few ways. You can think of others equally good.

---

1. How many days in 1 week and 6 days ?

2. If you sleep 8 hours to-night and 7 hours to-morrow night, how many hours will you sleep in both nights ?

3. One hen had 9 chickens and another hen had 7. How many chickens did both have ?

4. If you had 7 dollars and your brother had 9, how many dollars would you both have ?

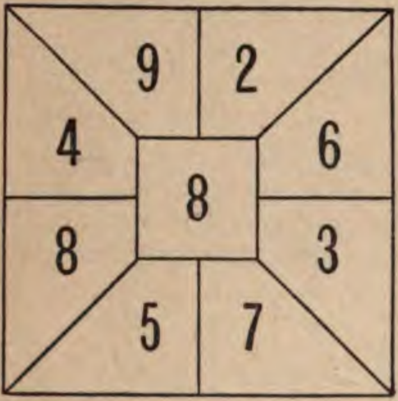
5. Sam ate 6 cherries in the forenoon, and 9 in the afternoon. How many did he eat that day ?

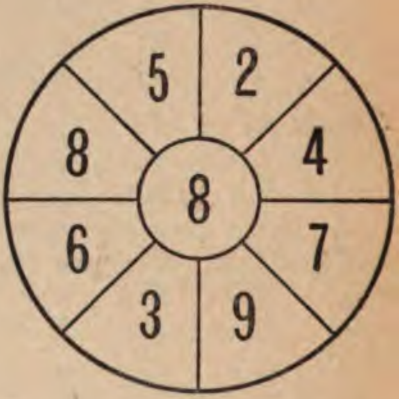
Counting out of Place in Addition.

Be careful to keep the habit of counting down. Perhaps you were allowed to count until you found the sum, but you can readily see that if you know that 9 and 7 are 16 just as you know two 2's are 4, you can add more quickly. So you must drill the children *to think the sum at once* when they see any of these combinations.

DRILLS.

Draw on a sheet of paper a large square and a circle. Divide them like these. Put, say the figure 8, in the center with the other numbers around it.





Write any figure where the 8 is in the center. Then drill, by pointing to the figures around it, while the child names at once the sum of the center figure and the one pointed at.

Use exercises like this frequently, and give many problems, using the combinations you find the child has difficulty with.

Exercises in which the child counts by 2's, by 3's, etc., to 100 are valuable.

ADDING NUMBERS OF MORE THAN ONE FIGURE.

TALK:

When the pupil comes to add numbers of more than one figure he will have no trouble, if his early number training has been properly done.

He will know that 13 is made up of 1 ten and 3 units, that 28 is made up of 2 tens and 8 units, etc.

Give him practice in writing numbers in columns, with the units' figures in one column, the tens' figures to the left in a column, etc. Then let him add examples like this:—

$$\begin{array}{r}32\\ \underline{44}\\ 76\end{array}$$

He adds, "4 and 2 are 6." Write the 6 under that column. "4 and 3 are 7." Write the 7 under that column.

He may say, "4 units and 2 units are 6 units. 4 tens and 3 tens are 7 tens."

After he has had a number like this, give him some where the sum of the units is more than ten, like this:—

$$\begin{array}{r}47\\ \underline{26}\end{array}$$

He adds, "6 and 7 are 13, which is 1 ten and 3 units. Write the 3 units under the units' column and add the 1 ten to the tens' column. Write the sum of the tens' column, 7, under it. The sum is 73."

CARRYING.

TALK:

Teach the pupil to remember the figure to be carried in these short additions rather than have him write it above or in the column to which it is to be added.

Some teachers allow the number to be written above, until the pupil comes to know where it is to go. This is only teaching something to be untaught later. Let him remember it from the start.

After some practice in adding two numbers give three, as here: —

$$\begin{array}{r}26\\14\\32\\\hline\end{array}$$

Then give four numbers. Then more. The arithmetics used in school give many of these.

Teach to add hundreds in the same way as tens and units.

LATER METHOD.

$$\begin{array}{r}7\\9\\6\\3\\4\\2\\\hline 31\end{array}$$

After some practice as above, when the child understands the work, lead him to do his work rapidly by omitting to name the numbers added each time. Teach him to add a column like the one given here in this way:

"Six, nine, fifteen, twenty-four, thirty-one." Let him from this time on do all of his work in addition that way. *Make this work rapid.*

Later he may be taught to see at once the sum of two groups of two figures each, as: —

$$\begin{array}{l}\left.\begin{array}{l}\left.\begin{array}{r}4\\5\end{array}\right\}9\\\left.\begin{array}{r}6\\3\end{array}\right\}9\end{array}\right\}18\\\hline 18\end{array}$$

When he adds horizontal columns teach him to begin on the left, as the eye is accustomed to travel from left to right. While in ordinary life this kind of addition is used but little, there are many positions that demand ability to do it quickly.

PROBLEMS.

TALK:

Give problems that lead the child to think. Make them easy at first, then gradually give those more difficult.

Here are a few that illustrate the kind you may *give when the child is adding numbers containing tens and units:—*

1. May has 31 days and April 30. How many days in both months?
2. Find the number of days in June, July, August, and September.
3. Find the sum of 25¢, 29¢, and 27¢.

WORK AND EXPLANATIONS.

Many of these problems can be solved orally by the pupil. When the work is written, it may be done in this way:—

(For the first problem given.)

31 days
30 days

61 days. After reading the problem let the pupil continue:—

"Both months have the sum of 31 days and 30 days which is 61 days."

Have the pupil explain as many problems in each of the operations as are necessary to show that he understands clearly what he does.

Where there are a great many numbers to be added the expression may be:—

"The sum of all these —— which is ——."

LARGE ADDITIONS.

The pupil who can take a set of numbers like those given on the next page and add them quickly and correctly, has accomplished what many pupils in our public schools never do.

Drill on many similar to those given.

It matters little whether the correct result is written below before the drill begins. If it is there the pupil knows as soon as he adds a column if he has done it correctly.

In real life the correct result is not found written below and he had best learn that way:—

342678	74286
172495	89324
546937	96989
295699	17640
187564	50963
443250	38296
987209	71724
643278	47898
895469	50465
956987	32985
734658	79685
6206224	650255

Note.—Give him some actual ledger pages to add.

RAPID ADDING EXERCISES.

When pupils have learned to add they quickly acquire skill by an occasional rapid adding exercise. A few minutes serves the purpose.

It may be made competitive by each pupil writing the numbers and at a given signal beginning the addition.

Slow and correct work is preferable to rapid work that is incorrect. Incorrect work is of no value here.

CIVIL SERVICE METHOD OF ADDING.

Employees under the Civil Service Bureau, bank clerks, and others who perform additions while surrounded by people engaged in conversation, use the following method in their additions.

This method is safe because one can detect and change an error very quickly.

$5280	16
9760.50	21
349	19
4006.75	23
6522.89	18
132.12	24
	$26,051.26

THE BANKER'S METHOD OF ADDING.

This method is the same as that used by the Civil Service employees, with one change. In adding the columns the number carried is "added in" to each partial sum.

$5280	16
9760.50	22
349	21
4006.75	25
6522.89	20
132.12	26
	$26,051.26

The Banker's Method is an improvement on the Civil Service Method as it does not require a second addition. The number to be carried is always in sight in the Banker's Method, being the tens' figure of the last partial sum written. The sum can be written at once.

PROVING ADDITION.

You may early teach how to prove an addition. There are a number of ways of doing this. Use the way the child's teacher uses. It is a good way.

One Way:

Add the columns down. If the result is the same as when the columns were added up, the result is correct.

Second Way:

Write the numbers in two groups. Find the sum of each group. Add the sums of the two groups. If the result is the same as when the numbers were added in one group, the result is correct.

Illustration of second way:—

$$\begin{array}{rcrr}
\left.\begin{array}{r}273\\456\end{array}\right\} & \text{———} & \begin{array}{r}273\\ \underline{456}\end{array} & \\
 & & 729 & 729\\
\left.\begin{array}{r}629\\ \underline{384}\end{array}\right\} & \text{———} & \begin{array}{r}629\\ \underline{384}\end{array} & \\
1742 & & 1013 & \underline{1013}\\
 & & & 1742
\end{array}$$

The Accountant's Proof.

This is a modification of the plan called *casting out the nines.*

ILLUSTRATION:

$$\begin{array}{l}
\left.\begin{array}{r}7926—24—6\\4356—18—9\\1824—15—6+\\9263—20—2\\ \underline{7856—26—8}\end{array}\right\}31—4\\
31225—13—4
\end{array}$$

EXPLANATION:

First add the columns in the regular way. Add each column horizontally, left to right. The results are 24, 18, 15, etc. Add these horizontally and you have 6, 9, 6, etc.

You have now reduced all to one column. Take the sum of this column. It is 31. This added horizontally is 4.

Add the sum 31225 horizontally until you have but one figure in it and you also have 4. This proves your answer correct.

ANOTHER ILLUSTRATION:

This illustrates where it is necessary to carry the horizontal additions a step further:—

$$\begin{array}{l} \left.\begin{array}{r} 986745-39-12-3\\ 726324-24-\ 6-6\\ 916348-31-\ 4-4\\ 992973-39-12-3 \end{array}\right\}+\ 16-7. \\ \hline 3622390-25-\ 7-7 \end{array}$$

Note.— With some practice it will be necessary to write the last line of figures only.

PROBLEMS.

TALK:

Throughout the work in addition many practical problems will be given. Pupils should need no help with these. If they do, aid them in understanding what to do. When the numbers are properly written no more aid should be given.

A few problems with a talk on each are given here.

PROBLEM:

In an encyclopedia there are four volumes. Vol. I has 638 pages, Vol. II has 617 pages, Vol. III has 596 pages, and Vol. IV has 626 pages. How many pages in the encyclopedia?

Talk on Above Problem:

If the child has been taught correctly to this point, he at once sees that this is a simple problem in addition.

If he has trouble and a few questions do not clear it up for him, show him what to do by making a practical illustration with a few of his own books.

Have him make other problems like this one. Let him use actual books. This leads him to realize that problems with big numbers are as real as those with small numbers. His early number story work comes to his aid now.

PROBLEM:

A farmer bought an outfit as follows: A plow for $9, a team of horses for $326, a harvester for $187, a thresher for $1229, and wagons for $279. What did all cost him?

WORK:

Have it written in this way:—

$ 9	cost	of	plow
326	"	"	horses
187	"	"	harvester
$1229	"	"	thresher
279	"	"	wagons
$2030	"	"	all.

NOTE.—If there is any difficulty teach as in the preceding problem.

PROBLEM:

In a city there were 965 children in the first ward, 786 in the second ward, and 1524 in the third ward. How many children in all?

WORK:

965	children	in	1st	ward
786	"	"	2d	"
1524	"	"	3d	"
3275	"	"	all.	

TALK:

Have them write the work for all problems as shown before. When they make problems themselves, let them find the answers in the same way.

When they understand the way to go about problems correctly they may write merely the actual work. Call for an explanation occasionally. Don't waste time on explanations, when they are so well known as to be monotonous to the child. When that is done no wonder he grows dull.

Let him make and solve many problems of his own.

PROBLEM:

Jones bought a farm for $3625. He built a house on it for $7500 and a stable that cost $1240. He paid $125 insurance on it and $85 for taxes. He wishes to sell it so as to gain $1700. What must he get?

WORK:

$3625	cost of farm
7500	cost of house
1240	cost of stable
125	insurance
85	taxes
1700	gain
$14,275	selling price.

Talk on Above:

Lead the pupil to see that he must first get back all of the money he has spent upon it, before there can be any gain. In addition to all that, he must get $1,700.

Use some familiar illustrations employing small numbers, if he has difficulty in seeing the reason of it.

PROBLEM:

Add twelve thousand forty-five, six hundred ninety-eight, twenty-four thousand eight hundred eighty-seven, sixty-three thousand four hundred forty-eight, nine hundred ninety, one thousand five hundred seventy-seven, sixteen hundred two, and twenty thousand nine.

WORK:

12045
698
24887
63448
990
1577
1602
20009
125,256

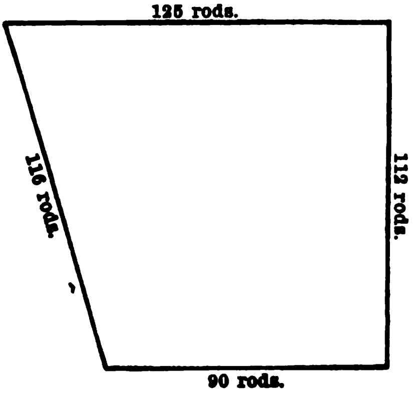
Talk on Above:

If there is any difficulty with such problems, it is because the pupil does not know how to read and write numbers. The only way to get rid of the difficulty is to review thoroughly this part of his work.

PROBLEM:

What is the distance around a piece of land that is 125 rods on the north, 112 rods on the east, 90 rods on the south, and 116 rods on the west?

WORK:



125 rods on N.
112 rods on E.
90 rods on S.
116 rods on W.
443 rods in all.

REMARK:—Illustrate such problems at first as shown here. Give others.

NEATNESS.

Great care should be taken that all written work be done neatly. The figures should be neat, as well as their arrangement.

The work in arithmetic furnishes an excellent place to cultivate neatness. Occasionally exhibit to the pupils a piece of work that you consider a model.

Sometimes young pupils do not have clear ideas of what really is neat work. In consequence their effort produces the opposite of what you want. Discover if the effort was earnest and, if so, suggest tactfully, what changes will improve it.

SUBTRACTION.

TALK:

The preliminary primary work that the pupil has done, fits him to go on with his work in subtraction easily and naturally. Remember his work here is carried on in connection with his work in the other operations. Not all of any one at once but a little of all all the time.

As he learned the forty-five facts of addition that he might become expert in addition, so now he must learn the combinations in subtraction to do the work well.

On the next page are the combinations on which he should be drilled until he knows them thoroughly.

Let him know what you want by illustrating it to him, if need be. After that teach him to name the difference of any combination on sight.

COMBINATIONS IN SUBTRACTION.

1	2	3	4	5	6	7	8	9	10	
− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	− 1	
2	3	4	5	6	7	8	9	10	11	
− 2	− 2	− 2	− 2	− 2	− 2	− 2	− 2	− 2	− 2	
3	4	5	6	7	8	9	10	11	12	
− 3	− 3	− 3	− 3	− 3	− 3	− 3	− 3	− 3	− 3	
4	5	6	7	8	9	10	11	12	13	
− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	− 4	
5	6	7	8	9	10	11	12	13	14	
− 5	− 5	− 5	− 5	− 5	− 5	− 5	− 5	− 5	− 5	
6	7	8	9	10	11	12	13	14	15	
− 6	− 6	− 6	− 6	− 6	− 6	− 6	− 6	− 6	− 6	
7	8	9	10	11	12	13	14	15	16	
− 7	− 7	− 7	− 7	− 7	− 7	− 7	− 7	− 7	− 7	
8	9	10	11	12	13	14	15	16	17	
− 8	− 8	− 8	− 8	− 8	− 8	− 8	− 8	− 8	− 8	
9	10	11	12	13	14	15	16	17	18	
− 9	− 9	− 9	− 9	− 9	− 9	− 9	− 9	− 9	− 9	
10	11	12	13	14	15	16	17	18	19	20
− 10	− 10	− 10	− 10	− 10	− 10	− 10	− 10	− 10	− 10	− 10

The differences of all these combinations should be learned so thoroughly that the pupil will be able to name the difference of any combination *on sight.* This will take time, but it is time well spent.

Use devices similar to those used in the drills on the addition table.

Give many problems for oral work. Let the pupil make many problems and give the difference in each case.

Illustrative Problems:

1. If I pay 4¢ for a lead pencil, how much change should I get from a dime?

2. John is 18 years old. His sister is 10. How many years older is John?

3. If you pay $16 for a suit and $8 for an overcoat, what is the difference in their cost?

4. We drove 7 miles on Saturday and 12 on Sunday. How many miles more did we drive on Sunday than on Saturday?

NOTE.—Teach the child to talk the first one in any of these ways:—

1. A dime is ten cents. I should get 10¢ less 4¢, which is 6¢.

2. I get the pencil and enough money to make in all 10¢ in value. Then I should get 6¢ in change.

3. I should get the difference between 10¢ and 4¢, which is 6¢.

When the pupil learns to talk these oral problems, do not waste time in continuing it. Let him say at once to such problems as the first:—

"I should get 6¢ change." That is all there is to it. The above sentence shows the right thought process in getting the result.

Remember:

Teach the pupil to think at once the figure which added to the lesser number gives the greater.

TERMS DEFINED.

TALK:

Be in no hurry to teach definitions here or elsewhere in arithmetic. Lead the pupil to see what is wanted and when he becomes familiar with it, let him tell you in his own words what he has done. If you can better it, do so by suggesting the better form.

Somewhere in the work in subtraction the teacher will teach these definitions; perhaps not as given here, but in a way equally good:

Subtraction is the process of taking one number from another.

It is evident to any pupil that he cannot take cows from horses, pens from pencils, etc.

The names of the numbers in a subtraction are:

The minuend, which is the greater of the two numbers.

The subtrahend, which is the smaller of the two numbers.

The remainder, or **difference,** which is the number left when the subtraction has been done.

ILLUSTRATIVE EXAMPLE:

$$\begin{array}{rl} 9 & \textit{minuend} \\ -\underline{7} & \textit{subtrahend} \\ 2 & \textit{difference} \text{ or } \textit{remainder}. \end{array}$$

WRITTEN SUBTRACTION OF NUMBERS GREATER THAN 10.

Remember:

Write the numbers so that the units' figures are in a column, the tens' figures in a column, etc.

EXAMPLE:

What is the difference between 79 and 32?

WORK:

$$\begin{array}{r} 79 \\ -32 \\ \hline 47 \end{array}$$

EXPLANATION:

Beginning with the units' figures, the difference between 9 and 2 is 7. Write the 7 under units. The difference between 7 tens and 3 tens is 4 tens. Write the 4 under tens.

OTHER EXAMPLES:

$$\begin{array}{rrrrr} 49 & 89 & 29 & 35 & 69 \\ -23 & -45 & -17 & -21 & -12 \\ \hline 26 & 44 & 12 & 14 & 57 \end{array}$$

$$\begin{array}{rrrr} 224 & 764 & 898 & 6495 \\ -112 & -341 & -146 & -1382 \\ \hline 112 & 423 & 752 & 5113 \end{array}$$

Give others to the pupil like the above, until he is familiar with them. As soon as he has explained a sufficient number to show that he understands the work, let him simply write the difference without talk.

Next Step.

The next step is to teach him to do what is commonly called "borrowing." It is best not to call it that. Talk it as shown here.

EXAMPLE:

What is the difference between 91 and 28?

WORK:

$$\begin{array}{r} 91 \\ -28 \\ \hline 63 \end{array}$$

EXPLANATION:

Since we cannot take 8 units from 1 unit, we take one of the 9 tens and change it to units. This is 10 units. 10 units added to 1 unit are 11 units. Now we can take 8 units from 11 units and the remainder is 3. Write the 3 under units.

Since we changed 1 ten of the 9 tens to units, there is but 8 tens left. 2 tens from 8 tens leaves 6 tens. Write the 6 under tens.

As soon as the child gets familiar with the way the work is done, omit the talk and let him simply write results.

In the way indicated before, explain what to do when there are three or more figures in the minuend and the subtrahend, and where the difficulty occurs in the tens' place, or any other than units'. He simply takes one of the next higher order and changes it to the order he is considering, then proceeds as in the one explained before.

EXAMPLE:

Find the difference between 729 and 342.

WORK:

$$\begin{array}{r} 729 \\ -342 \\ \hline 387 \end{array}$$

EXPLANATION:

When you get to the tens' place you cannot take 4 tens from 2 tens. Take one of the 7 hundreds and change it to tens. This gives 10 tens, which added to 2 tens makes 12 tens. Now 4 tens from 12 tens leaves 8 tens, and 3 hundreds from 6 hundreds leaves 3 hundreds.

OTHER EXAMPLES:

$$\begin{array}{r} 634 \\ -251 \\ \hline 383 \end{array} \qquad \begin{array}{r} 2438 \\ -1245 \\ \hline 1193 \end{array} \qquad \begin{array}{r} 3729 \\ -1816 \\ \hline 1913 \end{array} \qquad \begin{array}{r} 4524 \\ -1613 \\ \hline 2911 \end{array}$$

Let the child go over these and explain how each figure of the remainders was obtained. Then let him try a few where the remainders are not written. Be careful to present no new difficulty here.

A 0 in the Minuend.

What is the difference between 90 and 34?

WORK:

90
−34
56

EXPLANATION:

You cannot take $\overset{4}{5}$ units from 0 units. Take one of the 9 tens and change it to units. This makes 10 units. 4 units from 10 units leaves 6 units. Write the 6. 3 tens from 8 tens leaves 5 tens. Write the 5.

Additional Examples:

80	30	405	5406	3042
−25	−14	−132	−1234	−1422
55	16	273	4172	1620

Two or More 0's in the Minuend.

Find the difference between 4009 and 1327.

WORK:

$\overset{(10)}{4009}$
−1327
2682

EXPLANATION:

When he gets to the tens' place there are 0 tens from which he can take 2. Neither are there any hundreds from which he may take 1 hundred and change it to tens. So he crosses to the 4 thousands and takes 1 thousand. He changes this to 10 hundreds. (Indicated in the parenthesis above.) Now he takes 1 of these 10 hundreds and changes it to tens. Then 2 tens from 10 tens leaves 8 tens. 3 hundreds from 9 hundreds leaves 6 hundreds and 1 thousand from 3 thousands leaves 2 thousands.

Additional Examples:

5000	7005	3005	3007
−3240	−1204	−1624	−1529
1760	5801	1381	1478

TALK:

Some teachers allow pupils to indicate at first the unit of a higher order changed to the one below, by writing it in parentheses as shown here:—

$$\begin{array}{r} \overset{(10)(10)}{4002} \\ -1329 \\ \hline \end{array}$$

There is no objection to this plan until the pupil is familiar with the process. Then discard it.

Teach to pay no attention to 0's that may occur at the left of the subtraction.

THE COMPUTER'S METHOD.

TALK:

In making change, the method below is nearly always used. Computers find it the more practical way. It gives greater rapidity and accuracy than the other method.

Many of the best teachers use it alone in teaching subtraction.

It is based on the principle that the sum of the remainder and subtrahend equals the minuend.

This plan is called "The Austrian Method" by some.

EXAMPLE:

Find the difference between 725 and 432.

WORK:

$$\begin{array}{r} 725 \\ \hline 432 \\ 293 \\ \hline \end{array}$$

EXPLANATION:

Let the pupil imagine he is adding with the sum written above. He inserts the missing figures to form the sum as he works, 3, 9, 2. He thinks of 2 and *3* as 5; 3 and *9* as 12; 5 and *2* as 7. He may talk it this way: 3 and *2*, 5; 9 and *3*, 12; 3 and *4*, 7.

ANOTHER EXAMPLE:

$$\begin{array}{r} 73{,}426 \\ -\,14{,}315 \\ \hline 59{,}111 \end{array}$$

EXPLANATION:

5 and *1* are 6; 1 and *1* are 2; 3 and *1* are 4; 4 and *9* are 13; 2 and *5* are 7.

Use that method with these examples:—

$$\begin{array}{rrrr} 42{,}936 & 2034 & 7825 & 6243 \\ -17{,}245 & -1456 & -1504 & -1428 \\ \hline 25{,}691 & 578 & 6321 & 4815 \end{array}$$

NOTE.—With practice the pupil will come to prefer this method and should be helped in its use. It is practical since conducive to both *rapidity* and *accuracy*.

PROOFS OF SUBTRACTION.

To prove a subtraction *add* the *subtrahend* to the *remainder*. The result will be the *minuend*, if the work is correct.

EXAMPLE:	**PROOF:**
425 *minuend*	173
−173 *subtrahend*	252
252 *remainder.*	425

425 is the same as the minuend, hence the work is correct.

ANOTHER PROOF:

Take the *remainder* from the *minuend*. If the work is correct the subtrahend will be left.

If in the last example given we write

$$\begin{array}{r} 425 \\ -252 \\ \hline \end{array}$$

we get 173 which is the subtrahend used above, hence the work is correct.

NOTE.—Either proof may be used. Show the child why these two proofs are true. Use small numbers to do this like **7-2.** Later use larger ones.

TALK:

Again impress the importance of accuracy. A wrong result in arithmetic is worth nothing in actual life.

Remember that above arithmetic and above everything else the object of yourself and the teacher is to make the child intelligent.

PROBLEMS.

PROBLEM:

The Missouri river is 4506 miles long and the Amazon is 3994 miles. How many miles longer is the Missouri than the Amazon ?

WORK:

$$\begin{array}{rl} \mathbf{4506} & \text{miles, length of Mo.} \\ -\,\mathbf{3994} & \text{miles, length of Amazon} \\ \hline \mathbf{512} & \text{miles difference.} \end{array}$$

SOLUTION:

The Missouri river is as much longer than the Amazon as the difference in their lengths, which is 512 miles.

Talk on Above:

If the pupil has trouble in knowing what to do with such a problem, it is because he does not make use of his previous teaching. Lead him to see what is wanted and how to go about it, by using a familiar illustration. Take two sticks, one 16 inches in length and the other 30 inches. Have him find the difference in length.

Give other illustrations, if needed. Follow this with other problems similar to the one above.

PROBLEM:

The greatest depth of water ever measured is 29,400 feet. The highest mountain peak is Mount

Everest, 29,062 feet. How much greater is the distance to the bottom of the sea than to the top of Mount Everest?

WORK:

$$\begin{array}{rl} 29{,}400 & \text{feet} \\ -\,29{,}062 & \text{feet} \\ \hline 338 & \text{feet.} \end{array}$$

EXPLANATION:

The distance to the bottom of the sea is as much greater than the distance to the top of Mount Everest as the difference in the distances, which is 338 feet.

Talk on Above:

In such problems explain to the pupil that the measurement used begins at sea level.

One great trouble in the solution of problems is that pupils do not understand the conditions.

PROBLEM:

From Buffalo to Boston by rail is 499 miles. From Buffalo to Chicago by rail is 539 miles. How much farther is it from Buffalo to Chicago than from Buffalo to Boston?

WORK:

$$\begin{array}{rl} 539 & \text{miles} \\ -\,499 & \text{miles} \\ \hline 40 & \text{miles.} \end{array}$$

SOLUTION:

The distance from Chicago to Buffalo is as much farther than that from Buffalo to Boston, as the difference between 539 miles and 499 miles, which is 40 miles.

NOTE.—If the pupil does not readily see what to do with such problems, illustrate the principle by the use of short distances that are familiar to him.

PROBLEM:

I had $4295 in the bank and drew out $1764. How much remained in the bank?

WORK:

$4295
-1764
2531

SOLUTION:

I had left in the bank the difference between $4295 and $1764, which is $2531.

NOTE.—Few pupils will have trouble with this problem. If there be any, use an illustration of a few cents in place of the numbers above.

PROBLEM:

In 1900 the population of Illinois was 4,821,550 and that of Texas was 3,048,710. How many more people were there in Illinois than in Texas?

WORK:

4,821,550
-3,048,710
1,772,840

SOLUTION:

There were as many more people in Illinois as the difference between 4,821,550 and 3,048,710, which is 1,772,840.

NOTE.—If there is any trouble here, illustrate by taking the number of pupils in two rooms of your school, the number of people in two well-known families, etc.

PROBLEMS INVOLVING ADDITION AND SUBTRACTION.

PROBLEM:

Gardner went into business with $7000. The first year he earned $925. His expenses were, $325 for rent and $732 for living. What was he worth at the end of the year?

WORK:

$7000
925
$7925
-1057
$6868

$ 732
325
$1057

NOTE.—Lead the pupil to see that at the end of the year, had he spent nothing, he would have $7000 + $925.

Next lead him to find what the total expense was; $325 + $732 = $1057. It is an easy matter for him to see that what Gardner is now worth is the difference between $7925 and $1057, which is $6868.

PROBLEM:

A bin holds 925 bushels. On Sept. 20, I put 425 bu. of wheat into it and on Sept. 23, 325 bushels. How much more will it hold?

EXPLANATION:

1st Query. How many bu. does the bin hold?
925.
2d Query. How many bu. are in it now?
425 + 325 = 750.
3d Query. How much more will it hold?
925 bu. − 750 bu. = 175 bu.

NOTE.—This illustrates how by a series of queries the pupil may be led to see the successive steps required in the solution of a problem.

PROBLEM:

I bought a horse for $170 and a carriage for $125. I sold both for $300. Did I gain or lose, and how much?

SOLUTION:

I paid for the horse	**$170**
I paid for the carriage	**$125**
I paid for both	**$295**

I sold both for $300, so I gained $5.

PROBLEM:

I paid $2500 for a lot, built a house on it for $3750, and paid $755 for grading the lot. I sold all for $8000. Did I gain or lose, and how much?

SOLUTION:

The lot cost	**$2500**
The house cost	**3750**
The grading cost	**755**
All cost	**$7005**

I gained the difference between the selling price $8000 and the cost $7005=$995.

EXERCISES IN ADDITION AND SUBTRACTION.

The following tables will furnish material for much drill work in addition and subtraction. A few of the ways in which they may be used are suggested: —

1. Find the areas of the different divisions.

2. Compare the different divisions.

3. Compare your state with other well-known states. Find how much larger or smaller it is than any other state you may select.

TABLES FOR PROBLEMS IN ADDITION AND SUBTRACTION:

North Atlantic States.

Name.	**Area in sq. miles.**
Maine	29,895
New Hampshire	9,005
Vermont	9,135
Massachusetts	8,040
Rhode Island	1,058
Connecticut	4,845
New York	47,620
New Jersey	7,455
Pennsylvania	44,985
Total	

South Atlantic States.

Name.	Area in sq. miles.
Delaware	1,960
Maryland	9,860
Virginia	40,125
W. Virginia	24,645
N. Carolina	48,580
S. Carolina	30,170
Georgia	58,980
Florida	54,240
Total	

Southern Central States.

Name.	Area in sq. miles.
Kentucky	40,000
Tennessee	41,750
Alabama	51,540
Mississippi	46,340
Louisiana	45,420
Texas	262,290
Indian Territory	31,000
Oklahoma	38,830
Arkansas	53,045
Total	

Before attempting any of this work give the child an idea of how large a square mile is, if possible. If he lives in the city, use as many blocks in the vicinity of his home as will make a square mile, approximately. In the country some well-known farm, or farms, that approximates a square mile in area will do.

Northern Central States.

Name.	**Area in sq. miles.**
Ohio	40,760
Indiana	35,910
Illinois	56,000
Michigan	57,430
Wisconsin	54,440
Minnesota	79,205
Iowa	55,475
Missouri	68,735
N. Dakota	70,195
S. Dakota	76,850
Nebraska	76,840
Kansas	81,700
Total	

Western States and Territories.

Name.	**Area in sq. miles.**
Montana	39,159
Wyoming	20,789
Colorado	194,327
N. Mexico	119,565
Arizona	40,440
Utah	143,963
Nevada	109,740
Idaho	84,290
Washington	66,880
Oregon	94,560
California	155,980
Alaska	531,410
Total	

MULTIPLICATION.

Multiplication is a short process of finding the sum when the addends are equal.

When this sum has been found in the short way it is called the **product.**

The number representing the addends is called the **multiplicand.**

The number showing how many addends is called the **multiplier.**

TALK:

Some teachers may find fault with these definitions. Yet the above seems to be a logical, and easily understood, sequence of the child's previous work. He comes to look at it this way naturally. He is not frightened by the bugaboo that he has come to a place where he is to begin a new and difficult part of arithmetic.

Pay no attention to definitions until they are needed. Let him enter on the work here outlined in as informal a manner as possible.

If the early number work has been done properly, the pupil will be pleased with this way of finding the sum when the numbers are alike, and will not think it difficult.

Give many problems with small numbers until he is thoroughly familiar with the idea.

A thorough knowledge of the multiplication tables is absolutely necessary. They must be memorized to do good work in arithmetic, from now on.

Watch for the parts of the tables that the children have trouble with and emphasize those parts by extra drill.

Vary these drills. Don't let them get monotonous; but drill! drill! drill!

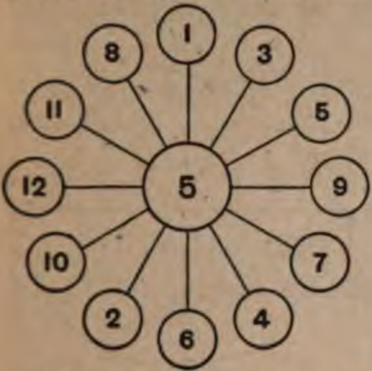
The suggestions given here as to drill work are but a few of what may be used.

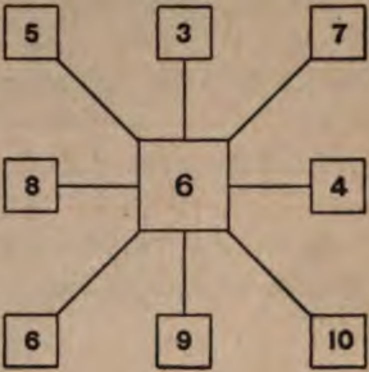
DRILLS.

Draw a diagram like this one and let the pupil fill in the squares with the proper products. Fix a time limit. Have results put in quickly.

1	2	3	4	5	6	7	8	9
2								
3								
4						28		
5								
6								
7					42			
8								
9								
10								
11			44					
12								108

The drill below is very good and effective, if a rapid answering exercise is made of it. Products should be given at once.





To vary it use squares.

4 X 1 = 4
4 X 2 = 8
4 X 3 = 12

Make diagrams like these, and let the pupils fill them in.

In this work it is not intended that the pupil should at first learn "his tables" in order, the 2's first, then the 3's, then the 4's, etc. He remembers many of the facts of multiplication from his primary work in number. These drills are to complete and fix firmly in the pupil's mind the completed multiplication tables.

7 X 1 = 7
7 X 2 = 14
7 X 3 = 21

The arithmetics are full of devices that may be used with profit in this connection. The teacher's own ingenuity will suggest others.

Let the pupils suggest some of these devices.

To Show That Multiplication Is Only a Short Way of Adding.

Examine some examples like this one:—

$$225 \times 4.$$

This means that we are to find what number is equal to 225 taken four times. By addition we have:

$$\begin{array}{r} 225 \\ 225 \\ 225 \\ 225 \\ \hline 900 \end{array}$$

By the short way we write it in this form:—

$$\begin{array}{r} 225 \\ \times 4 \\ \hline 900 \end{array}$$

The pupil at once sees that this means much less writing, for if he were to try to find what number equals 225 taken one hundred times, he would first have to write 225 one hundred times in a column and then add. That would take too long.

Explain the Short Way to Him as Below:

$$\begin{array}{r} 225 \\ \times 4 \\ \hline 900 \end{array}$$

4 times 5 ones = 20 ones; 20 ones = 2 tens and no ones; write 0 to show no ones.

4 times 2 tens = 8 tens; 8 tens and the 2 tens from the 20 ones = 10 tens; 10 tens = 1 hundred and 0 tens; write 0 to show no tens.

4 times 2 hundreds = 8 hundreds; 8 hundreds and 1 hundred = 9 hundreds.

Now we have written 900 which is the same number we had from adding the long way.

From this the pupil sees that *multiplying is merely repeating a number a certain number of times, and finding the sum.*

TALK:

Have the pupil find the product in several cases like the one given. Ask him to find the sum of 225, 143, 346, and 236 in the short way. He sees that it cannot be done.

Remember:

1. Use the short way of adding only when the numbers to be added are alike.

2. Use the long way when the numbers are not alike.

Teach him that instead of calling the short way *addition* we call it *multiplication.*

He now sees wherein they are alike, and where different, and will have little trouble.

Ask him which way he would use, and what he would call it in each of these cases, and why:—

1. Find the sum of 124, 24, 163, and 75.
2. Find the sum of 75, 75, 75, and 75.

Take others. Let him make some and tell which way he would use.

When it becomes necessary to teach technical terms have the pupil—

Remember:

The number repeated is the Multiplicand.

The number that shows how many times the multiplicand is repeated is the Multiplier.

The number that shows the result of the multiplication is the Product.

ILLUSTRATION OF THE NAMES USED:

7 multiplicand
×5 multiplier
35 product.

Let the pupil write examples and name the parts until he is familiar with them.

NO CARRYING NECESSARY.

Give many examples and problems where no "carrying" is necessary.

LONG WAY:

42 multiplicand
×2 multiplier
2×2 units 4
2×4 tens 80
8 tens + 4 units = 84 product.

SHORT WAY:

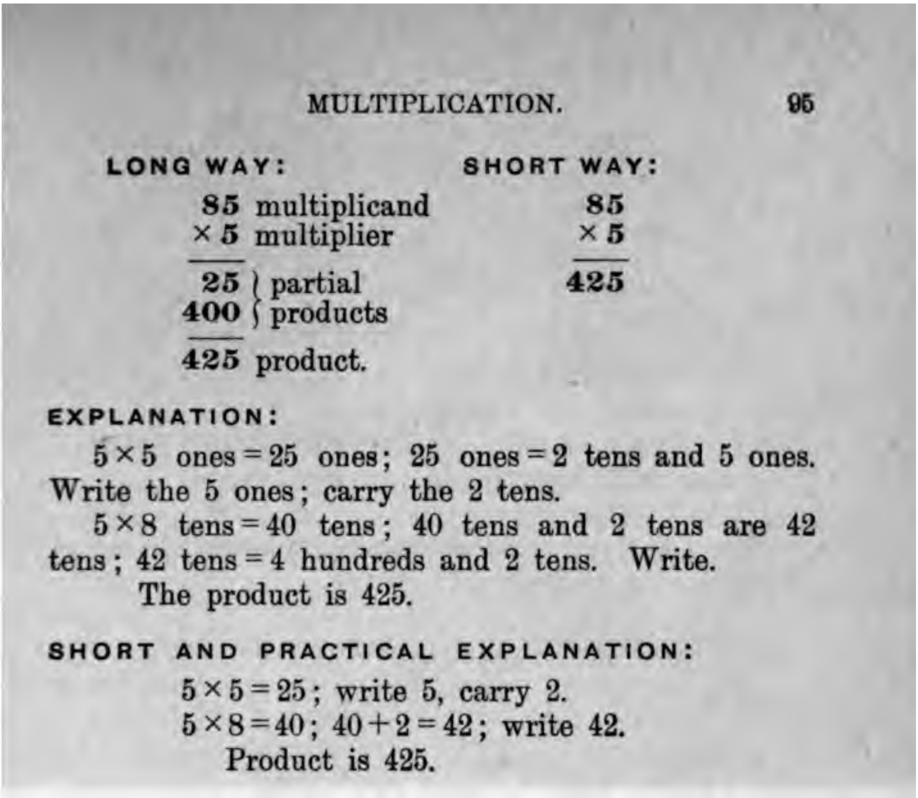
42 multiplicand
×2 multiplier
84 product.

EXPLANATION OF SHORT WAY:

2×2 units = 4 units. Write the 4.
2×4 tens = 8 tens. Write the 8.
Product 84.

CARRYING NECESSARY.

He has become familiar with the process where no "carrying" is necessary. Now he is ready for examples and problems in which it is necessary to "carry."



MULTIPLICATION. 95

LONG WAY:	SHORT WAY:
85 multiplicand	85
× 5 multiplier	× 5
25 } partial	425
400 } products	
425 product.	

EXPLANATION:

5 × 5 ones = 25 ones; 25 ones = 2 tens and 5 ones. Write the 5 ones; carry the 2 tens.

5 × 8 tens = 40 tens; 40 tens and 2 tens are 42 tens; 42 tens = 4 hundreds and 2 tens. Write.

The product is 425.

SHORT AND PRACTICAL EXPLANATION:

5 × 5 = 25; write 5, carry 2.
5 × 8 = 40; 40 + 2 = 42; write 42.
Product is 425.

WHERE THE MULTIPLIER IS GREATER THAN TEN.

LONG WAY:	SHORT WAY:
42	42
× 23	× 23
3 × 42 = 126	126
20 × 42 = 840	84
23 × 42 = 966	966

Remember:

1. **Always place the right-hand figure of each partial product under the figure you multiply by.**
2. **Add the partial products for the complete product.**

EXAMPLE:

612 × 402 = ?

LONG WAY:

$$\begin{array}{r} 612\\ \times 402\\ \hline 1224\\ 000\\ 2448\\ \hline 246{,}024 \end{array}$$

SHORT WAY:

$$\begin{array}{r} 612\\ \times 402\\ \hline 1224\\ 2448\\ \hline 246{,}024 \end{array}$$

EXPLANATION OF SHORT WAY:

It is not necessary to multiply by the 0. Keep in mind that the first figure of each partial product should be written directly *under the figure multiplied by.*

That saves the work of writing the row of 0's.

TO MULTIPLY BY ANY NUMBER.

EXAMPLE:

$$27 \times 20 = ?$$

LONG WAY:

$$\begin{array}{r} 27\\ \times 20\\ \hline 00\\ 54\\ \hline 540 \end{array}$$

SHORT WAY:

$$\begin{array}{r} 27\\ \times 20\\ \hline 540 \end{array} \quad \text{or} \quad \begin{array}{r} 27\\ \times 20\\ \hline 540 \end{array}$$

NOTE.—Do not have the pupil use the long way at all. It is put here merely for explanation.

Merely write the 0 and multiply by 2. Do not multiply through by 0.

LONG WAY:

$$\begin{array}{r} 625\\ \times 200\\ \hline 000\\ 000\\ 1250\\ \hline 125{,}000 \end{array}$$

SHORT WAY:

$$\begin{array}{r} 625\\ \times 200\\ \hline 125{,}000 \end{array} \quad \text{or} \quad \begin{array}{r} 625\\ \times 200\\ \hline 125{,}000 \end{array}$$

LONG WAY:

500
× 125
2500
1000
500
62,500

SHORT WAY:

125
× 500
62,500

Teach the pupil now that he may write the numbers without reference as to which is the true multiplicand or multiplier. This may shorten his work.

In the preceding example the work is made simple and short by reversing the relation, as shown by the short way.

Use many illustrations to show that this makes no difference in the product.

It makes the work practical and, after a pupil understands the reason, practical work is what counts.

PROOFS IN MULTIPLICATION.

THE MULTIPLICATION:

42
× 25
210
84
1050

123
× 475
615
861
492
58,425

THE PROOF:

25
× 42
50
100
1050

475
× 123
1425
950
475
58,425

The proof consists merely in reversing the positions of the multiplicand and multiplier, and performing the multiplication in the usual way. If the product is the same in both cases the work is correct.

PROBLEMS IN MULTIPLICATION.

TALK:

The child's early work in numbers has taught him how to find the cost of a number of things when he has the cost of one.

So in his work with measures, he has learned to find the number of small units in any number of larger units by multiplying, though perhaps he did it without calling it multiplication.

Many facts similar to these have been made familiar to him in his work with small numbers. When he comes to deal with problems involving large numbers he may be confused. At least some children will be. All do not grasp ideas equally well at the same stage of the work. Then, too, pupils are not equally bright every day.

Simple problems should be given the pupil throughout his work on any of the operations. Large numbers may be used as he gets familiar with their use in examples.

For pupils who have difficulty with problems in multiplication, the following pages are included. The simple, oral ones at first, with solutions, may be valuable for the slower pupils. All may be benefited by the solutions to the written problems given here.

Impress that where the cost of one of anything is given, to find the cost of many of the same, multiply. Explain why, frequently or until all understand.

NOTE.— In this book the term EXAMPLE embraces any work merely indicated, as, $\frac{42 \times 25}{4+2+6}$ =, etc. When a concrete question for computation is asked it is termed a PROBLEM.

ORAL PROBLEMS.

PROBLEM:

Yesterday I bought a pencil for 3¢. To-day I want 5 more. What will they cost me?

SOLUTIONS:

1. Three pencils will cost me 3 times 5¢ = 15¢.
2. They will cost me 3 × 5¢ = 15¢.

NOTE.—Either one is good.

PROBLEM:

I buy a book for 30¢. What will three more of the same kind cost me?

SOLUTION:

They will cost me 3 × 30¢ = 90¢.

WORK:

$$\begin{array}{r} 30\text{¢} \\ \times 3 \\ \hline 90\text{¢} \end{array}$$

PROBLEM:

In one week there are 7 days. How many days in 4 weeks?

SOLUTION:

In 4 weeks there are 4 × 7 days = 28 days.

PROBLEM:

How many days in 9 weeks?

SOLUTION:

There are 9 × 7 days = 63 days.

PROBLEM:

I have 7 marbles. John has 5 times as many. How many has John?

SOLUTION:

He has 5 × 7 marbles = 35 marbles.

PROBLEM:

What is the cost of 20 sheep at $6 each?

SOLUTION:

They cost 20 × $6 = $120.

WORK:

$$\begin{array}{r} \$6 \\ \times\ 20 \\ \hline \$120 \end{array}$$

WRITTEN PROBLEMS.

PROBLEM:

Find the cost of 85 chickens @ 40¢.

SOLUTION:

They cost 85 × 40¢ = $34.

WORK:

$$\begin{array}{r} \$.40 \\ \times\ 85 \\ \hline \$34.00 \end{array} \qquad \begin{array}{r} 85 \\ \times\ \$.40 \\ \hline \$34.00 \end{array}$$

PROBLEM:

What is the cost of 45 A. of land @ $35?

SOLUTION:

45 acres will cost 45 × $35 = $1575.

WORK:

```
  $35
 × 45
 ----
  175
 140
 ----
$1575
```

PROBLEM:

What will Jones pay for 640 A. of land @ $34?

SOLUTION:

He will pay 640 × $34 = $21,760.

WORK:

```
    $34
   × 640
 -------
   1360
  204
 -------
$21,760
```

PROBLEM:

What is the yield from 187 A., if the average yield is 17 bushels?

NOTE.—Such problems as this often give difficulty because the child does not understand the meaning. Be careful to explain all such.

SOLUTION:

The yield will be 187 × 17 bu. = 3179 bu.

WORK:

```
  187
 × 17 bu.
 ----
 1309
 187
 ----
 3179 bu.
```

PROBLEM:

The area of Belgium is 11,373 square miles. It has 551 people to the square mile. How many people in Belgium?

SOLUTION:

There are 11,373 × 551 people = 6,266,523 people.

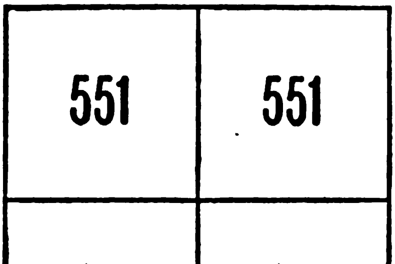
WORK:

$$\begin{array}{r} 11{,}373 \\ \times\, 551 \\ \hline 11373 \\ 56865 \\ 56865 \\ \hline 6{,}266{,}523 \end{array}$$

NOTE.—If there is trouble here lead up to an understanding of it as before. How many people to 2 square miles? 3 square miles?

It may be illustrated to the pupils in this way:—

Let each square represent a square mile. Find how many people on 4 square miles, etc.



Devices like this at this time should not be necessary. If they are, use them.

Explain how the result in such a problem is only approximately the population of Belgium.

PROBLEM:

From Albany to Buffalo is 297 miles. It takes 158 tons of steel rails to lay one mile of track. How many tons of rails will it take to lay a track from one city to the other?

SOLUTION:

It will take 297 times 158 tons = 46,926 tons.

WORK:

```
   158 tons
  ×297
  ----
  1106
 1422
 316
 -----
 46,926 tons
```

Note.— If there is trouble with problems like this ask such questions as, "How many tons will it take to lay 2 miles of tracks? 3 miles?" In a moment the pupil will see.

TALK:

These few problems merely suggest the plan to be followed. Don't tell the pupil when you can lead him to see for himself.

To feel that he has made a discovery by the use of a few judicious questions gives him more confidence than if you tell him every word he is to say.

It helps his mind training. That is the essential thing.

DIVISION.

TALK:

The child begins division naturally and easily. He sees that it is only the inverse of multiplication. He knows that 2 times 4 makes 8, and then 8 divided by 2 equals 4, comes as a matter of course.

Do not enter into the work of division formally, as then the child looks for something new and therefore difficult, when it is not at all difficult if he understands multiplication.

Division is more troublesome than either addition or subtraction. The reason for this is because there are three distinct uses of division. These uses are explained here. There is nothing gained in confusing the pupil with them if he does not need them in his school work.

Partition is a term that need not be used with the *child at all. Teach the way to find the equal parts of a number and say no more about it.*

The teacher who teaches partition as partition to the pupil has missed something in pedagogical training.

THE THREE USES OF DIVISION.

FIRST USE EXPLAINED: (Partition.)

What is $\frac{1}{4}$ of 24 cents?

Here 24 cents is the sum of money to be divided. It is to be divided by 4. The result shows the size of each part:—

Number of equal parts **4)24¢** money to be divided
6¢ size of each part.

SECOND USE EXPLAINED: (Division by Measurement.)

How many times is 8 cents contained in 32 cents?

Here 32 cents is the sum of money to be divided. The size of each part is 8 cents, and the result gives the number of equal parts.

Size of each part **8¢)32¢** money to be divided
4 number of equal parts.

THIRD USE EXPLAINED: (Ratio Idea.)

There are 36 acres in one lot and 9 acres in another. The first lot is how many times as large as the second?

From division we know that 36 acres contains 9 acres 4 times. Then 36 acres is 4 times as large as 9 acres; or, 36 acres is to 9 acres as 4 acres is to 1 acre.

NOTE.—This on the ratio idea leaves the impression that the other work does not embrace the ratio principle. Any comparison of numbers embraces the ratio idea. More is said of this under the head of *Ratio and Proportion.*

FACTS ABOUT DIVISION.

TALK:

The child will realize the facts given below early in his work in division.

Don't confuse him by loading him with all this at once. He must get it gradually.

In General:

The number divided is the dividend.

The number we divide by is the divisor.

The result of dividing is the quotient.

The number sometimes left over after dividing is the remainder.

If you wish to give exact definitions for dividend, divisor, and quotient, as used in each of the three uses of division, allow those made by the pupil.

The pupil may notice the problem and work given under the explanation of the *Second Use of Division.* Here the —

Divisor is the number that shows the size of each part.

Dividend is the number to be divided.

Quotient is the number that shows the number of equal parts.

You will see that the above definitions are more definite than the general ones given before. Therefore, they may be more satisfactory to you.

You might have the child define the terms used in the third kind of division (Ratio) in the same manner. This will be later, when definitions become valuable to him.

Things to Notice:

I. When the quotient shows the number of times, or number of equal parts, it has no name.

II. When the divisor has no name, the quotient and dividend have the same name.

III. When the divisor and dividend have the same name, the quotient has no name.

IV. The remainder is always like the dividend.

NOTE.—Many problems should be given to bring out the above facts. Proceed slowly. Review frequently.

PROBLEM:

I have 15 apples to distribute among 5 boys. How many shall I give each boy?

WORK:

Number of equal parts **5)15** apples to be divided
3 apples. Size of each part, or what each boy gets.

SOLUTION:

If 15 apples are to be divided among 5 boys, each boy will get 1 fifth of 15 apples, or 3 apples.

PROBLEM:

At $4 each, how many hats can I buy for $40?

WORK:

Size of each part **$4)$40** to be divided
10 number of equal parts, or number of hats.

SOLUTION:

I can buy as many hats as $4 is contained times in $40, which is 10. I can buy 10 hats for $40.

Ways of Expressing Division:

(1) $36 \div 9$ is a very common way and is usually read, "36 divided by 9."

(2) $\frac{36}{9}$ is another common method and is read, "36 divided by 9," "The ratio of 36 to 9," or "36 ninths."

(3) $36 : 9$ is the ratio method of expression and is read, "The ratio of 36 to 9," or "36 is to 9."

(4) $\frac{1}{9}$ of 36 is usually read, "One-ninth of 36." This is the fractional method of expressing division.

(5) The sign $\underline{)\quad\quad}$ is used to indicate division.

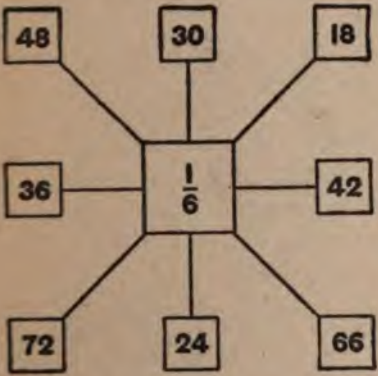
Thus $\begin{array}{r} 5\underline{)10} \\ 2 \end{array}$ means the same as $10 \div 5 = 2$, or $\frac{10}{5} = 2$.

DIVISION DRILLS.

TALK:

It is well to use drills of different kinds to help fix the facts of the division table in mind. At no time, at this stage, are you sure that the pupil remembers from one day to the next.

Diagram drills are given here as suggestive of what may be made use of. Use other numbers in order to get drill on the whole division table.



Be sure to make the drill work brisk and of short duration.

THE DRILL:

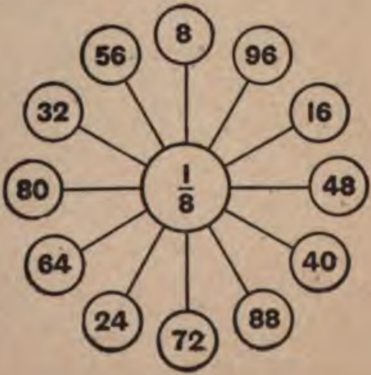
The teacher points to any of the numbers in the small squares and the child answers simply the quotient number. Skip about rapidly. Draw the same diagram on the blackboard and frequently change the fraction in the center square.

THE DRILL:

Allow no "thinking out" of results. The child should have the table at his tongue's end.

Change center fraction repeatedly.

The supreme test of his mental activity is to think quickly and tell correctly.



For the Pupil to Do:

1. Count by 5's from 100 to 25.
2. Count by 7's from 63 to 49.
3. Count by 8's from 16 to 72.
4. Count by 2's from 36 to 2.
5. $6 \times 4 = ?$ $7 \times 8 = ?$ $5 \times \$4 = ?$ $3 \times 8 = ?$
6. $8 \times 9 = ?$ $6 \times \$4 = ?$ $9 \times 3 = ?$ $9 \times 7 = ?$
7. Make a problem in division with a remainder. Write it and show that the remainder is like the dividend.
8. Make a problem showing the divisor without a name.
9. Make one showing the quotient without a name.
10. *Tell the quotients quickly:*

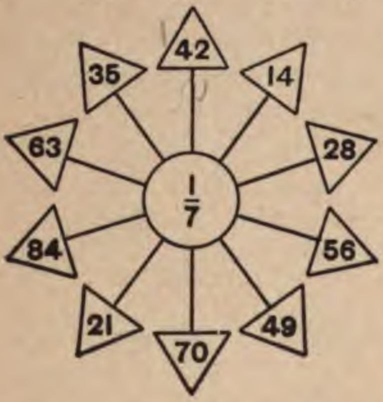
$$\frac{63}{7}, \frac{96}{8}, \frac{70}{7}, \frac{35}{5}, \frac{81}{9}, \frac{96}{12}, \frac{72}{8}.$$

$45 \div 5$
$64 \div 8$
$18 \div 2$
$100 \div 10$
$108 \div 9$

$132 \div 12$
$81 \div 9$
$77 \div 7$
$24 \div 3$
$24 \div 8$

$22 \div 11$
$120 \div 10$
$144 \div 12$
$72 \div 9$
$120 \div 10$

$63 \div 7$
$84 \div 12$
$120 \div 12$
$121 \div 11$
$15 \div 5$



THE DRILL.

Use this drill in the same manner as those shown before. It is given a place in order to illustrate another design that the teacher may use.

The usual method of using such drills may be varied by pointing to any number in the small triangles, asking the pupil to name the quotients for all the numbers involved in the drill, moving either to the right or left. Change the center number each time.

8 ÷ 8 = 1
16 ÷ 8 = 2
24 ÷ 8 = 3

To the Teacher:

Draw oblongs like these and have the pupil fill in with the division table for any number you may suggest.

After the work has been written in this way by the pupil, have him make other oblongs showing these same facts in the fractional form. Thus:—

$\frac{1}{8}$ of 8 = 1,
$\frac{1}{8}$ of 16 = 2,
$\frac{1}{8}$ of 24 = 3, etc.

The idea is to repeat the division table over and over again, but such repetitions should be of a pleasing nature to the child.

12 ÷ 12 = 1
24 ÷ 12 = 2
36 ÷ 12 = 3

SHORT DIVISION.

When the divisor is less than 12, the division is done mentally, and the process is called short division.

In short division the divisor should be written to the left of the dividend and the quotient below the dividend.

EXAMPLE:

Divide 84 by 7.

WORK:

$$\begin{array}{r|l} 7\,) & 84 \\ \hline & 12 \text{ quotient.} \end{array}$$

SOLUTION:

(1) 8 tens divided by 7 is 1, and 1 remaining. Write the 1 below the 8.

(2) The remainder 1 is tens, 1 ten or 10 units + 4 units = 14 units.

(3) 14 ÷ 7 = 2. Write the 2 below.

(4) Quotient = 12.

SHORT SOLUTION:

(To be used when the process is understood.)

(1) 8 divided by 7 is 1. Write the 1. One over.

(2) 14 divided by 7 is 2. Quotient 12.

What to Do with a Remainder:

Divide 185 by 6.

WORK:

$6\overline{)185}$

30 quotient, **$30\frac{5}{6}$** exact quotient.
5 remainder.

SOLUTION:

(1) 1 is smaller than 6, so we will use 18.

(2) 18 ÷ 6 = 3. Write the 3 below the 8.

(3) 5 is smaller than 6, so 6 is not contained in it. To show this, write 0 below the 5.

(4) The remainder is 5. Quotient is 30.

(5) Exact quotient is $30\frac{5}{6}$. Write the remainder over the divisor. The exact quotient is read, "Thirty and five-sixths."

SHORT WORK AND SOLUTION:

(To be used only after the process is understood.)

WORK:

$6\overline{)185}$

$30\frac{5}{6}$ quotient.

SOLUTION:

(1) 6 in 18, 3 times. Write the 3 below the 8.

(2) 6 in 5, 0 times. Write the 0.

(3) Quotient $30\frac{5}{6}$.

LONG DIVISION.

TALK:

Long division is nothing more or less than short division, only the products are written, instead of carried in the head.

Easy examples should be given at the beginning and then more difficult ones as the child overcomes the various difficulties.

A very sensible way is to give a series of problems embracing one kind of difficulty. Continue these problems until that difficulty is conquered. If you begin with any kind of problem, you may have several things to explain at the outset. Therefore, systematize and work out each difficulty with plenty of exercises before problems involving any other difficulty are used.

PARTIAL DIVIDENDS.

Perhaps the pupil will need some help in seeing *how much* of the dividend he should use in each division.

Some of the best instructors use the following to lead up to an understanding of this:—

1. Begin long division by using 25 as the divisor.

Ask questions like these—How many times is 25 contained in 100? in 99? in 97? in 85? in 79? in 105? in 122? in 200? in 195? in 187? in 205? in 220?

Change and divide by 24; by 23; by 26; by 27, etc.

The idea is to have the child think of 24, 23, 26, 27, etc., as being 25 (about).

2. Divide using 50 as the divisor.

Change and divide by 49; by 42; by 60; by 70, etc.

Have the child think of 49, 42, 60, 70, etc., as 50.

3. Proceed using 100 in a similar manner.

Practice Examples for Beginners in Long Division.

1. 625 ÷ 25 =?
2. 1275 ÷ 25 =?
3. 7225 ÷ 25 =?
4. 625 ÷ 23 =?
5. 1275 ÷ 26 =?
6. 7225 ÷ 20 =?

---

1. 6250 ÷ 50=?
2. 14750 ÷ 50=?

6250 ÷ 60 =?
14750 ÷ 45 =?

THE THREE DIFFICULTIES IN LONG DIVISION.

1. When the partial dividend contains one more figure than the divisor.

2. When the trial quotient figure is not the true quotient figure.

3. When the quotient contains 0's.

How to Overcome the First Difficulty.

A great number of examples like the following should be given and this first difficulty thoroughly overcome, before examples embracing the next difficulty are taken up at all.

Divide 345 by 23.

WORK:

$$\begin{array}{r} 1 \\ 23\overline{)345} \\ 23 \\ \hline 115 \end{array}$$

Here the partial dividend, 115, contains one more figure than the divisor 23.

EXPLANATION:

There is difficulty in telling exactly how many times 23 is contained in 115.

HOW TO DO IT:

Take the left-hand figure of the divisor and find how many times it is contained in the first two figures on the left of the partial dividend, 11.

The 2 is contained in 11 about five times. Then we say that 23 is contained in 115 five times. Multiply out and it is found to be correct. *This is the way to find the trial quotient figure.*

Hence, to Find the Trial Quotient Figure:

Divide the first or the first two figures on the left of the partial dividend by the left-hand figure of the divisor, and so continue.

To the Teacher:

In all problems given under the first difficulty, the trial quotient figure should be the true quotient figure.

How to Overcome the Second Difficulty:

Divide 342 by 26.

WORK:

$$\begin{array}{r} 1^{4} \\ 26\overline{)342} \\ 26 \\ \hline 82 \\ {\scriptstyle 104} \end{array}$$

Here the trial quotient figure, 4, is not the true quotient figure.

EXPLANATION:

2 is contained in 8 four times and naturally the pupil would write 4 as the quotient figure. But after multiplying, 4 is found to be too large, which means that the trial quotient figure is not the true quotient figure.

THE TRIAL QUOTIENT FIGURE:

1. When the remainder is smaller than the divisor, the trial quotient figure is the true quotient figure.

2. If the remainder is larger than the divisor, the trial quotient figure is *too small.*

3. If it is impossible to subtract, because the number to be subtracted is too large, the trial quotient figure is *too large.*

```
     13
26)342
   26
   --
    82
    78
    --
     4
```

Here the remainder, 4, is smaller than the divisor 26, so the trial quotient figure 3 is the true quotient figure.

Here the remainder, 30, is larger than the divisor 26, so the trial quotient figure, 2, is too small.

```
     12
26)342
   26
   --
    82
    52
    --
    30
```

```
     14
26)342
   26
   --
    82
   104
```

Here the number to be subtracted, 104, is too large. The trial quotient figure is too large.

If the right-hand figure of the divisor is large, the trial quotient figure should be estimated smaller than it apparently should be at sight.

Have the child show that in the above example if the divisor were 29, instead of 26, the true quotient figure would be less than 3; also, if the divisor be 20, instead of 26, the true quotient figure would be 4. Have the pupils explain and give reasons.

How to Overcome the Third Difficulty.

Divide 4896 by 48.

WORK:

```
    102
48)4896
   48
   --
     96
     96
     --
```

Here the quotient, 102, contains a 0.

EXPLANATION:

Everything proceeds all right until after the first subtraction. There is no remainder. Bring down the 9 but the child sees that 48 is not contained in it even once. Tell the child to write a 0 in the quotient to show that the divisor is contained no times, and *then bring down an additional figure.*

If, after bringing down the additional figure, the divisor still remains larger than the partial dividend, write another 0 in the quotient and bring down another figure.

Writing the Quotient:

The quotient in long division may be written either at the right or above the dividend.

Both methods have merit, therefore both ways are shown in the following example:—

EXAMPLE:

Divide 2372 by 14.

First Method.

$$\begin{array}{rll} & \mathbf{169\tfrac{6}{14}} & \\ \mathbf{14)} & \overline{\mathbf{2372}} & \\ & \underline{\mathbf{14}} & \\ & \mathbf{97} & \\ & \underline{\mathbf{84}} & \\ & \mathbf{132} & \text{quotient} = 169\tfrac{3}{7}. \\ & \underline{\mathbf{126}} & \\ & \mathbf{6} & \text{remainder.} \end{array}$$

Second Method.

$$\begin{array}{rll} \mathbf{14)} & \mathbf{2372(169\tfrac{6}{14}} & \\ & \underline{\mathbf{14}} & \\ & \mathbf{97} & \\ & \underline{\mathbf{84}} & \\ & \mathbf{132} & \text{quotient} = 169\tfrac{3}{7}. \\ & \underline{\mathbf{126}} & \\ & \mathbf{6} & \text{remainder.} \end{array}$$

FULL EXPLANATION:

14 is contained in 23 hundreds 1 (hundred) time and 9 hundreds remaining. Write 1 in the hundreds' place in the quotient.

9 hundreds or 90 tens + 7 tens = 97 tens. 14 is contained in 97 tens 6 (tens) times, and 13 tens remaining. Write the 6 in the tens' place in the quotient.

13 tens or 130 units + 2 units = 132 units. 14 is contained in 132 units 9 (units) times and 6 units remaining. Write the 9 in the units' place.

Write the remainder 6 over the divisor 14 to show that it is still to be divided. Reduce $\frac{6}{14}$ to lowest terms, $\frac{3}{7}$. Quotient $169\frac{3}{7}$.

SHORT EXPLANATION:

(To be used when process is understood.)

23 ÷ 14 = 1 and a remainder.
1 × 14 = 14. 23 − 14 = 9.

Bring down the 7.

97 ÷ 14 = 6. 6 × 14 = 84. 97 − 84 = 13.

Bring down the 2.

132 ÷ 14 = 9. 9 × 14 = 126. 132 − 126 = 6.

Quotient 169, remainder 6.
Complete quotient $169\frac{3}{7}$.

One place where the old way of writing the quotient to the right of the dividend seems advisable is in cases like the following, where it saves the rewriting of numbers:—

EXAMPLE:

47 × 64 ÷ 23 = ?

WORK:

```
       47
       64
      ───
      188
     282
    ──────
 23)3008(130 18/23
     23
      ──
      70
      69
      ──
      18
```

When the Divisor Ends in One or More O's.

The child should see that in dividing a number by ten, he is simply finding the number of tens in that number.

365 ÷ 10 = ?

Here the pupil will see at a glance that 365 has 36 tens and 5 units.

To divide by ten,

Strike off the units as a remainder.

Divide 523 by 100.

We see that there are 5 hundreds in 523, with 2 tens and 3 units as a remainder. Thus,

To divide by one hundred,

Strike off the tens and units as a remainder.

If we wish to divide by 200, we strike off the tens and units, and divide by 2; by 300, strike off the tens and units, and divide by 3, etc.

EXAMPLE:

Divide 4752 by 300.

WORK:

$$\begin{array}{r} 15\frac{252}{300} \text{ quotient.} \\ 3\cancel{00})\overline{47\cancel{52}}\phantom{\text{ quotient.}} \\ 252 \text{ remainder.} \end{array}$$

Note.—The remainder is the number remaining after the division, *plus the part struck off.*

EXAMPLE:

Divide 239,608 by 18,000.

WORK:

$$\begin{array}{r} 13\frac{5608}{18000} \text{ quotient.} \\ 18\cancel{000})\overline{239\cancel{608}}\phantom{\text{ quotient.}} \\ 18\phantom{\text{ quotient.}} \\ \hline 59\phantom{\text{ quotient.}} \\ 54\phantom{\text{ quotient.}} \\ \hline 5608 \text{ remainder.} \end{array}$$

Remember:

When the divisor ends in one or more 0's:—

1. Cut off the 0's on the right of the divisor.

2. Cut off the same number of figures on the right of the dividend.

3. Divide what remains of the dividend by what remains of the divisor.

PROOFS IN DIVISION.

THE BEST WAY:

The best way to prove work in division is to multiply the quotient by the divisor and add the remainder, if there is one.

The result will equal the dividend if the work is correct.

EXAMPLE:

Divide 600,181 by 937.

WORK:

$$
\begin{array}{rr}
 & 640\frac{501}{937} \\ \cline{2-2}
937) & 600181 \\
 & 5622 \\ \cline{2-2}
 & 3798 \\
 & 3748 \\ \cline{2-2}
 & 501
\end{array}
$$

PROOF:

$$
\begin{array}{r}
640 \\
937 \\ \hline
4480 \\
1920 \\
5760 \\ \hline
599680 \\
501 \\ \hline
600181
\end{array}
$$

PROBLEMS IN DIVISION.

TALK:

The early work has taught the child to know that when the cost of a number of articles is given, to find the cost of one, he should divide.

He has learned in his measure work that to reduce a number of one denomination to a higher denomination, he should divide.

He has learned many other important facts that he may apply to problems with large numbers. He has had problem work in connection with learning the operation of division, and now that he understands the operation in all its phases, his practice should include practical problems that develop reasoning power.

The succeeding problems are merely suggestive. Any text-book on arithmetic will supply many more for *practice and solution. The solutions given here are also merely suggestive.*

In the explanations of problems do not make the child a phonograph. If he understands one way of talking a problem better than another, let him use his own, so long as it is logical and concise.

Oral Problems.

PROBLEM:

A grocer charged me 60¢ for a dozen cocoanuts. What was the cost of each?

SOLUTION:

One Way.—The cost of one was $\frac{1}{12}$ of 60¢, or 5¢.

Another Way.—One cost as many cents as 12 is contained times in 60, which is 5.

NOTE.—The first solution is shorter and more logical.

PROBLEM:

I earn $3 a day. How long will it take to earn $24?

SOLUTION:

1. It will take as many days as 3 is contained times in 24, or 8.
2. It will take as many days as the ratio of $24 to $3, which is 8.

NOTE.—Either of the above is satisfactory.

PROBLEM:

I wish to cut a piece of twine 64 inches long into 8 equal pieces. How long will each piece be?

SOLUTION:

Each piece will be $\frac{1}{8}$ of 64 inches, or 8 inches.

WORK:

$$\begin{array}{r} 8\overline{)64}\text{ in.} \\ \overline{8}\text{ in.} \end{array}$$

To the Teacher:

When the child is dealing with numbers so small that he can make the computations without writing the work, have him do so.

After he understands the *work* for each kind of problem, let him give the oral solution.

PROBLEM: (Written or oral.)

A lot contains 440 square rods. One side is 22 rods long. What is the length of the other side?

WORK:

$$\begin{array}{r} 22\underline{)440} \\ 20 \end{array}$$

SOLUTION:

The other side is as many rods long as 22 is contained times in 440, or 20.

NOTE.—Such work should be written in the short division form. Here it is not necessary with young children to explain the philosophy of omitting to name the dividend and divisor in the work and solution.

When the pupil is older it can be done without a waste of time. Find the explanation under *Surface Measures.*

Written Problems.

PROBLEM:

My farm of 120 acres yields 2340 bushels of oats. What is the average yield per acre?

SOLUTION:

First Way.—The average yield is as many bushels as 120 is contained times in 2340, or 19½ bushels.

Second Way.—The average yield is $\frac{1}{120}$ of 2340 bushels, or 19½ bushels.

WORK:

$$\begin{array}{r} 19\tfrac{1}{2} \\ 12\cancel{0}\overline{)234\cancel{0}} \\ \underline{12} \\ 114 \\ \underline{108} \\ \tfrac{6}{12} = \tfrac{1}{2} \end{array}$$

PROBLEM:

There are 5280 feet in a mile. (Remember.) Mount Everest, the highest peak in the world, is 29,062 feet high. How many miles high is it?

SOLUTION:

It is as many miles high as 5280 feet is contained times in 29,062 feet, which is 5, and 2662 feet remaining, or nearly 5½ miles.

WORK:

$$\begin{array}{r} 5 \\ 528\cancel{0})\overline{2906\cancel{2}} \\ 2640 \\ \hline 2662 \end{array}$$

PROBLEM:

A farmer sold his oats for 32¢ a bushel and received $639.36. If he had 80 A. in his oat field, what was the average yield per acre?

SOLUTION:

He sold as many bu. as 32¢ is contained times in $639.36, or 1998.

The average yield was as many bushels as 80 is contained times in 1998, or 25 (nearly).

WORK:

$$\begin{array}{r} 1998 \\ .32)\overline{639.36} \\ 32 \\ \hline 319 \\ 288 \\ \hline 313 \\ 288 \\ \hline 256 \\ 256 \\ \hline \end{array}$$

$$\begin{array}{r} 24\tfrac{39}{40}. \\ 80)\overline{1998} \\ 160 \\ \hline 398 \\ 320 \\ \hline \tfrac{78}{80} = \tfrac{39}{40}. \end{array}$$

PROBLEM:

In North Dakota a rancher bought 2400 A. of land for $64,800. What was the price per acre?

SOLUTION:

The price per acre was as many dollars as 2400 is contained times in 64,800, or 27.

WORK:

$$\begin{array}{r} 27 \\ 24\cancel{00})\overline{648\cancel{00}} \\ 48 \\ \hline 168 \\ 168 \\ \hline \end{array}$$

To the Teacher:

In problems like the following, where two operations in division are involved, it may be necessary to help out with questions. The first thing to find is the number of bushels. That found, the rest of the problem is simple.

Help the pupil to see that the problem is just two simple problems in division combined. Let him make and work others like it.

PROBLEM:

In one day in 1902 there were 32,670 bushels of wheat received at Chicago. The average car holds about 330 bu. of wheat, and about 40 wheat cars make a train. How many cars were needed to bring the wheat to Chicago? How many trains?

SOLUTION:

It took as many cars as 330 is contained times in 32,670, or 99.

It took as many trains as 40 is contained times in 99, or 2, and 19 cars on a third train.

WORK:

$$\begin{array}{r} 99 \\ 33\cancel{0})\overline{3267\cancel{0}} \\ 297 \\ \hline 297 \\ 297 \\ \hline \end{array} \qquad \begin{array}{r} 40)\underline{99} \\ 2\frac{19}{40} \end{array}$$

PROBLEM:

When the last census was taken (1900), Massachusetts had a population of 2,805,346. The area of that state is 8040 sq. miles. How many people had it to the square mile?

WORK:

$348\frac{7426}{8040}$

$804\cancel{0})\overline{280534\cancel{6}}$

2412

3933

3216

7174

6432

7426

SOLUTION:

It had as many people to the square mile as 8040 is contained times in 2,805,346, or $348\frac{7426}{8040}$.

PROBLEM:

How many bushels in 2792 quarts?

WORK:

$87\frac{1}{4}$

$32)\overline{2792}$

256

232

224

$\frac{8}{32}=\frac{1}{4}$

SOLUTION:

There are as many bushels as 32 is contained times in 2792, or $87\frac{1}{4}$.

PROBLEM:

How many square feet in 324,619 square inches?

WORK:

$2254\frac{43}{144}$

$144)\overline{324619}$

288

366

288

781

720

619

576

$\frac{43}{144}$

SOLUTION:

There are as many square feet as 144 is contained times in 324,619, or $2254\frac{43}{144}$.

PROBLEM:

The area of Lake Superior is 31,200 square miles. The area of Connecticut is 4990 square miles. Lake Superior is how many times as large as Connecticut?

SOLUTION:

It is as many times as large as 4990 is contained times in 31,200, or $6\frac{126}{499}$.

WORK:

$$\begin{array}{r} 6\frac{126}{499} \\ 499\cancel{0}\,)\overline{3120\cancel{0}} \\ 2994 \\ \hline 126 \\ \hline 499 \end{array}$$

MISCELLANEOUS PROBLEMS.

Involving the Fundamental Operations.

PROBLEM:

Two trains travel at the rate of 25 miles and 30 miles an hour, respectively. If I make a trip of 900 miles, how many hours do I save by taking the fast train?

SOLUTION:

The fast train travels 900 miles in as many hours as 30 is contained times in 900, or 30.

The slow train travels 900 miles in as many hours as 25 is contained times in 900, or 36.

I save the difference between 36 hours and 30 hours, or 6 hours.

INDICATED WORK:

$$\begin{aligned} 900 \div 30 &= 30 \\ 900 \div 25 &= 36 \\ 36 - 30 &= 6. \end{aligned}$$

PROBLEM:

A merchant has a stock of goods that cost him \$5000. He wishes to sell it at a gain of \$10 on each \$100 of cost. What will his whole gain be?

SOLUTION:

In \$5000 there are as many times \$100 as 100 is contained in 5000, or 50.

If on \$100 he gains \$10, on 50 times \$100 he gains 50 × \$10, or \$500.

INDICATED WORK:

$$\$5000 \div \$100 = 50$$
$$50 \times \$10 = \$500.$$

PROBLEM:

Four men have between them \$8640. The first has \$2600, the second has \$1865, the third has \$2749. How much has the fourth?

SOLUTION:

The first three have the sum of \$2600, \$1865, and \$2749, or \$7214. The fourth has the difference between \$8640 and \$7214, or \$1426.

WORK:

$$\begin{array}{r} \$2600 \\ 1865 \\ 2749 \\ \hline \$7214 \end{array} \qquad \begin{array}{r} \$8640 \\ 7214 \\ \hline \$1426 \end{array}$$

PROBLEM:

If 264 be taken from the difference of two numbers, 186 remains. If the smaller number is 296, what is the larger?

SOLUTION:

The difference of the two numbers is 264 + 186 = 450. Since the smaller number is 296 and the difference is 450, the larger number is 296 + 450, or 746.

WORK:

$$264 + 186 = 450$$
$$450 + 296 = 746.$$

NOTE.—When the pupil understands the *why* in such a problem, the work may be done in the shortest way,—add the three numbers.

PROBLEM:

I sell my house and lot for $14,200. I get in exchange a farm worth $6000, cash $3750, and take a note for the rest. What is the face of the note?

NOTE.—By the *face* of a note, the sum for which it is drawn is meant.

SOLUTION:

The sum of the cash and farm value is $6000 + $3750, or $9750.

The note must be drawn for the difference between $14,200 and $9750, or $4450.

PROBLEM:

Two ocean steamers are 2600 miles apart. They travel toward each other, one at the rate of 300 miles a day, the other at the rate of 325 miles. How many miles will they be apart in 4 days?

SOLUTION:

In one day they travel toward each other 300 miles + 325 miles, or 625 miles.

In 4 days they travel 4 × 625 miles, or 2500 miles. They are then apart the difference between 2600 miles and 2500 miles, or 100 miles.

NOTE.—Work is indicated in solution.

PROBLEM:

Light travels at the rate of 186,000 miles a second. The sun is about 92,000,000 miles away. How long does it take the light to reach us?

SOLUTION:

It takes the light as many seconds as 186,000 is contained times in 92,000,000, which is $494\frac{58}{93}$ seconds, or 8 min. $14\frac{58}{93}$ sec.

PROBLEM:

The longer sides of a rectangular field are 80 rods each. The shorter are 40 rods each. What is the distance around the field?

SOLUTION:

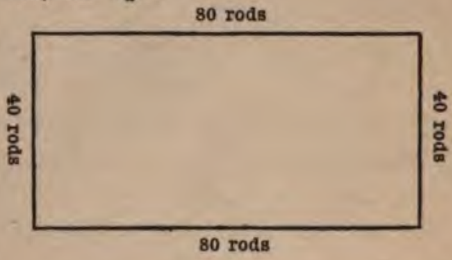
There are 2 sides of 80 rods each = 160 rods.

There are 2 sides of 40 rods each = 80 rods.

The length of the four sides = 160 rods + 80 rods = 240 rods.

1 mile = 160 rods. 240 rods = as many miles as 160 is contained times in 240, or $1\frac{1}{2}$.

A diagram like this will make such problems clear to dull pupils. Bright pupils need no diagram.



PROBLEM:

I bought 80 A. of land at $35 an acre. Two years later I sold it all for $3200. What was my gain per acre?

SOLUTION:

The selling price of one acre equals as many dollars as 80 is contained times in 3200, or 40.

I gain on each acre the difference between the cost and selling price, or $40–$35, which equals $5.

To the Teacher:

Many of these problems admit of other forms of solution equally good. If a pupil works one of them himself, give him credit. These are merely suggestive.

Don't stifle originality.

PROBLEM:

If 40 men can build a fence in 30 days, how many days will it take 100 men to build the same fence?

SOLUTION:

$$\frac{40}{100} = \frac{2}{5}$$

It will not take 100 men as long as 40 men. It will take only $\frac{2}{5}$ as long.

$\frac{2}{5}$ of 30 days = 12 days.

Hence, it will take 100 men 12 days to build the same fence.

PROBLEM:

Mr. Brown was mixing 4 grades of coffee while I was in his store this morning. He had some worth 31 cents per pound, some 19 cents, some 40 cents, some 42 cents. He used equal quantities of each kind. He wants to make a profit of 6 cents on each pound of the mixture. How will he sell it per pound?

SOLUTION:

A 4-lb. mixture is worth $31 + 19 + 40 + 42$, or 132 cents. One pound of the mixture is worth $\frac{1}{4}$ of 132 cents, or 33 cents. To make a profit of 6 cents on a pound, he must sell it for $33 + 6$, or 39 cents.

SHORT REVIEW:

1. What is the difference between addition and multiplication?
2. What is an addend? a sum?
3. Give two ways of proving an addition.
4. Name the difficulties you have found in long division.
5. The remainder always has what name?
6. Define what you mean by divisor, dividend, quotient, remainder, short division, long division.
7. How can you tell when a quotient figure is too large? too small? right?

PART THREE.

FACTORING—LEAST COMMON MULTIPLE — GREATEST COMMON DIVISOR— CANCELLATION.

"What is really wanted is to light up the spirit that is within a boy. In some sense and in some effectual degree, there is in every boy the material of good work in the world; in every boy, not only in those who are brilliant, not only in those who are quick, but in those who are solid, and even in those who are dull."

— *Gladstone.*

DEFINITIONS.

THE pupil should be made to feel that he has accepted nothing without a clear and demonstrative reason, to the entire exclusion of authority, tradition, prejudice or self-interest. — BAIN.

IN education the process of self-development should be encouraged to the uttermost. Children should be led to make their own investigations, and to draw their own inferences. They should be told as little as possible, and induced to discover as much as possible. . . . Any piece of knowledge which the pupil has himself acquired, any problem which he has himself solved, becomes by virtue of the conquest much more thoroughly his than it could else be. — SPENCER, "*Education.*"

FACTORING.

TALK:

Pupils should learn to factor numbers readily if they are to become expert in the operations of arithmetic. This knowledge is very essential in finding the Greatest Common Divisor and Least Common Multiple; also in fractions, percentage and its applications, and in the work in ratio and proportion.

Give the children a thorough knowledge of factoring and they will perform many of the problems they are given without the use of pencil.

Have them memorize the factors of the numbers to one hundred that they meet most frequently.

It should be part of their stock in trade.

The **factors of a number** are the numbers which, when multiplied together, equal that number.

The factors of 9 are 3 and 3, because 3 × 3 = 9.

A **factor of a number,** then, is a divisor of the number. 2 is a factor of 10, since it is a divisor of 10. Because we can divide any number by itself and 1 or unity, we seldom mention them as factors.

Factoring a number is separating it into its factors.

Some numbers, like 3, 5, 11, etc., can only be divided by themselves and unity. As they cannot otherwise be factored, we call them **prime numbers.**

Those numbers that can be factored, like 6, 10, 18, 22, etc., are called **composite numbers.** They are called so from the fact that they are thought of as composed of other numbers multiplied together.

An **exact divisor** of a number is one that is contained in it an exact number of times, with no remainder.

3 is an exact divisor of 18, since 3 is contained in 18 exactly six times.

An **even number** is one that is exactly divisible by 2.

4, 6, 8, 10, 12, etc., are even numbers. All numbers that end in 2, 4, 6, 8, or 0 are even.

An **odd number** is one that is not exactly divisible by 2, as 1, 3, 5, 7, 9, etc.

All numbers that end in 1, 3, 5, 7, or 9 are odd, and that means that they cannot be divided by 2 or any multiple of 2.

Teach the child to always recognize, at a glance, an even or odd number. Ask him how he knows.

Any number that is a prime number is a **prime factor.**

5 and 2 are factors of 10 and because they are prime numbers they are also prime factors.

Likewise, factors that are composite numbers are **composite factors.**

6 and 4 are composite factors of 24, and they are composite numbers.

PRIME FACTORS.

$$24 = 2 \times 2 \times 2 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$
$$28 = 2 \times 2 \times 7$$

The prime factors of 24 are three *twos* and one *three;* of 36, two *twos* and two *threes;* and of 28, two *twos* and one *seven.*

It is wrong to say that the prime factors of 24 are 2 times 2 times 2 times 3, as many do. Accuracy in statement means much, and carelessness is never more out of place than in arithmetic.

$$\left.\begin{array}{l}40 = 4 \times 10\\ 40 = 5 \times 8\\ 40 = 2 \times 4 \times 5\end{array}\right\}$$

These are factors of 40, but not all of them are prime factors.

$$40 = 2 \times 2 \times 2 \times 5\}$$

40 can have only one set of prime factors.

Show the child that while many numbers have two or more sets of factors, *there is only one set of prime factors.*

$$\begin{array}{l}24 = 2 \times 2 \times 2 \times 3\\ 27 = 3 \times 3 \times 3\\ 25 = 5 \times 5\end{array}$$

In factoring numbers many times we find the same factor in two or more of them. The factor 3 is found in 24, also in 27. Then 3 is a **common factor** of 24 and 27.

When we find no factor common to two or more numbers, we say the factors are **prime to each other.** The factors of 27 and 25 are prime to each other.

Are the factors of 25 and 24 prime to each other?

FACTORING SMALL NUMBERS.

Small numbers are factored by inspection.

The child should be drilled until he can name a prime factor of any small number that may be named.

Give him the number 14. He sees at a glance that a prime factor of 14 is 2.

Give him the number 135. He does not see without drill that 5 is a prime factor, also 3.

The thing to do is to drill on the principles given here, that the child may quickly tell if a prime factor is 2, 3, 5, 7, etc.

How to Tell One Prime Factor of a Number:

1. One factor is 2, if the number ends in 0, 2, 4, 6, 8.
2. One factor is 3, if 3 is a factor of the sum of the digits.
3. One factor is 5, if the number ends in 0 or 5.
4. One factor is 7, if the difference between the sum of the odd periods and the sum of the even periods is divisible by 7.

NOTE.—It requires about as much time to apply this last test as it does to divide by 7, so it is seldom used. The others are practical.

FACTORING LARGE NUMBERS.

There are two ways of factoring large numbers, both of them being used.

FIRST WAY.

RULE:

Find the composite factors first and then the prime factors of the composite ones.

WORK:

$$\begin{array}{c} 240 = 12 \quad \times \quad 20 \\ \| \qquad\qquad \| \\ \overbrace{2\times2\times3}\times\overbrace{2\times2\times5} \end{array}$$

$240 = 2\times2\times2\times2\times3\times5.$

SECOND WAY.

RULE:

Divide the given number by prime numbers.

WORK:

$$\begin{array}{r} 2\underline{)240} \\ 2\underline{)120} \\ 2\underline{)60} \\ 2\underline{)30} \\ 3\underline{)15} \\ 5 \end{array}$$

$240 = 2\times2\times2\times2\times3\times5.$

Things for the Child to Do:

1. Name the six factors of 24.
2. Is there any prime number larger than 2 that is not an odd number?
3. Find all the prime numbers from 3 to 63.
4. Name all the composite numbers from 28 to 60.

Prime Numbers from 1 to 1000.

[Use This Table for Reference.]

PRIME NUMBER TABLE.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	

GREATEST COMMON DIVISOR.

PRESENT IN THIS WAY:

What number will exactly divide 12 and 18? (*Child*) 2.

2 is then a divisor of 12 and of 18. It is a *common divisor* of 12 and 18. We also say it is a common factor of 12 and 18.

Is 2 the only common divisor of 12 and 18? (*Child*) No, 3 and 6 are also common divisors of 12 and 18.

What is the Greatest Common Divisor of 12 and 18? (*Child*) 6.

Who can tell what is meant by the Greatest Common Divisor of two or more numbers?

Name two common divisors of 10 and 20.

What is the Greatest Common Divisor of 28 and 64?

The **Greatest Common Divisor (G. C. D.)** of two or more numbers is the largest number contained in each of them a whole number of times.

NOTE.—Greatest Common Measure and Greatest Common Factor mean the same as Greatest Common Divisor and are used in some schools.

ILLUSTRATION:

Find the G. C. D. of 12, 18, 36.

The divisors of 12 are 2, 4, 6.

The divisors of 18 are 2, 3, 6, 9.

The divisors of 36 are 2, 3, 4, 6, 9, 12, 18.

2 is a common divisor of 12, 18, and 36, because it is found in each of the numbers as a factor; but *it is not the Greatest Common Divisor.*

6 is the G. C. D. of 12, 18, 36.

Review:

1. Review the definitions of prime and composite factors.
2. Drill the child to recognize prime factors of numbers.
3. Have him name all the prime divisors of 42; of 50; of 30; of 36.
4. Have him name all the divisors of the above numbers.
5. Name all their factors.
6. What is the difference between a factor and a divisor of a number?
7. Can there be more than one G. C. D. of a set of numbers?

Remember:

1. Every factor of a number is a divisor of that number.

2. Any number can be divided by the product of two or more of its prime factors.

FINDING THE G. C. D. OF SMALL NUMBERS.

We can find the G. C. D. of small numbers by writing the common divisors of the numbers and then selecting the greatest one, as we did in the first part of this lesson. But there is a better way.

ILLUSTRATION:

Find the G. C. D. of 12, 18, and 36.

$$12 = 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$

2 and 3 are common prime factors, or divisors, of 12, 18, and 36.

Multiply the common prime factors together and the result is the Greatest Common Divisor.

$2 \times 3 = 6$.

6 is the G. C. D. of 12, 18, and 36.

Principle.

The Greatest Common Divisor of two or more numbers is the product of their common prime factors.

ILLUSTRATION:

Find the G. C. D. of 24, 48, and 72.

FIRST WAY.

WORK:

$$24 = 2 \times 2 \times 2 \times 3$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

G. C. D. $= 2 \times 2 \times 2 \times 3 = 24$

NOTE.—It is a good plan to run a line through the common factors. Then one is less likely to miss a common factor.

SECOND WAY.

WORK:

$$\begin{array}{r|l} 2 & 24,\ 48,\ 72 \\ \hline 2 & 12,\ 24,\ 36 \\ \hline 3 & 6,\ 12,\ 18 \\ \hline 2 & 2,\ 4,\ 6 \\ \hline & 1,\ 2,\ 3, \end{array}$$

G. C. D. $= 2 \times 2 \times 3 \times 2 = 24$.

NOTE.—In this method divide by as many prime numbers as will divide *all the numbers*.

FINDING THE G. C. D. OF LARGE NUMBERS.

When the G. C. D. is to be found of numbers which cannot be readily factored, the following method is used:—

If the G. C. D. of two numbers is to be found, divide the larger by the smaller; then divide the divisor last used by the last remainder, and so on, until there is no remainder. The *final divisor* is the G. C. D.

ILLUSTRATION:

Find the G. C. D. of 592 and 333.

WORK:

```
333)592(1
    333
    ---
    259)333(1
        259
        ---
         74)259(3
            222
            ---
G. C. D.=   37)74(2
               74
               --
```

The final divisor, 37, is the G. C. D.

To the Teacher:

A very common mistake made by pupils who are learning to find the G. C. D. of two or more large numbers, is this:

They take the result obtained last for the G. C. D., instead of the final divisor. Notice whether or not your pupils are too observing to forget *which* is the G. C. D.

To find the G. C. D. of three or more numbers, the above process is carried out with all of them. The G. C. D. of any two is found first. Next find the G. C. D. of the third number and the G. C. D. (already found) of the other two.

Proceed in this way and *the last divisor will be the G. C. D. of all the numbers.*

TO ILLUSTRATE:

Find the G. C. D. of 3070, 2149, and 614.

WORK:

2149) 3070 (1
2149
921) 2149 (2
1842
307) 921 (3
921

2
G. C. D. = 307) 614
614

EXPLANATION:

Find the G. C. D of 3070 and 2149. It is 307.

Find the G. C. D. of 307 and 614. It is 307.

Then 307 is the G. C. D. of 3070, 2149, and 614.

307 = G. C. D. of 3070, 2149, 614.

Let the child try other examples like the above, until he is familiar with the process.

He is just as liable to be called upon to find the G. C. D. of large numbers in real life, as he is the G. C. D. of small ones.

Some numbers, as those given below, are found to be prime to each other only after performing the work.

ILLUSTRATION:

Find the G. C. D. of 1321 and 973.

WORK:

973) 1321 (1
973
348) 973 (2
696
277) 348 (1
277
71) 277 (3
213
64) 71 (1
64
7) 64 (9
63
G. C. D. = 1) 7 (7

These numbers are prime to each other because the G. C. D. is 1.

LEAST COMMON MULTIPLE.

PRESENT IN THIS WAY:

Name a number of which 2 is a factor. Name one of which 3 is a factor. Of which 5 is a factor.

Name a number which is exactly divisible by 5. (*Child*) 15. By 3. (*Child*) 9.

We call 15 a multiple of 5. We call 9 a multiple of 3. Can you tell what a multiple is now, in your own words?

Name a number that is exactly divisible by both 5 and 3. (*Child*) 15. Another. (*Child*) 30. Another. (*Child*) 45.

We call 15 a common multiple of 5 and 3. Name other common multiples of 5 and 3. What do you mean by common multiple?

Name the least number that is exactly divisible by both 5 and 3. (*Child*) 15.

We call 15 the Least Common Multiple of 5 and 3. Why?

A **multiple** of a number is a number that is exactly divisible by it.

When we multiply a number we get a larger number, so a multiple of a number is always larger than the number itself.

Some multiples of 2, 3, and 5.

Some multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 80, etc.

Some multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 30, etc.

Some multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 90, etc.

The pupil should notice, on looking over the multiples of 2 and 3, that they have some multiples in common. 6, 12, and 18 are common multiples of 2 and 3.

6 is the smallest common multiple, that is, it is the least common multiple of 2 and 3.

The **Least Common Multiple (L. C. M.)** of two or more numbers is the least number that is a multiple of each of them.

FINDING THE L. C. M. OF NUMBERS.

WE KNOW—

1. That the Least Common Multiple must be larger than any of the numbers.
2. That it must exactly contain each one of the numbers one or more times.

Therefore,

The Least Common Multiple of two or more numbers must contain every prime factor the greatest number of times it is found in any one of the numbers.

ILLUSTRATION:

What is the L. C. M. of 12, 18, and 24?

EXPLANATION:

The prime factors of 12 = 2, 2, 3.
The prime factors of 18 = 2, 3, 3.
The prime factors of 24 = 2, 2, 2, 3.
2 is found *three times* in 24.
3 is found *twice* in 18.
L. C. M. of 12, 18, and 24 = 2 × 2 × 2 × 3 × 3, or 72.

Principle:

The Least Common Multiple of two or more numbers is the product of the prime factors, each prime factor to be taken the greatest number of times it is found in any one number.

TO ILLUSTRATE:

Find the L. C. M. of 144, 240, 600.

FIRST WAY.

Work:

$$144 = \underline{2 \times 2 \times 2 \times 2} \times \underline{3 \times 3}$$
$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$
$$600 = 2 \times 2 \times 2 \times 3 \times \underline{5 \times 5}$$

L. C. M. =
$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$,
or 3600.

NOTE.—The series of factors underlined show the groups where the prime factors are taken the greatest number of times.

SECOND WAY.

Work:

2) 144, 240, 600
2) 72, 120, 300
2) 36, 60, 150
2) 18, 30, 75
3) 9, 15, 75
5) 3, 5, 25
3, 1, 5

Explanation:

In this method we go through a series of divisions by prime factors that will divide at least two of the numbers.

Numbers that cannot be divided should be brought down unchanged.

The L. C. M. is the product of the divisors and the final results.

FINDING THE L. C. M. OF LARGE NUMBERS.

EXAMPLE:

Find the L. C. M. of 2862 and 3498.

Work:

2862) 3498 (1
2862
636) 2862 (4
2544
G. C. D. = 318) 636 (2
636

2862 ÷ 318 = 9.

3498 × 9 = 31482 L. C. M.

Explanation:

To find the L. C. M. of large numbers, find the G. C. D. of any two of them, and then divide one of those numbers by the G. C. D.

The product of the obtained quotient and the other number, is the Least Common Multiple of those two numbers.

If there is a third number, take the L. C. M. already found and find the L. C. M. of that and the third number.

The foregoing method may be used with small numbers also, as,

EXAMPLE:

Find the L. C. M. of 18, 36, 72.

WORK:

G. C. D. = 18) 36 (2
36

18 ÷ 18 = 1

1 × 36 = 36

G. C. D. = 36) 72 (2
72

36 ÷ 36 = 1

1 × 72 = 72 L. C. M.

Example Showing Proof:

Find the L. C. M. of 42, 50, 72.

WORK:

2) 42, 50, 72
3) 21, 25, 36
7, 25, 12

L. C. M. = 2 × 3 × 7 × 25 × 12, or 12600.

PROOF:

12600 ÷ 42 = 300
12600 ÷ 50 = 252
12600 ÷ 72 = 175

12600 must be the L. C. M.

We have shown that each of the numbers is contained in it a whole number of times. Therefore, 12600 must be a common multiple of all three numbers.

It must be the least common multiple, because 300, 252, and 175 have no common factor.

TALK:

Some arithmetics give problems which involve the finding of the Least Common Multiple or Greatest Common Divisor of two or more numbers. Most of them are problems that have no practical application. They are deemed a waste of time here.

A good practical knowledge of how to find quickly the Least Common Multiple or Greatest Common Divisor is what is needed. This can be better accomplished by dealing with mere numbers, than in spending time on problems that do not occur in real life often enough to give compensation for the energy expended.

There is enough practical work to be done in arithmetic to give all mind training that might be secured by the use of impractical problems.

CANCELLATION.

PRESENT IN THIS WAY:

Divide 36 by 18. What is the quotient? (*Child*) 2.

Divide 9 times 4 by 9 times 2. What is the quotient? (*Child*) 2.

How often is 3 times 6 contained in 9 times 6? (*Child*) 3 times.

Divide 9 × 5 by 3 × 5. (*Child*) The quotient is 3.

Suppose we cut out or cancel the factor 5, because it is found both in the divisor and dividend, would the quotient be the same? (*Child*) Yes.

If the dividend and divisor both have common factors, the work can be made much easier by cancellation.

In order to cancel, write the dividend above the divisor, with a line between.

Remember:

Cancelling like factors from both dividend and divisor does not change the quotient, but makes the work easier.

EXAMPLE:

$18 \div 9 = 2$

Cancelling the factor 3 from both 18 and 9, we have—

$6 \div 3.$

$6 \div 3 = 2.$ The quotient is not changed.

ANOTHER FORM:

$$\frac{18}{9} = 2$$

Cancelling the factor 3 from both 18 and 9, we have—

$$\frac{\overset{6}{\cancel{18}}}{\underset{3}{\cancel{9}}} = \frac{6}{3} = 2.$$

NOTE.—In cancellation we use the last form.

To the Teacher:

Here is a good place to test the pupil on the effect of a 0 as a factor. Ask him what 8 × 2 × 0 × 4 will equal. Show him that 16 and 4 are incorrect results. Show him that whenever 0 is used as a factor, the product is always 0.

EXAMPLE:

Multiply 620 by 18 and divide the product by 20.

THE LONG WAY:

WORK:

$$\begin{array}{r} 620 \\ 18 \\ \hline 4960 \\ 620 \\ \hline 2\cancel{0})\,1116\cancel{0} \\ \hline 558 \end{array}$$

THE SHORT WAY:

WORK:

$$\frac{\overset{31}{\cancel{62}}\cancel{0} \times 18}{\cancel{2}\cancel{0}} = 558.$$

EXAMPLE:

17 × 13 × 15 divided by 65 × 10 = ?

WORK:

$$\frac{17 \times \cancel{13} \times \overset{3}{\cancel{15}}}{\underset{\cancel{5}}{\cancel{65}} \times 10} = \frac{51}{10} = 5\tfrac{1}{10}.$$

EXPLANATION:

13 into 65, 5 times. Cancel 13 and 65 and write 5 below 65. 5 into 15, 3 times. Cancel 5 and 15 and write 3 above 15. 17 × 3 = 51. The divisor is 10. Result = $\frac{51}{10}$ = $5\frac{1}{10}$.

Principle:

Dividing any one of a series of factors by a number divides their product by that number.

ILLUSTRATION:

Take the series 9 × 12 × 15. Suppose we wish to divide this series of factors by 3.

The pupil when asked how to do it may say, "Divide the 9, 12, and 15, each by 3." Show him that he is wrong.

9 × 12 × 15 is really a number broken up into factors, and if you divide even one factor, it is as if you divide the whole number by that divisor. $9 \times 12 \times 15 = 1620$. $1620 \div 3 = 540$.

Suppose we divide each of the factors 9, 12, and 15 by 3. We have left 3 × 4 × 5, or 60.

But, suppose we divide one factor by 3, say the first. We have then 3 × 12 × 15, or 540. This is dividing the whole number by 3. $1620 \div 3 = 540$, which is correct.

PROBLEM:

A farmer took to town 12 bu. of potatoes, which he sold at 35¢ per bu. In exchange he bought sugar at 4¢ per pound. How many pounds of sugar did he get?

SOLUTION:

His potatoes were worth 12 × 35¢.

He received as many pounds of sugar as 4¢ is contained times in 12 × 35¢, or 105.

WORK:

$$\frac{\overset{3}{\cancel{12}} \times 35}{\cancel{4}} = 105.$$

PART FOUR.

Common Fractions.

"The best education in the world is that got by struggling to get a living. What is defeat? Nothing but the first step to something better."

— *Wendell Phillips.*

FRACTIONS.

Since there is nothing new in the process of fractions, so in the teaching of fractions there is nothing essentially different from the familiar operations with whole numbers. If the idea of number as measurement has been made the basis of method in primary work and in the fundamental operations, the fraction idea must have been constantly used, and there is absolutely no break when the pupil comes to a formal study of fractions. There is only before him the easy task of examining somewhat more attentively the nature of the processes he has long been using. . . . The teacher must be loyal to principles, but a slave of no man's devices.

—McLellan and Dewey's
"*Psychology of Number.*"

FRACTIONS.

A GENERAL TALK:

For a long time the child has dealt with fractions without knowing them as fractions. He learned of them early in his school career along with whole numbers. Longer ago than that, even before he had reached the age for school, he learned of halves, thirds, fourths, etc., from his home experiences with apples and pies.

A noted author on arithmetic said once that "a very little child sees fractions in the form of parts of pies and apples." Later on he wants to unite these parts, delights in separating them, puzzles himself in grouping them, and cuts his fingers in dividing them. These are "home experiences."

The child begins work involving fractions. It is positively wrong to let him get the impression that he is entering a new field, because he is not. Rather tell him, "Here we have been breaking up whole things and talking about their parts, but now we are going to give names to these fractured parts, and we are going to call them fractions. Why is that a good name for the parts? etc."

So all the pupil has to do now is to write what he has been talking about. He will write one third of an apple, $\frac{1}{3}$ *of an apple, etc.*

At school the child writes on paper what he does in reality at home.

From his knowledge of parts of apples and pies acquired at home, he is taught to generalize after entering school. In that way the child realizes what the *figures mean, and does not see merely so many meaningless characters staring him in the face. He sees an action in each step performed, something that he could do himself with every expression. Fractions represent something to him.*

In beginning with fractions, do not tell the child that the number above the line is the numerator, and the one below is the denominator. Let them remain the number above the line and the number below the line.

The essential thing at this stage is to have the child see that every fraction asks him two questions: 1. "How large are my parts?" and 2. "How many parts have I taken?" He sees the number below the line answers his first question and the one above, the latter. Drill on this phase of the work by asking him what $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{6}$, *and* $\frac{7}{8}$ *mean to him.*

The work in fractions, above all things, should be void of rules and hard names. Have the child do the work as he understands it should be done, after a short review of previous subjects and a few leading questions. He should not perform any work because a rule guided him that way, nor after he has learned to explain the process parrotlike.

If he understands previous work, he is able to make a new step for himself, with some little preparation and help from the teacher. He should explain his work, telling what he has done and why he has done as he has. Then he may make a rule for himself.

Things to Remember in Teaching Fractions:

1. Begin with objects.
2. Make your talks short.
3. Allow no guessing results.
4. Do not teach rules.
5. Have the pupil tell what he has done, *after* he has completed assigned work.
6. Give numerous test problems.

HOW TO TEACH SMALL FRACTIONS.

TALK:

A very good method of teaching the combinations in small fractions is by the use of paper or cardboard disks (pupils may call them circles.)

Cut out a large number of them, and, in order to avoid trouble later on, it might be better to have the disks all of one size—about 4 inches in diameter.

Teaching the Fraction $\frac{1}{2}$ with Disks.

What to say—

I am going to cut this circle into two equal parts. What is this part called? What is this other part called? How many halves in the whole circle? One half and one half are what? One half taken away from one leaves what? If I take a half two times, what do I get? How many halves in a whole?

Now I will write these:—

$\frac{1}{2} + \frac{1}{2} =$ 1 divided by $\frac{1}{2} =$

1 less $\frac{1}{2} =$ $1 \div 2 =$

$2 \times \frac{1}{2} =$ $1 - \frac{1}{2} =$

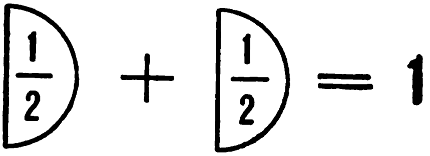
You may give me the answers and I will write them.

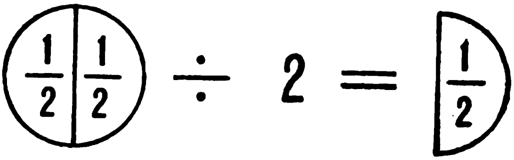
For the Child to Do:

1. Draw a line 1 inch long.
2. Divide it into halves.
3. Write what you have done.
4. Draw a square.
5. Divide it into halves.
6. Tell three stories about halves.
7. Here is one—"One half apple and one half apple are one apple."

To the Teacher:

Drawings showing the "placing" of disks for number stories can be made by the pupil; as,





Ask the pupil to make similar drawings to tell about halves.

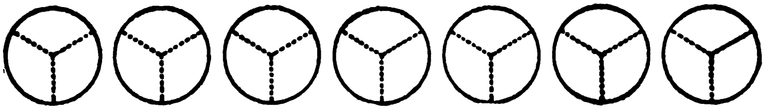
Proceed like this, "How many halves in a pie? If a pie cost 10 cents, what will half a pie cost? Who can tell other stories about halves? etc."

Teach fourths along with halves. Use quarters of a dollar and quarts in a gallon to illustrate.

Teaching the Fraction ⅓ with Disks.

TALK:

Cut several disks into thirds. Have the child practice on cutting, so that he will be able to make the three parts of each disk equal.



Then proceed like this—"What do you call each of these parts? Why are they called thirds? How many thirds in a circle? I am going to take a circle and cut it anyway, so as to make three parts; do I call these unequal parts thirds? Why not? Let me write one third on a piece of paper for you. (*Write,* ⅓.)

Draw a circle for me. Instead of cutting it, draw lines where you would cut it to make thirds. Write one third ($\frac{1}{3}$) on each third of a circle. I write this ($\frac{1}{3}+\frac{2}{3}$). Who can tell me what the answer is? Are two thirds and two thirds more than one? How much more? I have two thirds of an apple and give Mary one third, how much have I left? Who can give other story problems about thirds? Everybody try, etc."

Teach sixths along with thirds. Use disks, dots, marks, sticks, and inches to illustrate.

Remember that no advance should be made until each little part is understood.

Review every day on previous work.

Teaching the Fraction $\frac{1}{5}$ with Disks.

TALK:

Have fifths compared with fourths, thirds, and halves.

Be sure that the child sees that the number below the line shows the size of each part and the number above how many parts are taken.

Keep reviewing and repeating.

Hundreds of little story problems should be given along with the work.

Have the child illustrate every step with objects or drawings. If at any time he cannot answer results at sight, let him show the work with disks.

What to say:—

I am going to cut this circle into five equal parts. What is each part called? How many fifths in the whole circle? How many fifths here? (Show two fifths.) One fifth and one fifth are how many fifths? One less two fifths is what? If I take three fifths, how many times one fifth do I take? How many times one fifth are left? Are two fifths greater than one third?

Give and ask for story problems and number stories telling about fifths.

Review thirds, fourths, and halves.

Teach tenths along with fifths.

When twelfths are taught, show the relations between twelfths and sixths, fourths, thirds, and halves.

KINDS OF FRACTIONS.

TALK:

The pupil has learned that fractions are only equal parts of numbers.

It is improper but convenient to call anything larger than a whole, a fraction. Four thirds and three halves are fractions only in form, therefore we call them improper fractions.

One third, one half, and three fourths are each less than a unit and are proper fractions.

Fractions are equal parts of numbers.

The **numerator** of a fraction is the number above the line, and shows how many parts are taken.

The **denominator** is the number below the line, and shows the size of each part.

The numerator and denominator together are called **the terms of a fraction**.

A **proper fraction** is one in which the numerator is less than the denominator.

An **improper fraction** is one in which the numerator is not less than the denominator.

A proper fraction is less than 1 in value; an improper fraction has a value of one, or more than one.

A **mixed number** is a whole number and a fraction; as $3\frac{2}{5}$, $10\frac{1}{2}$, $6\frac{2}{3}$.

The mixed number means that there are whole things taken together with a fraction of another.

To the Teacher:

Write a number of proper fractions, improper fractions, and mixed numbers, and have the child pick out those of each kind; as,

$\frac{3}{8}$, $27\frac{1}{2}$, $\frac{5}{11}$, $\frac{19}{20}$, $\frac{20}{20}$, $\frac{18}{16}$, $\frac{11}{5}$, $3\frac{1}{2}$, $16\frac{2}{3}$.

Review — Why do we call $\frac{11}{5}$ an improper fraction? Should it be left that way?

Changing Improper Fractions to Whole or Mixed Numbers.

TALK:

The pupil knows that the improper fraction is composed of a whole number and a fraction.

Impress on him that improper fractions should never be left in that form in the result. They must be reduced to whole or mixed numbers.

The following explanations are given to show how the change is made in reducing improper fractions to whole or mixed numbers. Both are good. The child should use the one taught in his grade.

EXPLANATIONS OF WORK.

First Explanation:

Change $\frac{12}{2}$ to a whole number.

SOLUTION:

2 halves = 1.

12 halves = as many 1's as there are 2's in 12, or 6.

Second Explanation:

Change $\frac{13}{4}$ to a mixed number.

SOLUTION:

Since in 4 fourths there is 1 unit, in 13 fourths there are as many units as there are 4's in 13, or $3\frac{1}{4}$. Thus, $\frac{13}{4} = 3\frac{1}{4}$.

EXAMPLE:

Reduce $\frac{512}{13}$ to a mixed number.

WORK:

$$\begin{array}{r}
39\frac{5}{13}\\
13\,\overline{)\,512}\\
\underline{39}\\
122\\
\underline{117}\\
5
\end{array}$$

After the pupil has worked a number of examples, let him make a rule for changing improper fractions to whole or mixed numbers.

Changing Mixed Numbers to Improper Fractions.

TALK:

The pupil has learned to change improper fractions to whole or mixed numbers. Now all he will have to do is to reverse the process.

EXPLANATIONS OF WORK.

First Explanation:

Change $5\frac{1}{6}$ to an improper fraction.

SOLUTION:

$$1 = \frac{6}{6} \qquad 5 = 5 \times \frac{6}{6}, \text{ or } \frac{30}{6}$$

$$\frac{30}{6} + \frac{1}{6} = \frac{31}{6} \qquad \text{Hence, } 5\frac{1}{6} = \frac{31}{6}.$$

Second Explanation:

Change $4\frac{1}{5}$ to an improper fraction.

SOLUTION:

Since there are 5 fifths in 1, in 4 there are 4 times 5 fifths, or 20 fifths.

$$\frac{20}{5} + \frac{1}{5} = \frac{21}{5} \qquad \text{Hence, } 4\frac{1}{5} = \frac{21}{5}$$

NOTE.—Use either of the foregoing explanations, but not both.

Equal Fractions in Different Forms.

TALK:

Have the child see how fractions may differ in form but still remain the same in value.

Begin with his knowledge of smaller fractions, as $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, *and* $\frac{5}{10}$ *of an apple.*

Let him show by the use of drawings that fractions may have large or small terms but be equal in value.

Things for the Child to Do:

Fill out the numerators in the following fractions and prove each by a drawing:—

1. $\frac{1}{4} = \frac{2}{8} = \frac{4}{16} = \frac{8}{32} = \frac{16}{64}.$

2. $\frac{1}{5} = \frac{?}{10} = \frac{?}{15} = \frac{?}{20} = \frac{?}{30}.$

3. $\frac{1}{8} = \frac{?}{16} = \frac{?}{24} = \frac{?}{32} = \frac{?}{40}.$

4. $\frac{3}{4} = \frac{?}{8} = \frac{?}{12} = \frac{?}{16} = \frac{?}{24}.$

5. $\frac{2}{5} = \frac{?}{10} = \frac{?}{15} = \frac{?}{20} = \frac{?}{40}.$

Changing Fractions to Higher Terms.

TALK:

Review like this — What are the terms of a fraction? What term of a fraction names the equal parts? Which shows the number of parts taken? Which shows the size of each part? Which is larger, $\frac{1}{2}$ or $\frac{5}{10}$ of a dollar? Why? May fractions that are different in form be equal in value? Name three fractions equal to one third, but with higher terms.

After this short review, have the child make a general statement about the change that takes place if both numerator and denominator are multiplied by the same number. Let him see the effect of multiplying both numerator and denominator of $\frac{1}{2}$ by 2; by 3.

Have a statement made about the effect of multiplying the numerator and denominator by different numbers. Let him try it.

Remember:

If both numerator and denominator are multiplied by the same number, the form and not the value of the fraction is changed.

EXAMPLE:

Reduce $\frac{1}{3}$ to ninths.

WORK:

$$\frac{1}{3} = \frac{3}{9}.$$

EXPLANATION:

To change the denominator to ninths, multiply by 3. Since the denominator is multiplied by 3, the numerator must also be multiplied by 8, so as not to change the value of the fraction.

EXAMPLE:

Change 7 to eighths.

WORK:

$$7 = \frac{7}{1}$$

$$\frac{7}{1} = \frac{8 \times 7}{8 \times 1} = \frac{56}{8}$$

ANALYSIS:

Since in 1 there are 8 eighths, in 7 there are 7 times 8 eighths, or 56 eighths.

EXAMPLE:

Reduce $\frac{10}{17}$ to a fraction whose denominator is 85.

WORK:

$$\begin{array}{r} 5 \\ 17\overline{)85} \\ \underline{85} \end{array}$$

$$\frac{10}{17} = \frac{5 \times 10}{5 \times 17} = \frac{50}{85}.$$

Changing Fractions to Lower Terms.

TALK:

Have the pupil draw a square four inches on a side and divide it into fourths.

Let him show that $\frac{2}{4} = \frac{1}{2}$; *that* $\frac{4}{8}$, $\frac{6}{12}$, $\frac{9}{18}$, *and* $\frac{1}{2}$ *are all of the same value, only different in form.*

Ask him to tell in each case which has the lowest terms.

In telling when a fraction is in its lowest terms, it should be told something like this — A fraction is in its lowest terms when both numerator and denominator cannot be divided by the same number.

When we divide both numerator and denominator by the same number, we do not change the value of the fraction. Ask the pupil to tell what change is made.

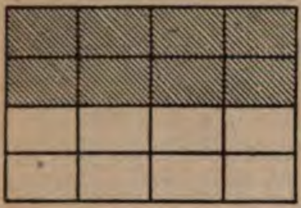
Have him show by written examples that he understands.

Remember:

Dividing both terms of a fraction by the same number reduces a fraction to lower terms but does not change its value.

EXAMPLE:

Reduce $\frac{8}{16}$ to halves.



$\frac{8}{16} = \frac{1}{2}$

WORK:

$$\frac{8}{16} = \frac{1}{2}$$

EXPLANATION:

Divide both numerator and denominator by 8.

Result, $\frac{1}{2}$. Have the pupil prove that the result is in its lowest terms.

In reducing large fractions to lowest terms, it is difficult sometimes to divide at sight. In that case, we find the G. C. D. of numerator and denominator and divide each term by it.

EXAMPLE:

Reduce $\frac{93}{217}$ to lowest terms.

WORK:

The G. C. D. of 93 and 217 is 31.

$$\frac{93 \div 31}{217 \div 31} = \frac{3}{7}.$$

NOTE.—Another method used much in practice is that of cancelling factors common to both numerator and denominator.

EXAMPLE:

Reduce $\frac{132}{900}$ to lowest terms.

WORK:

$$\frac{\overset{\overset{11}{\cancel{33}}}{\cancel{132}}}{\underset{\underset{75}{\cancel{225}}}{\cancel{900}}} = \frac{11}{75}.$$

EXPLANATION:

We cancelled by the common factor 4, and then by 3. Result $= \frac{11}{75}$.

Getting Fractions Ready to be Added or Subtracted.

TALK:

Present like this—Can you add 4 cabbages and 5 cauliflower? Why not?

Can you add 3 apples, 5 pears, and 2 plums?

Suppose we give the apples, pears, and plums the common name fruit, can we then add 3 fruits, 5 fruits, and 2 fruits?

Suppose we wish to add 4 gallons and 12 quarts, how would we do it?

What change must we make before we can add?

Can we add 3 fourths and 1 half? Why not?

Suppose we change the half to fourths, then can we add the 2 fourths and the 3 fourths?

Make a general rule.

We will write what we have said:—

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{2}{4} + \frac{3}{4} = \frac{5}{4} = 1\tfrac{1}{4}.$$

You said that cabbages and cauliflower must have a common name, the same name, before we could add.

Which is it that gives the name to the fraction, the numerator or denominator?

What does the word denominator mean?

The denominators must always have the same name before the fractions can be added or subtracted. In other words, the fractions must be changed to fractions having the Least Common Denominator (*L. C. D.*)

Fractions as well as whole numbers must have a common name before they can be added or subtracted.

The Least Common Multiple of the denominators is the Least Common Denominator. Thus, the Least Common Denominator of the fractions $\frac{6}{12}$, $\frac{3}{36}$, $\frac{2}{24}$ will be the Least Common Multiple of 12, 36, and 24, or 72. Review on finding L. C. M. of numbers.

EXAMPLE:

Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$ to L. C. D.

WORK:

The L. C. M. of 2, 4, and 6 is 12.

$$\frac{1}{2} = \frac{6\times1}{6\times2} = \frac{6}{12}$$

$$\frac{3}{4} = \frac{3\times3}{3\times4} = \frac{9}{12}$$

$$\frac{5}{6} = \frac{2\times5}{2\times6} = \frac{10}{12}$$

EXPLANATION:

Change the fractions so that they have the common name *twelfths.* They are then ready to be added.

ANOTHER WAY:

Get $\frac{9}{16}$, $\frac{3}{4}$, and $\frac{3}{8}$ ready to be added.
The L. C. M. of 16, 4, and 8, is 16.

$$16\,)\,\underline{16}\quad 1 \qquad 9\times1=9 \qquad \frac{9}{16}=\frac{9}{16}$$

$$4\,)\,\underline{16}\quad 4 \qquad 3\times4=12 \qquad \frac{3}{4}=\frac{12}{16}$$

$$8\,)\,\underline{16}\quad 2 \qquad 3\times2=6 \qquad \frac{3}{8}=\frac{6}{16}$$

NOTE.—Fractions should be reduced to lowest terms before the L. C. D. is found.

ADDITION OF FRACTIONS.

TALK:

Present like this—Ask for answers to these statements and then write them as follows:—

3 apples and 4 apples are 7 apples.

3 fifths and 4 fifths are 7 fifths.

$\frac{3}{5}$ *and* $\frac{4}{5} = \frac{7}{5}$.

Tell the child to notice that we added the numerators and put the sum over the common denominator.

For the Pupil:

Can we add 5 eighths and 3 fourths? What shall we do?

3 fourths equal how many eighths? Let us write what we do:

$$\frac{3}{4} = \frac{6}{8} \qquad\qquad \frac{5}{8} + \frac{6}{8} = \frac{11}{8}$$

To the Teacher:

Have the pupil make a rule telling how to add fractions. The one below is only a model for you, not for the pupil. With little or no aid the pupil will give you as good a rule as the one below and one that he understands.

To Add Fractions:

1. Get the fractions ready to be added by giving them a common name, that is, reduce them to a L. C. D.

2. Add the fractions by adding the numerators. Place the sum over the L. C. D.

3. Change the result to its simplest form.

There are so many different methods of arranging the work in addition and subtraction of fractions that it necessitates giving two or three forms of work below. The child should be taught only one method, and the one used in your school should be the one.

The mechanical work needed to get the Least Common Multiple and to reduce the fractions to L.C.D., need not be kept.

1st Method of Arrangement.

Find the sum of $\frac{5}{7}$, $\frac{3}{5}$, $\frac{9}{14}$.

WORK:

The L.C.M. of 7, 5, and 14, is 70.

$$\frac{5}{7}=\frac{50}{70}$$

$$\frac{3}{5}=\frac{42}{70}$$

$$\frac{9}{14}=\frac{45}{70}$$

$$\frac{50}{70}+\frac{42}{70}+\frac{45}{70}=\frac{137}{70}=1\frac{67}{70}.$$

2d Method of Arrangement.

(A shorter way.)

Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$.

WORK:

L.C.D. = 24.

$$\frac{2}{3}+\frac{3}{4}+\frac{7}{8}=\frac{16+18+21}{24}=\frac{55}{24}=2\frac{7}{24}.$$

3d Method of Arrangement.

Add $\frac{9}{16}$, $\frac{3}{4}$, $\frac{3}{8}$.

WORK:

L.C.D. = 16.

$$\text{Numerators}\begin{cases} 9 \\ 12 \\ 6 \end{cases} \quad \text{Sum of fractions} = \frac{27}{16} = 1\frac{11}{16}.$$

$$\text{Sum of numerators} = 27$$

ADDING MIXED NUMBERS.

1st Method of Arrangement.

Add $2\frac{11}{14}$, $\frac{1}{9}$, $10\frac{1}{7}$.

Work:

L. C. D. of fractions = 126.

$$\frac{11}{14} = \frac{99}{126}$$

$$\frac{1}{9} = \frac{14}{126}$$

$$\frac{1}{7} = \frac{18}{126}$$

Sum of fractions = $\frac{131}{126} = 1\frac{5}{126}$

Sum of whole numbers = 12

Sum of mixed numbers = $13\frac{5}{126}$

2d Method of Arrangement.

Add $2\frac{11}{14}$, $\frac{1}{9}$, $10\frac{1}{7}$.

Work:

L. C. D. of fractions = 126.

$$\text{Numerators}\begin{cases} 99 \\ 14 \\ 18 \end{cases}$$

$$\text{Sum of numerators} = 131$$

In adding mixed numbers, add the fractions and integers separately and find the sum of the results.

Sum of fractions = $\frac{131}{126} = 1\frac{5}{126}$.

Sum of integers = 12

Sum of all = $13\frac{5}{126}$.

PROBLEMS IN ADDITION OF FRACTIONS.

TALK:

All problems in fractions involve the same processes of reasoning as those in whole numbers. The manipula-

tions of the fractional parts, the computations, embrace the only difference. Consequently, it is not deemed worth while here to give problems after each operation.

If the pupil has received the proper drill in the working of problems involving the four fundamental operations, he should have no trouble now, unless it be in the actual computation. For that he must have constant drill and faithful reviews in examples and problems.

For those who do have difficulty, a few problems in the different operations are included at the end of this chapter.

SUBTRACTION OF FRACTIONS.

TALK:

Ask for answers to these statements and then write them as follows—15 apples less 8 apples are 7 apples. 15 tenths less 8 tenths are 7 tenths.

$$\frac{15}{10} - \frac{8}{10} = \frac{7}{10}. \qquad \frac{7}{5} - \frac{2}{5} = \frac{5}{5}.$$

Tell the child to notice that we found the difference of the numerators and wrote it over the common denominator.

For the Pupil:

Can we subtract 2 thirds from 7 ninths? What shall we do?

How do we get fractions ready to subtract?

Let us write what we *have* to do and then show the work.

$$\frac{7}{9} - \frac{2}{3} = ? \qquad \frac{2}{3} = \frac{6}{9} \qquad \frac{7}{9} - \frac{6}{9} = \frac{1}{9}.$$

To the Teacher:

Have the pupil make a general rule for the subtraction of fractions.

To Subtract Fractions:

1. **Get the fractions ready to be subtracted by giving them a common name, that is, reduce them to L. C. D.**

2. Find the difference of their numerators and place the difference over the L. C. D.

3. Simplify the result.

1st Method of Arrangement.

Take $\frac{8}{9}$ from $\frac{25}{27}$.

Work:

The L. C. D. = 27.

$$\frac{8}{9} = \frac{24}{27};\quad \frac{25}{27} - \frac{24}{27} = \frac{1}{27}.$$

2d Method of Arrangement.

1. $\frac{9}{10} - \frac{3}{4} = ?$

L. C. D. = 20.

$$\frac{9}{10} - \frac{3}{4} = \frac{18 - 15}{20} = \frac{3}{20}.$$

2. $19\frac{7}{8} - 16\frac{2}{3} = ?$

Work:

L. C. D. = 24.

$$\begin{array}{r} 19\frac{7}{8} = 19\frac{21}{24} \\ 16\frac{2}{3} = 16\frac{16}{24} \\ \hline 3\frac{5}{24} \end{array}$$

The only difference in this arrangement is that after the fractions are reduced, the numerators are placed together over one writing of the common denominator.

Subtracting Mixed Numbers Where the Fraction in the Minuend Is Smaller than the One in the Subtrahend.

Subtract $7\frac{8}{9}$ from $19\frac{1}{7}$.

1ST WAY.

Work:

L. C. D. of fractions = 63.

$$\begin{array}{r} 19\frac{1}{7} = 19\frac{9}{63} = 18\frac{72}{63} \\ 7\frac{8}{9} = 7\frac{56}{63} = 7\frac{56}{63} \\ \hline 11\frac{16}{63} \end{array}$$

Explanation:

Take 1 from 19 and add this 1, or $\frac{63}{63}$, to $\frac{9}{63}$, making $\frac{72}{63}$.

Subtract the fractions and integers separately.

2D WAY

Work:

L. C. D. of fractions = 63.

$$\begin{array}{r} 19\frac{1}{7} = 19\frac{9}{63}, + \frac{7}{63} = 19\frac{16}{63} \\ 7\frac{8}{9} = 7\frac{56}{63}, + \frac{7}{63} = 8 \\ \hline 11\frac{16}{63} \end{array}$$

Explanation:

Add $\frac{7}{63}$ to both minuend and subtrahend to get rid of the fraction $\frac{56}{63}$.

Does it alter the value of the result to add $\frac{7}{63}$ to both minuend and subtrahend?

MULTIPLICATION OF FRACTIONS.

MULTIPLYING FRACTIONS AND INTEGERS.

TALK:

Present like this— We have learned that multiplication is only a short way of adding whole numbers. It is also true that multiplication of fractions is only a short way of adding them. We will write this out and you may fill in as I ask for results:—

Write—$8 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$.

8 × 4 men = 32 men.
8 × 4 feet = 32 feet.
8 × 4 parts = 32 parts.
8 × 4 fifths = 32 fifths.
$8 \times \frac{4}{5} = \frac{32}{5}$.

Tell the child that $8 \times \frac{4}{5}$ *is read "8 times 4 fifths" or, "multiply 4 fifths by 8," and it means that there are 8 groups, each group containing 4 fifths, to be reduced to one group containing 32 fifths.*

32 fifths equal 6 and 2 fifths.

Ask for a general rule after several examples have been explained by the pupil.

To Multiply Fractions and Integers:

1. Make a fraction of the integers by giving each the denominator 1.

2. CANCEL IF POSSIBLE. Multiply the numerators for a new numerator and the denominators for a new denominator.

3. Reduce the result to its simplest form.

EXAMPLES:

1. $6 \times \frac{2}{3} = ?$

WORK:

$6 = \frac{6}{1}$

$\frac{\overset{2}{\cancel{6}}}{1} \times \frac{2}{\cancel{3}} = \frac{4}{1} = 4.$

2. What is $\frac{5}{9}$ of 225?

Work:

$\frac{5}{7} \times \frac{225}{1} = \frac{1125}{7} = 160\frac{5}{7}$.

Explanation:

$\frac{1}{7}$ of $225 = \frac{225}{7}$

$\frac{5}{7}$ of $225 = 5 \times \frac{225}{7}$, or $160\frac{5}{7}$.

Principle:

Cancel always when you can, multiply only when you must.

MULTIPLYING FRACTIONS BY FRACTIONS.

TALK:

Present like this— One half of $8 equals how many dollars? One half of one dollar equals what?

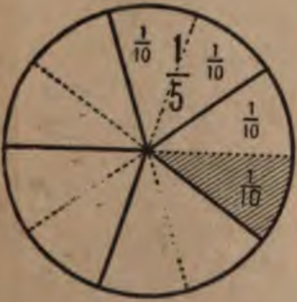
One half of one half dollar equals what?

Many times we get parts of things and many times we get parts of parts of things.

Show by a drawing of a square that 1 half of 1 half equals 1 fourth.

For the Pupil:

Here I will show by a circle that 1 half of 1 fifth equals 1 tenth.



$\frac{1}{2}$ of $\frac{1}{5} = \frac{1}{10}$

Show by drawings that—

$\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$

$\frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$

To the Teacher:

Tell the pupil to notice that here we found a part of a part, or we multiplied a fraction by a fraction.

To Multiply Fractions by Fractions:

1. Cancel if possible.

2. Then multiply the numerators for a new numerator and the denominators for a new denominator.

3. Simplify the result.

EXAMPLES.

1. Multiply $\frac{9}{16}$ by $\frac{8}{27}$.

The Right Way:

$$\frac{9}{\underset{2}{16}} \times \frac{8}{\underset{3}{27}} = \frac{1}{6}$$

The Wrong Way:

$$\frac{9}{16} \times \frac{8}{27} = \frac{72}{432} = \frac{1}{6}$$

2. $\frac{9}{10} \times \frac{21}{25} \times \frac{11}{18} \times \frac{62}{12} = ?$

Work:

$$\frac{9}{10} \times \frac{\overset{7}{21}}{25} \times \frac{11}{\underset{2}{18}} \times \frac{\overset{31}{62}}{\underset{4}{12}} = \frac{2387}{1000} = 2\tfrac{387}{1000}$$

3. What is the product of $2\frac{2}{5} \times 3\frac{1}{9} \times 6\frac{3}{10}$?

Work:

$$2\tfrac{2}{5} = \frac{12}{5}. \quad 3\tfrac{1}{9} = \frac{28}{9}. \quad 6\tfrac{3}{10} = \frac{63}{10}.$$

$$\frac{\overset{6}{12}}{5} \times \frac{28}{9} \times \frac{\overset{7}{63}}{\underset{5}{10}} = \frac{1176}{25} = 47\tfrac{1}{25}.$$

Explanation:

Reduce the mixed numbers to improper fractions and then multiply, employing cancellation.

4. Multiply $8\frac{1}{9}$ by 15.

Work:

$$\begin{array}{r} 8\frac{1}{9} \\ 15 \\ \hline 1\frac{2}{3} \\ 120 \\ \hline 121\frac{2}{3} \end{array}$$

Explanation:

Multiply the fraction by the integer, then the integer by the integer, and add the products.

The pupil may explain the work thus:—

(1) Write the mixed number, with the whole number below the integer of the mixed number

(2) $15 \times \frac{1}{9} = \frac{15}{9} = 1\frac{6}{9}$; $1\frac{6}{9} = 1\frac{2}{3}$; Write $1\frac{2}{3}$.

(3) $15 \times 8 = 120$; Write 120 below the 1. Add.

(4) $120 + 1\frac{2}{3} = 121\frac{2}{3}$. Product is $121\frac{2}{3}$.

DIVISION OF FRACTIONS.

TALK:

Present like this:—
How many thirds in 1?
1 divided by 1 third equals what?
1 third is contained in 1 how many times?
1 third is contained in 2 how many times? in 6?
Then, 6 divided by 1 third equals what?
Let us write a few facts, you helping me:—

$1 \div \frac{1}{3} = 3$

$6 \div \frac{1}{3} = 18$

$\frac{6}{3} \div \frac{1}{3}$ *reads the same as 6 yds.* ÷ *1 yd.*

6 yds. ÷ *1 yd. = 6*

$\frac{6}{3} \div \frac{1}{3} = 6$

$\frac{3}{5} \div \frac{1}{5} = ?$

$\frac{3}{5} \div \frac{1}{5}$ *reads the same as 3 yds.* ÷ *1 yd.*

3 yds. ÷ *1 yd. = 3.*

$\frac{3}{5} \div \frac{1}{5} = 3.$

$\frac{5}{6} \div \frac{1}{2}$ *cannot be divided in that form.*

The fractions must have the same name.

$\frac{1}{2} = \frac{6}{12}$; $\frac{5}{6} = \frac{10}{12}$.

$\frac{10}{12} \div \frac{6}{12} = 1\frac{4}{6}$, *or* $1\frac{2}{3}$.

The child should now be able to tell these facts.

ONE WAY TO DIVIDE IN FRACTIONS:

1. The fractions must have a common name, or common denominator, before we can divide.
2. Divide the numerator of the dividend by the numerator of the divisor.
3. Simplify the result.

The preceding manner of dividing in fractions is not the most practical. It is not used in practice.

The Easiest and Best Way to Divide in Fractions:

1. **Invert the divisor, and**
2. **Proceed as in multiplication of fractions.**

Thus, $\frac{2}{3} \div \frac{1}{5} = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3} = 3\frac{1}{3}$.

Why We Invert the Divisor.

$1 \div \frac{1}{3} = \frac{3}{1}$ $\qquad 1 \div \frac{5}{6} = \frac{6}{5}$

$1 \div \frac{1}{5} = \frac{5}{1}$ $\qquad 2 \div \frac{5}{6} = 2 \times \frac{6}{5} = \frac{12}{5}$

$1 \div \frac{2}{9} = \frac{9}{2}$ $\qquad 5 \div \frac{5}{6} = 5 \times \frac{6}{5} = \frac{30}{5}$.

One divided by a fraction is the fraction inverted, or the *reciprocal* of the fraction.

Remember:

Any number larger or smaller than one is so many times (as expressed by that number) larger or smaller than the fraction inverted. This is the reason we invert the divisor.

$1 \div \frac{1}{8} = \frac{8}{1}$ But, $9 \div \frac{1}{8} = 9 \times \frac{8}{1} = \frac{72}{1} = 72$.

$1 \div \frac{2}{3} = \frac{3}{2}$ But, $4 \div \frac{2}{3} = 4 \times \frac{3}{2} = \frac{12}{2} = 6$.

EXAMPLES:

1. Divide 12 by $\frac{3}{4}$.

WORK:

$1 \div \frac{3}{4} = \frac{4}{3}$

$12 \div \frac{3}{4} = 12 \times \frac{4}{3} = \frac{48}{3} = 16$.

In practice we shorten the work thus:—

$12 \div \frac{3}{4} = \overset{4}{12} \times \frac{4}{3} = 16$.

NOTE.—Ask the child again and again to explain why he inverts the terms of the divisor. He should be asked to write an explanation something similar to the one given above, to illustrate the successive steps.

2. Divide $\frac{3}{4}$ by $\frac{5}{6}$.

Work:

$$1 \div \frac{5}{6} = \frac{6}{5}$$

$$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}.$$

In practice the work is shortened thus:—

$$\frac{3}{4} \div \frac{5}{6} = \frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{9}{10}.$$

3. $\frac{2}{9} \div 4 = ?$

Work:

$$1 \div 4 = \frac{1}{4}$$

$$\frac{2}{9} \div 4 = \frac{2}{9} \times \frac{1}{4} = \frac{2}{36} = \frac{1}{18}.$$

SHORTENED PROCESS:

$$\frac{2}{9} \div 4 = \frac{2}{9} \times \frac{1}{4} = \frac{1}{18}.$$

A Good Way to Divide a Mixed Number by an Integer.

Divide $16\frac{2}{3}$ by 11.

Work:

$$\begin{array}{r} 11\,)\,16\frac{2}{3} \\ \hline 1\frac{17}{33} \end{array}$$

First, divide the integer by the divisor, then join the remainder, if any, to the fraction, and divide this sum by the divisor.

"Clearing of Fractions" Method in Long Division.

This is really dividing an integer by a mixed number.

EXAMPLE:

Divide 500 by $33\frac{1}{3}$.

Work:

$$\begin{array}{r} 33\frac{1}{3}\,)\,500 \\ 3 \qquad\quad 3 \\ \hline 100\,)\,1500 \\ \hline 15 \end{array}$$

Multiply both dividend and divisor by the denominator of the fraction, and that clears of fractions. Divide through.

COMPLEX FRACTIONS.

A **complex fraction** is one which has a fraction in either its numerator or denominator, or both. Thus:—

$\dfrac{\frac{1}{2}}{\frac{3}{4}}$, $\dfrac{2\frac{1}{2}}{10}$, $\dfrac{4}{6\frac{2}{3}}$ are all complex fractions.

Complex fractions should not be left complex but should be simplified. The line between merely indicates division, which operation should be performed.

The given complex fractions are read in this way: $\frac{1}{2}$ divided by $\frac{3}{4}$; $2\frac{1}{2}$ divided by 10; 4 divided by $6\frac{2}{3}$.

EXAMPLES.

1. Change $\frac{\frac{1}{2}}{\frac{3}{4}}$ to a simple fraction.

Work:

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{\overset{2}{4}}{3} = \frac{2}{3}.$$

2. Change $\frac{2\frac{1}{2}}{10}$ to simpler form.

Work:

$$2\frac{1}{2} = \frac{5}{2} \qquad \frac{5}{2} \div 10 = \frac{5}{2} \times \frac{1}{\underset{2}{10}} = \frac{1}{4}.$$

To Find What Part One Number Is of Another.

TALK:

Present like this:—

What part of 2 is 1?

1 is what part of 3?

1 day is what part of 30 days?

1 is what part of 10?

3 is what part of 10?

The pupil should be led to see that the number that shows the part is used for the numerator and the number denoting the whole as the denominator. Thus:—

4 is what part of 9? The whole, or 9, is used as the denominator, and 4, which shows the part, as the numerator.

EXAMPLES.

1. 5 is what part of 7?

Work:

$1 = \frac{1}{7}$ of 7.

$5 = 5 \times \frac{1}{7}$ of 7, or $\frac{5}{7}$ of 7.

2. $\frac{3}{4}$ is what part of 9?

Work:

$1 = \frac{1}{9}$ of 9.

$\frac{3}{4} = \frac{3}{4}$ of $\frac{1}{9}$ of 9, or $\frac{1}{12}$ of 9.

Hence, $\frac{3}{4} = \frac{1}{12}$ of 9.

Remember:

1. Use the number denoting the part for the numerator.
2. Use the number showing the whole as the denominator.

MISCELLANEOUS PROBLEMS IN FRACTIONS.

To the Teacher:

Many problems should be given throughout the work in fractions. These may be both oral and written. A few illustrative problems follow:—

PROBLEM:

$\frac{1}{2}$ of a pole is in water and $\frac{1}{3}$ of it is in mud. rest is in the air. What fraction of the whole is the air?

SOLUTION:

In the water and mud there is $\frac{1}{2}$ of the pole $\frac{1}{3}$ of it, or $\frac{5}{6}$ of it. The rest, or $\frac{1}{6}$ of the pole, is in the air.

WORK:

$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$

$\frac{6}{6}-\frac{5}{6}=\frac{1}{6}$

No pencil should be used in such problems.

PROBLEM:

How much silk is there in remnants measuring $6\frac{3}{4}$ yds., $12\frac{2}{3}$ yds., and $7\frac{1}{2}$ yds.?

SOLUTION:

In all there is the sum of $6\frac{3}{4}$ yds., $12\frac{2}{3}$ yds., and $7\frac{1}{2}$ yds., or $26\frac{11}{12}$ yds.

WORK:

$$\begin{array}{rcr} 6\frac{3}{4} & = & 6\frac{9}{12} \\ 12\frac{2}{3} & = & 12\frac{8}{12} \\ 7\frac{1}{2} & = & 7\frac{6}{12} \\ \hline & & 26\frac{11}{12} \end{array}$$

A problem like this ought to be worked mentally.

PROBLEM:

What is the cost of 10 bbls. flour at $4½ per bbl.?

SOLUTION:

The cost is 10 times $4½, or $45.

PROBLEM:

In a quart, liquid measure, there are 57¾ cubic inches. How many cubic inches in a gallon?

SOLUTION:

In a gallon there are 4 times 57¾ cubic inches, or 231 cubic inches.

WORK:

$$\begin{array}{r l} 57\tfrac{3}{4} & \text{cu. in.} \\ 4 & \\ \hline 3 & \\ 228 & \\ \hline 231 & \text{cu. in.} \end{array}$$

NOTE.—Here the work is written merely to indicate the process. It should be done mentally by the pupil.

PROBLEM:

An electric car goes a mile in 7 minutes and 10 seconds. How long will it take to run 8 miles?

SOLUTION:

10 seconds = ⅙ min. To run 8 miles, it will take 8 times 7⅙ min., or 57⅓ min.

PROBLEM:

My house is worth $8000. I insured it for ⅘ of its value. For how much is it insured?

SOLUTION:

It is insured for ⅘ of $8000, or $6400.

WORK:

$$\frac{4}{5} \times \frac{\overset{1600}{\cancel{8000}}}{1} = 6400.$$

PROBLEM:

I have $32\frac{1}{2}$ lbs. of seeds to be put in 1 oz. packages. How many packages will I have?

SOLUTION:

1 lb. will make 16 packages. $32\frac{1}{2}$ lbs. will make $32\frac{1}{2}$ times 16 packages, or 520 packages.

WORK:

$$\begin{array}{r} 32\frac{1}{2} \\ \underline{16} \\ 8 \\ 192 \\ \underline{32} \\ 520 \end{array}$$

PROBLEM:

A train ran $87\frac{1}{2}$ miles in an hour. What was the average speed per minute?

SOLUTION No. 1:

The average speed was as many miles as 60 is contained times in $87\frac{1}{2}$, or $1\frac{11}{24}$.

WORK:

$$87\frac{1}{2} \div 60 = \frac{\overset{35}{\cancel{175}}}{2} \times \frac{1}{\underset{12}{\cancel{60}}} = \frac{35}{24} = 1\frac{11}{24}.$$

SOLUTION No. 2:

The average speed per minute was $\frac{1}{60}$ of $87\frac{1}{2}$ miles, or $1\frac{11}{24}$ miles.

WORK:

$$\frac{1}{60} \text{ of } 87\frac{1}{2} = \frac{1}{\underset{12}{\cancel{60}}} \times \frac{\overset{35}{\cancel{175}}}{2} = \frac{35}{24} = 1\frac{11}{24}.$$

PROBLEM:

The scantling in a woodshed will be placed $1\frac{1}{3}$ ft. apart. If the woodshed is 16 ft. × 24 ft., how many scantling are needed, no allowance being made for the door, windows, or gables?

SOLUTION:

The length of all the walls will be 2 × 16 ft., plus 2 × 24 ft., or 80 ft. It will take as many scantling as $1\frac{1}{3}$ is contained times in 80, or 60.

NOTE.—Illustrate by a diagram.

PROBLEM:

How many scantling will it take for a wall supported at the ends, on one side of a lot 160 ft. long? The scantling are to be 2 ft. apart.

SOLUTION:

It will take as many scantling as 2 is contained times in 160, or 80. One extra is needed to support one end, making 81 in all.

NOTE.—Illustrate by diagram that one extra is needed.

PROBLEM:

A farmer raises $93\frac{3}{4}$ bu. of oats on $3\frac{1}{2}$ acres. What is the average yield per acre?

SOLUTION:

The average yield is as many bushels as $3\frac{1}{2}$ is contained times in $93\frac{3}{4}$, or $26\frac{11}{14}$.

WORK:

$$93\tfrac{3}{4} \div 3\tfrac{1}{2} = \frac{375}{4} \div \frac{7}{2} = \frac{375}{\underset{2}{\cancel{4}}} \times \frac{\cancel{2}}{7} = \frac{375}{14} = 26\tfrac{11}{14}.$$

NOTE.—The second step in the above work need not be written when the pupil is familiar with the process

---

PROBLEM:

A freight train travels $19\frac{5}{8}$ miles an hour. A passenger train on the same road travels $42\frac{1}{4}$ miles per hour. The freight train is 314 miles ahead of the passenger train. How far apart are they in 12 hours?

SOLUTION:

The passenger train gains in 1 hour the difference between $42\frac{1}{4}$ miles and $19\frac{5}{8}$ miles, or $22\frac{7}{12}$ miles. In 12 hours, it gains 12 times $22\frac{7}{12}$ miles, or 271 miles. The distance they are apart then is the difference between 314 miles and 271 miles, or 43 miles.

WORK:

(1) $42\frac{1}{4} - 19\frac{5}{8} = 42\frac{3}{12} - 19\frac{8}{12} = 22\frac{7}{12}$.
(2) $12 \times 22\frac{7}{12} = 271$.
(3) $314 - 271 = 43$.

PROBLEM:

Which is greater, $\frac{2}{3}$ of $\frac{19}{20}$, or $\frac{1}{2}$ of $\frac{58}{70}$, and how much?

SOLUTION:

$$\frac{2}{3} \text{ of } \frac{19}{\underset{10}{20}} = \frac{19}{30}. \quad \frac{1}{2} \text{ of } \frac{\overset{29}{58}}{70} = \frac{29}{70}.$$

Reducing $\frac{19}{30}$ and $\frac{29}{70}$ to a common denominator, we have $\frac{133}{210}$ and $\frac{87}{210}$. $\frac{133}{210}$ is greater than $\frac{87}{210}$. It is $\frac{46}{210}$, or $\frac{23}{105}$ greater.

Hence, $\frac{2}{3}$ of $\frac{19}{20}$ is $\frac{23}{105}$ greater than $\frac{1}{2}$ of $\frac{58}{70}$.

NOTE.—Such problems are not of practical value, but children will find them in their text-books.

REVIEW OF THE PRINCIPLES OF FRACTIONS.

1. A fraction's value is the quotient obtained by dividing the numerator by the denominator.

$\frac{6}{2} = 3.$ 3 is the value of $\frac{6}{2}$.
$\frac{2}{3} = \frac{2}{3}.$ $\frac{2}{3}$ is the value of $\frac{2}{3}$.

2. Multiplying the denominator of a fraction divides the fraction by that number.

$$\frac{1}{2 \times 4} = \frac{1}{8}. \qquad \frac{3}{7 \times 3} = \frac{3}{21}. \qquad \frac{2}{3 \times 9} = \frac{2}{27}.$$

3. Dividing the denominator of a fraction multiplies the fraction by that number.

$$\frac{3}{8 \div 4} = \frac{3}{2}. \qquad \frac{10}{9 \div 3} = \frac{10}{3}. \qquad \frac{3}{10 \div 5} = \frac{3}{2}.$$

4. Multiplying the numerator of a fraction multiplies the fraction by that number.

$$\frac{2 \times 2}{3} = \frac{4}{3}. \qquad \frac{1 \times 8}{9} = \frac{8}{9}. \qquad \frac{5 \times 3}{8} = \frac{15}{8}.$$

5. Dividing the numerator of a fraction divides the fraction by that number.

$$\frac{4 \div 2}{7} = \frac{2}{7}. \qquad \frac{12 \div 12}{16} = \frac{1}{16}. \qquad \frac{3 \div 3}{7} = \frac{1}{7}.$$

6. Multiplying both numerator and denominator of a fraction by the same number does not change the value of the fraction.

$$\frac{1 \times 3}{3 \times 3} = \frac{3}{9} = \frac{1}{3}. \qquad \frac{6 \times 2}{7 \times 2} = \frac{12}{14} = \frac{6}{7}.$$

7. Dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.

$$\frac{12 \div 3}{15 \div 3} = \frac{4}{5} = \frac{12}{15}. \qquad \frac{18 \div 9}{27 \div 9} = \frac{2}{3} = \frac{18}{27}.$$

TALK:

Many practical problems should be given involving the four operations in fractions.

Don't give fool problems like some arithmetics. Make them practical.

These from arithmetics in use are not practical:—

1. A man who weighs 155 lbs. wishes to buy a bicycle that weighs $\frac{1}{7}$ of his weight. What will the bicycle weigh?

No sane man buys a bicycle in that way.

2. The thermometer showed $34\frac{7}{8}$ degrees at 10 A.M. and $12\frac{9}{32}$ degrees at 8 P.M. What was the difference in temperature at the two periods?

Who ever read thirty-seconds of a degree on a thermometer used as this suggests?

PART FIVE.

UNITED STATES MONEY.
DECIMAL FRACTIONS.
PRICE MARKS.
BILLS.

> *"He who is taught to live upon little owes more to his father's wisdom than he that has a great deal left him does to his father's care."*
>
> — *William Penn.*

RULES.

It is just as unpsychological to begin the teaching of arithmetic by a mass of inherited rules, as it is senseless to try to teach language by means of mere rules of speech.

— Tillich.

Whoever would bring his pupils to intelligent computation should develop no rule, but should wait until the children themselves discover it. — Fetzga.

Give me leave to take notice of one thing I think a fault in the ordinary method of education, and that is, the charging of children's memories, upon all occasions, with rules and precepts, which they often do not understand, and constantly as soon forget as given. — Locke.

UNITED STATES MONEY.

Bronze:

One-cent piece.

Silver:

Dollar = $1.00.
Half-Dollar = 0.50.
Quarter-Dollar = 0.25.
Dime = 0.10.

Nickel:

Five-cent piece.

Gold:

Double Eagle = $20.00.
Eagle = 10.00.
Half Eagle = 5.00.
Quarter Eagle = 2.50.

TALK:

United States money has been treated in this book before. The purpose of the previous pages on this subject is to acquaint the pupil with the common U. S. metal coins, that he may become familiar with change.

When dollars and cents are written by the pupil, he is using the decimal notation. When he adds and subtracts dollars and cents, he adds and subtracts decimals. For that reason, the subject of U. S. money may be taken up just before the pupil enters on work in decimals.

Besides the coins spoken of in a previous chapter, there are gold coins representing the values of $2.50, $5, $10, and $20.

Then there is paper money, founded on credit. It represents value, but in itself has no value.

This paper money is made up of paper promises to pay the amounts named, in gold or silver, on demand.

It includes bank bills, U. S. treasury notes, government bonds, etc.

These notes are used in the place of coin, and are preferred in many sections of the country, because they are light, and are more easily carried than either gold or silver. They represent the values $1, $2, $5, $10, $20, $50, $100, $500, $1000.

NOTE.—It is interesting to know that in the eastern part of the U. S., paper money is almost exclusively used for amounts of $1 and above, while on the Pacific coast gold and silver are used in nearly every transaction.

The standard of U. S. money is the gold dollar. Gold is used because in itself it has great worth and little bulk, and because it varies very little in value.

WRITING DOLLARS AND CENTS.

Dollars and cents are written together. Thus, two dollars and sixteen cents is written, $2.16.

The dollars are separated from the cents by a period. If the number of cents is less than ten, the tens' place is filled by a 0. Thus, we write twenty dollars and two cents, $20.02.

Mills, or tenths of a cent, are written to the right of the cents. Five dollars, six cents, four mills is written, $5.064.

Read the period as "and."

Give much drill on reading dollars and cents.

READ:

$102.40 $417.62 $108.09 $49.13 $100.023

WRITE IN FIGURES:

Four hundred two dollars.
Two hundred dollars and forty cents.
Six hundred eight dollars and one cent.

ADDING DOLLARS AND CENTS.

In adding dollars and cents, write the numbers as for ordinary addition, taking care to write the cents under cents, and the dollars under dollars. The numbers may then be added as if the periods were not there.

Add $33.18, $166.02, $133.63.

WORK:

$$\begin{array}{r} \$\ 33.18 \\ 166.02 \\ 133.63 \\ \hline \$332.83 \end{array}$$

SUBTRACTING DOLLARS AND CENTS.

Write the dollars under dollars and cents under cents, and subtract as in simple numbers. Put the period in the remainder to show the cents.

EXAMPLE:

From $402.24 take $157.35.

WORK:

$$\begin{array}{r} \$402.24 \\ 157.35 \\ \hline \$244.89 \end{array}$$

EXAMPLE:

From $170 take $25.45.

WORK:

$$\begin{array}{r} \$170.00 \\ 25.45 \\ \hline \$144.55 \end{array}$$

EXAMPLE:

From $26.45 take $9.

WORK:

$$\begin{array}{r} \$26.45 \\ 9.00 \\ \hline \$17.45 \end{array}$$

MULTIPLYING DOLLARS AND CENTS.

Multiply as in simple numbers, and put a period in the product to show the cents.

EXAMPLE:

Multiply $35.04 by 27.

Work:

$$\begin{array}{r}\$35.04\\ 27\\ \hline 24528\\ 7008\\ \hline \$946.08\end{array}$$

DIVIDING DOLLARS AND CENTS.

Divide as in simple numbers, and put a period in the quotient when you get to the period in the dividend.

EXAMPLE:

Divide $450.25 by 25.

Work:

$$\begin{array}{r l}& \$18.01\\ \cline{2-2} 25 \,) & \$450.25\\ & 25\\ \cline{2-2} & 200\\ & 200\\ \cline{2-2} & 25\\ & 25\\ \cline{2-2}\end{array}$$

HISTORY OF OUR COINAGE.

The American Indians used strings of shell beads, called wampum, as money. If one Indian bought a blanket from another, he gave so many of these shells in exchange.

The colonists at that time used musket balls, furs, pelts, and tobacco as money. To put a price on a horse in the southern colonies was to estimate its worth in so many pounds of tobacco.

In 1652, the colony of Massachusetts decided that lighter and better money be coined, and John Hull was made mint-master. He struck off silver shillings, sixpences, and threepences for the colony, and in pay-

.10
3,00 %

60

.60
.300

ment was allowed to make a certain number for himself.

In 1659, Lord Baltimore caused sixpences and groats to be coined in Maryland. This was the second colony to have a mint.

Florida and Louisiana had mints of their own before they became parts of the United States.

In 1789, the constitution of the United States was ratified, and in it was an article providing for the establishment of a United States mint. No state was to make its own money. The mint was established at Philadelphia in 1792, and a year later was coining gold and silver money.

Other mints have since been established in New Orleans, San Francisco, Carson City, and Denver. Where are these places?

Derivation of Names of United States Money:

It may be interesting to know that the word *dollar* is supposed to have come from *Dale*, the name of a small town where dollars were first coined.

Dime is from the French word *disme*, which means tenth.

Cent comes from the Latin word *centum*, meaning hundred.

Mill is also from the Latin, coming from *mille*, a thousand.

Eagles were named after our national bird.

DECIMAL FRACTIONS.

TALK:

By the time the teacher takes up the actual work with decimal fractions, the children are familiar with writing dollars and cents.

Thus, the children have been using the decimal sys-

tem in a simple way, only they did not call it so. Since our money system is based on the decimal scale, they can perhaps approach the decimal fraction more easily from that direction.

The children know that,

1 cent is one hundredth of a dollar.
10 cents are ten hundredths of a dollar.
25 cents are twenty-five hundredths of a dollar.

Instead of writing the 10 cents as $\frac{10}{100}$ of a dollar, they have been writing the number 10 with a period before it, and the sign $ before the whole, as $4.10. They read this "four dollars and ten cents," which they readily see means the same as, "four and ten-hundredths dollars."

This has made the child familiar with writing simple decimals as far as hundredths, only he has not called them decimals. His work has perhaps included much that is given on the next few pages.

SIMPLE DECIMAL WORK.

Almost as soon as pupils become familiar with easy common fractions, they may be shown that *one tenth* may be written in two ways, thus:—

$\frac{1}{10} = .1$
and $\frac{11}{10}$ may be written 1.1
$\frac{45}{100}$ may be written .45
$\frac{125}{100}$ may be written 1.25.

At first, write only tenths in the two ways, and give plenty of examples until pupils are familiar with them.

Use examples like these:—

$$\begin{array}{r} 4\frac{1}{10} \\ +7\frac{3}{10} \\ \hline 11\frac{4}{10} \end{array} \qquad \begin{array}{r} 4.1 \\ +7.3 \\ \hline 11.4 \end{array} \qquad \begin{array}{r} 6\frac{3}{10} \\ +7\frac{2}{10} \\ \hline 13\frac{5}{10} \end{array} \qquad \begin{array}{r} 6.3 \\ +7.2 \\ \hline 13.5 \end{array} \qquad \begin{array}{r} 26\frac{1}{10} \\ +14\frac{7}{10} \\ \hline 40\frac{8}{10} \end{array} \qquad \begin{array}{r} 26.1 \\ +14.7 \\ \hline 40.8 \end{array}$$

$$\begin{array}{r} 7\frac{7}{10} \\ -\,3\frac{3}{10} \\ \hline 4\frac{4}{10} \end{array} \qquad \begin{array}{r} 7.7 \\ -\,3.3 \\ \hline 4.4 \end{array} \qquad \begin{array}{r} 6\frac{9}{10} \\ -2\frac{2}{10} \\ \hline 4\frac{7}{10} \end{array} \qquad \begin{array}{r} 6.9 \\ -\,2.2 \\ \hline 4.7 \end{array}$$

Pupils will soon prefer to add and subtract such fractions in the decimal way. This helps to drive away the idea that decimal fractions are something to be feared.

Have the children read such expressions as 4.5, in two ways: *four and five-tenths,* and *forty-five tenths.* Show them that both expressions have the same value.

Use many simple examples to make this clear.

They will enjoy examples like these:—

$$\begin{array}{r} 3\frac{1}{10} \\ \times\,2\phantom{\frac{1}{10}} \\ \hline 6\frac{2}{10} \end{array} \qquad \begin{array}{r} 3.1 \\ \times\,2 \\ \hline 6.2 \end{array} \qquad \begin{array}{r} 4\frac{3}{10} \\ \times\,3\phantom{\frac{1}{10}} \\ \hline 12\frac{9}{10} \end{array} \qquad \begin{array}{r} 4.3 \\ \times\,3 \\ \hline 12.9 \end{array} \qquad \begin{array}{r} 7\frac{2}{10} \\ \times\,4\phantom{\frac{1}{10}} \\ \hline 28\frac{8}{10} \end{array} \qquad \begin{array}{r} 7.2 \\ \times\,4 \\ \hline 28.8 \end{array}$$

$$\begin{array}{r} 2\,\underline{)\,4\frac{2}{10}} \\ 2\frac{1}{10} \end{array} \qquad \begin{array}{r} 2\,\underline{)\,4.2} \\ 2.1 \end{array} \qquad \begin{array}{r} 3\,\underline{)\,6\frac{9}{10}} \\ 2\frac{3}{10} \end{array} \qquad \begin{array}{r} 3\,\underline{)\,6.9} \\ 2.3 \end{array} \qquad \begin{array}{r} 2\,\underline{)\,8.6} \\ 4.3 \end{array} \qquad \begin{array}{r} 2\,\underline{)\,8\frac{6}{10}} \\ 4\frac{3}{10} \end{array}$$

The results in all such work should be given from inspection. With a little practice, it can be made a very rapid and pleasant exercise.

Let the pupils copy, and write results for similar examples.

Dictate many numbers to write, using the decimal point, as,

four and five tenths,
forty-five tenths,
two hundred four tenths.

Write numbers in figures and have them read in the two ways mentioned, as,

1.4
.4
.6
7.2
1.8
3.4

NOTE.— The child's mind is not equally active at all times. Some of this work will require more drill than other parts. Give enough. Review often.

When the teacher is teaching hundredths, and the pupil needs more individual attention, help him in this way:—

1. Review his work in dollars and cents.
2. Teach him to write hundredths decimally.

You will have little difficulty if he is asked such questions as,

What part of a dollar is 27 cents? etc.

Write for him $\frac{27}{100}$ in the two ways—

$\frac{27}{100}$ and .27.

Have him write many more, as,

$\frac{8}{100} = .08$
$\frac{19}{100} = .19$
$\frac{50}{100} = .50$

Teach him to read expressions like 3.24 in two ways, thus:—

three and twenty-four hundredths.
three hundred twenty-four hundredths.

Give many of these:—

$$\begin{array}{rrrr} 24.02 & 24\frac{2}{100} & 12.03 & 12\frac{3}{100}\\ +1.03 & +1\frac{3}{100} & +10.24 & +10\frac{24}{100}\\ \hline 25.05 & 25\frac{5}{100} & 22.27 & 22\frac{27}{100}\end{array}$$

$$\begin{array}{rrrr} 16.08 & 16\frac{8}{100} & 27\frac{9}{100} & 27.09\\ -4.03 & -4\frac{3}{100} & -13\frac{3}{100} & -13.03\\ \hline 12.05 & 12\frac{5}{100} & 14\frac{6}{100} & 14.06\end{array}$$

.24 + .11 = ? Read, 24 hundredths plus 11 hundredths are 35 hundredths.

Read in the same way:—

.14 + .09
.35 + .14
.63 + .22

.14 − .07
.39 − .06
.25 − .13

Note.—Let the pupil make others and read them.

.09 × 3 = .27. Read, 9 hundredths multiplied by 3 are 27 hundredths.

Do the same with these and others:—

.14 × 2
.04 × 6
.07 × 3
.11 × 3
.08 × 5
.07 × 9.

.25 ÷ 5 = .05. Read, 25 hundredths divided by 5 are 5 hundredths.

.16 ÷ 4
.27 ÷ 3
.18 ÷ 6
.54 ÷ 9
.32 ÷ 8
.60 ÷ 4.

To write a quotient of two or more decimal places, when the divisor is a whole number and the dividend has more than one decimal place, the pupil need only remember his work in dividing dollars and cents.

The results to those below should be given from inspection. Some pupils are not able to do this, and may be allowed to write part of the work, or enough to determine the answer.

Write results as rapidly as possible for:—

4.34 ÷ 14
14.14 ÷ 14
28.42 ÷ 14
42.56 ÷ 14
1.54 ÷ 14
1.554 ÷ 14
2.94 ÷ 14

31.5 ÷ 15
31.65 ÷ 15
30.15 ÷ 15
45.45 ÷ 15
61.5 ÷ 15
94.5 ÷ 15
79.5 ÷ 15

48.48 ÷ 16
16.64 ÷ 16
97.6 ÷ 16

100.8 ÷ 16
86.4 ÷ 16
48.96 ÷ 16

44.40 ÷ 20
70.20 ÷ 20
46.20 ÷ 20

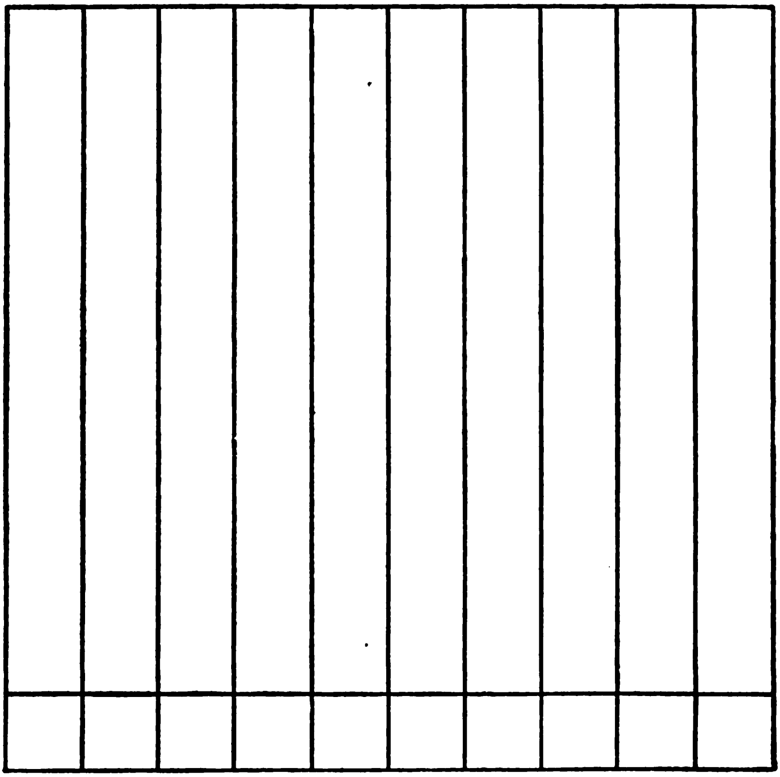
48.24 ÷ 24
72.96 ÷ 24
78.24 ÷ 24

DECIMAL FRACTIONS FORMALLY.

TALK:

The child's work has included the essence of much on the next few pages. They connect his work with the decimal fraction as the decimal system.

Most teachers have some good plan for doing this. Some find one similar to the following good:—



A GOOD PLAN.

Take a piece of cardboard and rule it as shown here, ten strips and ten equal parts to each strip.

Let the pupil have it.

Ask Questions Like These:

How many strips in the large square? (*Pupil.*) 10.

One strip is what part of the large square? (*Pupil.*) $\frac{1}{10}$.

What part of the large square are three strips? (*Pupil.*) $\frac{3}{10}$.

Take out $\frac{6}{10}$ of the large square. What remains? (*Pupil.*) 4 strips, or $\frac{4}{10}$.

Take out $\frac{8}{10}$. How many strips is that? (*Pupil.*) 8 strips.

Take out $\frac{10}{10}$. (*Pupil.*) All of it.

How many small squares in each strip? (*Pupil.*) 10.

In two strips? (*Pupil.*) 20.

In all the large square? (*Pupil.*) 100.

What part of the large square is one of the small squares? (*Pupil.*) $\frac{1}{100}$.

Three of the small squares? (*Pupil.*) $\frac{3}{100}$.

Take out $\frac{10}{100}$ of the large square. (*Pupil.*) 1 strip.

Take out $\frac{1}{10}$ of the large square. (*Pupil.*) 1 strip.

Take out $\frac{1}{10}$ and $\frac{4}{100}$ of the large square. (*Pupil.*) One strip and 4 parts of another.

How many hundredths of the large square in the $\frac{1}{10}$ and $\frac{4}{100}$? (*Pupil.*) $\frac{14}{100}$.

Write the one-tenth in figures. (*Pupil.*) $\frac{1}{10}$.

What is the denominator? (*Pupil.*) 10.

We can write the fraction $\frac{1}{10}$ in another way, like this, .1.

We simply write 1 and put a dot before it, .1.

Now we call it a **decimal fraction.**

We call the dot a **decimal point.** Such fractions as we have been using here we call **decimal fractions,** because their denominators are all ten or a multiple of ten.

Read these decimals: —

.3, .4, .7, .9, .5.

Show how much of the large square each one represents.

Write in decimal fractions:—

six tenths,
eight tenths,
four tenths.

WRITING HUNDREDTHS.

Now pick up four hundredths of the large square. We write that in a decimal fraction this way:—

.04

You see we put a 0 in the place where tenths were above, and write the 4 in the next place to the right.

Read these:—

.07, .06, .02,

How many tenths and how many hundredths in .24?

Read it and call it all hundredths. (*Pupil.*) 24 hundredths.

Illustrate in this way the writing and reading of tenths and hundredths. Continue until it is clear to the pupil.

THOUSANDTHS.

In the same way, if necessary, you may illustrate the decimal fraction as far as thousandths.

Many teachers secure good results without illustrating it in the practical way suggested. Perhaps your child's teacher is one of these.

Teach that the third place to the right of the decimal point is called thousandths.

Write five thousandths for him as a decimal fraction. .005.

Ask him to write seven thousandths as a decimal fraction. .007.

He knew when he took ten hundredths he had one tenth, so now, in the same way, if he takes ten thousandths, he will have one hundredth.

Write ten thousandths for him as a decimal. .010.

Explain that as in common fractions we reduce to lowest terms to simplify, so in decimal fractions, when we get ten parts of one name, we drop that name, and call it one of the next higher order.

So, to write ten thousandths as a decimal simplified, we leave out the last 0, and write it .01. Read, *one hundredth.*

NOTE.—You may say that the 0, when at the right of a decimal fraction, may be omitted, because it has no value. It simply shows a decimal not in its lowest terms. This is true only when it is not followed by another figure.

After drill similar to that outlined here, he is ready to be given the "decimal places," as they are called, and their names.

The first place is **tenths.**
The second place is **hundredths.**
The third place is **thousandths.**
The fourth place is **ten-thousandths.**
The fifth place is **hundred-thousandths.**
The sixth place is **millionths.**
The seventh place is **ten-millionths, etc.**

Illustration of Fraction of Each Order:

.2 - - - two tenths
.02 - - - two hundredths
.002 - - - two thousandths
.0002 - - two ten-thousandths
.00002 - - two hundred-thousandths
.000002 - two millionths
.000000002 - two billionths.

Let the child write other numbers for each place and read them to you.

Explain in a nice way how easy it is in this manner to write fractions that have ten, or some multiple of ten, for a denominator. The figures of the numerator put in the right place and the decimal point, do all.

The same law holds good here that the child found true in writing integers:—

Ten units of any order make one unit of the next higher order.

The place of the right-hand figure in a decimal fraction shows the name of the denominator, as,

.256

The name of the figure to the right in the third place is thousandths, so the denominator is thousandths.

Have the child name the denominators of many decimal fractions, as,

.03
.12
.021
.634
.0413

.3724
.5
.61004
.000203
.7120.

NOTE.—Go over every step taken so far, until you are sure the pupil understands all clearly. If he does, the remainder of the work will be so much easier for him. Be sure that he gets each step.

Go slow and be thorough.

WHOLE NUMBERS WITH DECIMAL FRACTIONS.

The child will have no trouble reading whole numbers with decimal fractions attached to them.

You need only tell him to read the whole number as if there were no decimal fraction, say "and," and read the decimal fraction as if there were no whole number attached to it. The "*and*" connects them.

EXAMPLES.

4.2
Read, *four* and *two tenths.*
5.25
five and *twenty-five hundredths.*

Practice with These:

14.3
25.02
6.342
8.9
785.625
4.0032
24.786

Some teachers have the children read such numbers in two ways:—

14.3
(*a*) one ten, four units, and three tenths.
(*b*) fourteen, and three tenths.

USE OF WORD "AND."

You often hear people read a number like 347 in this way:—

"Three hundred *and* forty-seven."

Why not read it—

"Three hundred forty-seven," which is the correct way.

Omit the "*and*" in every whole number or mere decimal fraction.

EXAMPLES.

696	six hundred ninety-six.
7240	seven thousand two hundred forty.
.206	two hundred six thousandths.

When there is a whole number with a decimal fraction, read "*and*" at the decimal point.

EXAMPLES.

32.4	thirty-two *and* four tenths.
624.112	six hundred twenty-four *and* one hundred twelve thousandths.

Remember:

Call the decimal point "and," in reading numbers having decimal fractions.

With this idea firmly fixed and carried out in practice, the child will know just where the decimal point comes, when a number is read to him.

He is now ready to write numbers with decimal fractions, as well as to read them.

VALUABLE EXERCISES.

Any part of a number may be read by giving the part read the name of its last figure, as, 4.35 is forty-three tenths and five hundredths.

26.143 is two thousand six hundred fourteen hundredths and three thousandths.

Read the following as indicated above:—

3.14 ——— tenths ——— hundredths.
3.256 ——— hundredths ——— thousandths.
143.25 ——— tenths ——— hundredths.

Teach that any number may be read by giving the name of the right-hand figure to the whole number, as,

135 is one hundred thirty-five units.
4.27 is four hundred twenty-seven hundredths.

Read these in that way:—

4.15	7.1342
2.604	.114

A good way to teach this is given here:—

36 = 3 tens + 6 units = 36 units.

2.5 = 2 units + 5 tenths = 25 tenths.

.025 = 2 hundredths + 5 thousandths = 25 thousandths.

3.145 = 3 units + 1 tenth + 4 hundredths + 5 thousandths = 3145 thousandths.

Try many examples like these, using simple ones until the pupil sees what is wanted.

Do this now and the pupil will have less trouble in understanding division of decimals.

Be sure the work is thorough.

Reduction of Decimal Fractions to Common Fractions.

TALK:

This work may be made a pleasure to the pupil. The only feature of it he has not had previously is the connection between the decimal, as such, and its final form as a common fraction in its lowest terms. Call his attention to this.

Have the child take the decimal fraction and after reading it aloud, write it as a common fraction.

This done, let him reduce the new form to lowest terms.

$$.5 = \frac{5}{10} = \frac{1}{2}.$$

He may talk it this way:—

The decimal fraction, *five tenths*, equals the common fraction, *five tenths*. This reduced to lowest terms is *one half*. Give many like the following:—

$$.8 = \frac{8}{10} = \frac{4}{5}$$
$$.25 = \frac{25}{100} = \frac{1}{4}$$
$$.625 = \frac{625}{1000} = \frac{5}{8}.$$

Follow the above work with this:—

Reduce 42.5 to a mixed number.

Have the pupil pay no attention to the whole number. Taking the decimal .5, he treats it as in the previous exercise. It equals $\frac{1}{2}$. Now add it to the whole number. The result is $42\frac{1}{2}$.

EXAMPLES.

1. $4.7 =$

$.7 = \frac{7}{10}$

$4 + \frac{7}{10} = 4\frac{7}{10}.$

2. $5.25 =$

$.25 = \frac{25}{100} = \frac{1}{4}$

$5 + \frac{1}{4} = 5\frac{1}{4}.$

Practice on these:—

1.2 4.3

25.45 18.05

4.045 6.125

Give practice on such forms as,

$$.37\tfrac{1}{2} = \frac{87\frac{1}{2}}{100} = \frac{75}{200} = \tfrac{3}{8}.$$

$$.33\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{100}{300} = \tfrac{1}{3}.$$

$$.16\tfrac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{50}{300} = \tfrac{1}{6}.$$

$$.83\tfrac{1}{3} = \frac{83\frac{1}{3}}{100} = \frac{250}{300} = \tfrac{5}{6}.$$

$$.36\tfrac{7}{8} = \frac{36\frac{7}{8}}{100} = \frac{295}{800} = \tfrac{59}{160}.$$

Annexing 0's to the Right of a Decimal.

We can annex as many 0's to the right of a decimal as we please, and the form, but not the value is changed.

Have the pupil state a principle now, in his own words.

Illustrate in this way:—

$.4 = \frac{4}{10}$ $\qquad$ $.7 = \frac{7}{10}$

$.40 = \frac{40}{100} = \frac{4}{10}$ $\qquad$ $.70 = \frac{70}{100} = \frac{7}{10}$

$.400 = \frac{400}{1000} = \frac{40}{100} = \frac{4}{10}.$ $\qquad$ $.700 = \frac{700}{1000} = \frac{70}{100} = \frac{7}{10}.$

NOTE.—Do as much of this work by inspection, as possible. Don't, however, look for the impossible in oral work, as some teachers do.

Reducing Decimals to a Common Denominator.

At first, the pupil need only know that he may do this by—

Annexing 0's to the right of each decimal until all have the same number of decimal figures.

EXAMPLE:

Reduce .5, .04, and .025 to a common denominator.

.025
.500
.040

Simply write the number with most decimal figures first. Write the others under it and fill in with 0's, as shown.

The pupil from this can readily see that 0's annexed to the right of a decimal do not change its value.

ADDITION OF DECIMALS.

There is just one thing to keep in mind here and that is:—

Write the numbers so that the decimal points are in a column.

This is not even new to the child, for his work with United States money has made him familiar with it.

EXAMPLE:

Add 3.702, 2.3, 143.275, 60.756, 73.0046, 2736, .0042.

Work:

$$\begin{array}{r@{.}l} 3 & 702 \\ 2 & 3 \\ 143 & 275 \\ 60 & 756 \\ 73 & 0046 \\ 2736 & \\ & 0042 \\ \hline 3019 & 0418 \end{array}$$

Explanation:

Write the numbers so that the decimal points form a column, and add as in whole numbers.

Note.—Since the 0's in an addition do not affect the result, there is nothing new in the addition.

It is an easy matter to explain to the child, or let him explain, why the numbers should be written with the decimal points in a column. It places units of the same value in a column.

SUBTRACTION OF DECIMALS.

Remember:

Write the numbers so that the decimal point of the subtrahend is directly under the decimal point in the minuend.

Then subtract as in integers, placing a decimal point in the remainder, directly under the decimal points above.

EXAMPLE:

From 42.375 take 5.025.

WORK:

$$\begin{array}{r} 42.375 \\ -5.025 \\ \hline 37.350 \end{array}$$

For convenience, at first, it is well to write the decimals with a common denominator, before subtracting.

EXAMPLE:

From 7.5 take .002.

WORK:

$$\begin{array}{r} 7.500 \\ -\ .002 \\ \hline 7.498 \end{array}$$

Write a number of examples in this way, until the pupil becomes familiar with the principle. After that, he need not actually write the 0's to make a common denominator. He may imagine them written.

EXAMPLES.

1. From 2.753 take 1.4.

WORK:

$$\begin{array}{l} 2.753 \\ -1.4 \\ \hline 1.353 \end{array}$$

2. From 7.3 take 2.003.

WORK:

$$\begin{array}{l} 7.3 \\ -2.003 \\ \hline 5.297 \end{array}$$

MULTIPLICATION OF DECIMALS.

Perhaps the simplest way to help the child to know the place of the decimal point in the product, is the one given here:—

Ask him to find the product of the common fractions, $\frac{5}{10}$ and $\frac{3}{10}$.

$$\frac{5}{10} \times \frac{3}{10} = \frac{15}{100}.$$

Now, let him write the fractions as decimals, and multiply. He gets 15. He will see at once where to put the decimal point to make it hundredths, .15.

Give other examples like:—

$$1.\ \begin{cases}\frac{3}{10}\times\frac{7}{10}=\frac{21}{100}.\\ .3\times .7=.21.\end{cases}$$

$$2.\ \begin{cases}\frac{9}{10}\times\frac{5}{10}=\frac{45}{100}.\\ .9\times .5=.45.\end{cases}$$

.07
3
021

Then give some like these:—

$$\frac{3}{10}\times\frac{7}{100}=\frac{21}{1000}.$$
$$.3\times .07=.021.$$

He will see that, in order to write the 21 as a decimal fraction, whose denominator is thousandths, he must put a 0 before the 2, and the decimal point before that.

Give many like these:—

$$1.\ \begin{cases}\frac{7}{100}\times\frac{7}{10}=\frac{49}{1000}.\\ .07\times .7=.049.\end{cases}$$

$$2.\ \begin{cases}\frac{3}{10}\times\frac{5}{100}=\frac{15}{1000}.\\ .3\times .05=.015.\end{cases}$$

$$3.\ \begin{cases}\frac{13}{100}\times\frac{9}{100}=\frac{117}{10000}.\\ .13\times .09=.0117.\end{cases}$$

After drill on these, the pupil is ready to tell you, or to be told (it matters little, as he is ready to remember it), that—

The product always has as many decimal places as both the multiplicand and multiplier.

Another Way:

For those who prefer it, the reason for the position of the decimal point in the product may be explained as in the following:—

EXAMPLE:

Multiply 27.3 by 14.

Work:

$$\begin{array}{r} 27.3 \\ \times\ 14 \\ \hline 109.2 \\ 273.0 \\ \hline 382.2 \end{array}$$

Explanation:

We multiply the 3 tenths by 4 and get 12 tenths, or, 1 and 2 tenths. We write the 2 tenths. Multiplying 7 by 4, we get 28; 28 + 1 = 29. Write the 9. Multiplying 2 by 4, we get 8; 8 + 2 = 10. Write the 10.

Multiplying the 3 tenths by 10, we get 30 tenths; 30 tenths = 3, and 0 tenths. Write 0 in tenths' place. Multiplying 7 by 10, we get 70. 70 + 3 = 73. Write 3. Multiplying 2 by 10, we get 20. 20 + 7 = 27. Write 27.

After adding, we find we have 1 decimal place in the product.

By using a few examples like this, the truth of the fact regarding the place of the decimal point in the product, may be established.

Note.—The explanation by means of the use of both common fractions and decimals, is to be preferred in most cases. Older and more matured pupils may grasp the latter plan. Young pupils will have difficulty with it for a time.

EXAMPLES.

1. $.025 \times .4 = ?$

Work:

$$\begin{array}{r} .025 \\ \times\ .4 \\ \hline .01\cancel{00} \end{array}$$

2. $4.25 \times 1.9. = ?$

Work:

$$\begin{array}{r} 4.25 \\ \times\ 1.9 \\ \hline 3825 \\ 425 \\ \hline 8.075 \end{array}$$

3. $14.02 \times 6.13 = ?$

Work:

$$\begin{array}{r} 14.02 \\ \times\ 6.13 \\ \hline 42\,06 \\ 140\,2 \\ 8412 \\ \hline 85.9426 \end{array}$$

4. $.007 \times .002 = ?$

Work:

$$\begin{array}{r} .007 \\ \times\ .002 \\ \hline .000014 \end{array}$$

Note.— Work like the last may be done by inspection.

Multiplying by 10, 100, 1000, etc.

Give the pupil a few examples to write and work like these:—

ILLUSTRATIONS:

3.2 × 10 = 32.0.
4.25 × 10 = 42.5.
63.42 × 10 = 634.2.
734.5 × 10 = 7345.

He will soon see that in each product he has simply rewritten the multiplicand with the decimal point one place farther to the right.

Give him examples like these:—

34.4 × 100 = 3440.
3.145 × 100 = 314.5.
.2 × 100 = 20.0.
6.0041 × 100 = 600.41.
70.91 × 100 = 7091.

Then these:—

4.3 × 1000 = 4300.0.
7.156 × 1000 = 7156.
6.0402 × 1000 = 6040.2.
96.145 × 1000 = 96145.

It is not long until he discovers that—

1. *To multiply a decimal by 10,*
   Remove the decimal point one place to the right.
2. *To multiply a decimal by 100,*
   Remove the decimal point two places to the right.
3. *To multiply a decimal by 1000,*
   Remove the decimal point three places to the right.

In General:

To multiply a decimal by 10, or any power of 10,

Remove the decimal point to the right in the multiplicand as many places as there are 0's in the multiplier.

Multiplying by Any Multiple of 10.

Let the pupil write the work for a number of these:—

$$
\begin{aligned}
3.4 \times 20 &= 68.0.\\
3.4 \times 200 &= 680.0.\\
3.4 \times 2000 &= 6800.0.\\
2.7 \times 30 &= 81.0.\\
2.7 \times 300 &= 810.0.\\
2.7 \times 3000 &= 8100.0.
\end{aligned}
$$

He will soon discover that—

To multiply a number by any multiple of 10,

1. Remove the decimal point in the multiplicand as many places to the right as there are 0's in the multiplier.

2. Multiply the result by the number to the left of the 0's in the multiplier.

EXAMPLE:

$$4.2 \times 300 = 420 \times 3 = 1260.$$

EXPLANATION:

There are two 0's in the multiplier.

Remove the decimal point two places to the right,— the result is 420.

Multiply by 3, the number before the 0's in the multiplier, $420 \times 3 = 1260$.

DIVISION OF DECIMALS.

TALK:

Briefly review multiplication and division of dollars and cents.

$2.50 × 4 is what? (Pupil.) $10.00.

$10.00 ÷ 4 is what? (Pupil.) $2.50.

The pupil knows that the dividend is equal to the product of the divisor by the quotient.

He also knows that in multiplication of decimals the product has as many decimal places as both the multiplicand and multiplier.

Hence, it follows that the dividend must have as many as both divisor and quotient.

Have the pupil prove this.

Remember:

1. Before beginning the division, the dividend must have at least as many decimal places as the divisor.

2. If the dividend lacks in the number of decimal places, annex 0's until the number of places in both is the same.

3. Divide as in whole numbers, paying no attention to 0's that may be on the left of the dividend or divisor.

4. After dividing, if the number of decimal places used in the dividend is the same as the number used in the divisor, the quotient is an integer.

5. After dividing, if the number of decimal places used in the dividend is greater than the number used in the divisor, the number of decimal places in the quotient must equal that excess.

1st Method — The Ordinary Plan.

EXAMPLE:

$25.45 \div 5 = ?$

WORK:

$$\begin{array}{r} 5\,)\underline{25.45} \\ 5.09 \end{array}$$

EXPLANATION:

The dividend has two decimal places. The divisor has none. Hence, the quotient must have two, that the number in divisor and quotient may equal those in the dividend.

Give examples with no decimal point in the divisor.

The pupil will discover that—

When the divisor is a whole number, we write the decimal point in the quotient as soon as we come to it in the dividend.

EXAMPLES.

1. $\begin{array}{r} 6)\underline{7.02} \\ 1.17 \end{array}$

2. $\begin{array}{r} 12\,)\underline{.4812} \\ .0401 \end{array}$

```
          4.2                   32.22
3.  16)67.2          4.  25)805.50
       64                   75
       --                   --
       32                   55
       32                   50
       --                   --
                             55
                             50
                             --
                              50
                              50
                              --
```

WHERE BOTH DIVIDEND AND DIVISOR HAVE AN EQUAL NUMBER OF DECIMAL PLACES.

Give examples where the number of places in dividend and divisor are equal, as:—

```
        123
.25)30.75
     25
     --
      57
      50
      --
       75
       75
       --
```

Here the pupil knows that since the divisor has as many decimal places as the dividend, there will be none in the quotient. He knows this from his work in multiplication. Hence, with such examples, he has no difficulty.

Give practice examples to make sure of this.

2d Method—Making the Divisor a Whole Number.

Some teachers write examples that seem difficult in such a way that the decimal point is removed altogether from the divisor. They then point off as in the first examples given in division here.

TO ILLUSTRATE:

$$42.346 \div 1.24 = ?$$

The pupil sees by inspection that if he multiplies the divisor above by 100, he removes the decimal point. Since he must treat the dividend in the same way to keep the value unchanged, we have—

$$4234.6 \div 124$$

Work:

$$\begin{array}{r} 34.1 \\ 124\,\overline{)\,4234.6} \\ 372 \\ \hline 514 \\ 496 \\ \hline 186 \\ 124 \\ \hline 62 \end{array}$$

Note.—This removes all difficulty in such cases.

3d Method—The Check Plan.

The result in the preceding case is also accomplished by counting to the right from the decimal point in the dividend, as many places as there are decimal places in the divisor, and inserting a check mark there to indicate when the decimal point is to be placed in the quotient.

EXAMPLES.

1. $44.263 \div 1.25 = ?$

Work:

$$\begin{array}{r} 35.4 \\ 1.25\,\overline{)\,44.26\checkmark 3} \\ 3\,75 \\ \hline 676 \\ 625 \\ \hline 513 \\ 500 \\ \hline 13 \end{array}$$

2. $3.1416 \div .2734 = ?$

Work:

$$\begin{array}{r} 11 \\ .2734\,\overline{)\,3.1416\checkmark} \\ 2\,734 \\ \hline 4076 \\ 2734 \\ \hline 1342 \end{array}$$

If the number of decimal places in the dividend is less than the number in the divisor, annex 0's to the dividend until it has as many places as there are in the divisor.

EXAMPLES.

1. 124.2 ÷ 3.725 = ?

Work:

```
               33
    3.725 ) 124.200✓
            111 75
            ------
             12 450
             11 175
             ------
              1 275
```

2. 2 ÷ .1204 = ?

Work:

```
               16
    .1204 ) 2.0000✓
             1 204
            ------
              7960
              7224
              ----
               736
```

In the last example explain, or let the pupil explain, the value of 2.0000. Show that the number is still read 2. That the 0's add nothing to its value, but are merely a convenience for the operation of division.

Note.—In the work for the two preceding examples, the line separating the quotient from the dividend is omitted. For pupils who thoroughly understand division it is not needed.

ADDITIONAL EXAMPLES.

1. Divide .79586 by .13.

Work:

```
        6.122
     ---------
.13 ) .79✓586
       78
       --
        15
        13
        --
         28
         26
         --
          26
          26
          --
```

Explanation:

By the Check Plan, the decimal point is readily placed in the quotient here.

By the Ordinary Plan, we say the difference in the number of decimal places in the dividend and divisor is 3. Hence, the quotient has 3 decimal places.

Work:

2. $.000242 \div .11 = ?$

$$\begin{array}{r} .11\,\underline{)\,.00'0242} \\ .0022 \end{array}$$

The Check Plan places the decimal point in the quotient at once.

By the other plan, when the division is performed, paying no attention to the 0's on the left of the dividend, the quotient is 22. Since there are four more decimal places in the dividend than in the divisor, we prefix two 0's to the 22 and place the decimal point.

Note.—The Check Plan is to be preferred. It saves time and avoids confusion.

4th Method—The Austrian Plan.

The Austrian Plan is an improvement on the 2d Method given in this chapter for pupils beginning division of fractions.

The divisor is made a whole number, before division is attempted, and the entire remainder is brought down each time with the decimal point preserved.

This is a very good method to follow until pupils become familiar with handling decimal fractions. Then the "bringing down" of the entire remainder each time can be dispensed with.

EXAMPLE:

$10.912 \div 3.2 = ?$

Work:

$$3.2\overline{)10.912}$$

$$\begin{array}{r} 3.41 \\ 32\overline{)109.12} \\ 96 \\ \hline 13.12 \\ 12.8 \\ \hline .32 \\ .32 \\ \hline \end{array}$$

Note.—In the quotient, the child should place the decimal point directly above where he finds it in the dividend.

THE REMAINDER.

By annexing 0's to the dividend, the quotient may be carried to any number of decimal places, either until the remainder disappears, or becomes too small to be considered.

EXAMPLES.

1. Divide 2.413 by 26 to five decimal places.

Work:

```
      .09280+
  26)2.√41300
     2 34
     ----
       73
       52
       --
       210
       208
       ---
         20
```

2. 275.31 ÷ 4.25 to three decimal places.

Work:

```
          64.778
  4.25)275.31√000
       255 0
       -----
        20 31
        17 00
        -----
         3 310
         2 975
         -----
          3350
          2975
          ----
           3750
           3400
           ----
            350
```

3. 15 ÷ 25 = ?

Work:

```
         .6
  25)15√0
     15 0
     ----
```

4. 243 ÷ 7256 to four decimal places.

Work:

```
          .0334
  7256)243√000
        21768
        -----
         25320
         21768
         -----
          35520
          29024
          -----
           6496
```

Dividing by 10, 100, 1000, etc.

Have the pupil write the work for a number of examples like these: —

42.5 ÷ 10 = 4.25	715.4 ÷ 10 = 71.54
42.5 ÷ 100 = .425	715.4 ÷ 100 = 7.154
42.5 ÷ 1000 = .0425	715.4 ÷ 1000 = .7154

He readily discovers that —

1. To divide by 10,

Remove the decimal point one place to the left.

2. To divide by 100,

Remove the decimal point two places to the left.

3. To divide by 1000,

Remove the decimal point three places to the left, etc.

In General:

Remove the decimal point as many places to the left as there are 0's in the divisor.

Dividing by a Multiple of 10, 100, 1000, etc.

If we divide by 20, it is the same as if we divide by 10 and then by 2.

If we divide by 300, it is the same as if we divide by 100 and then by 3.

The preceding general rule states how to divide by 10, 100, 1000, etc., now we will state another for multiples of 10, 100, 1000, etc.

To divide by any multiple of 10, 100, 1000, etc.,

Remove the decimal point as many places to the left as there are 0's in the divisor, and divide by the number on the left of the 0's in the divisor.

EXAMPLES.

WORK:

1. Divide 24.3 by 20.

$$2\,\overline{)\,2.43}$$
$$1.215$$

EXPLANATION:

There is one 0 in the divisor. Removing the decimal point one place to the left in the dividend, we have 2.43. Divide by the number on the left of the 0 in the divisor, which is 2, and the quotient is 1.215.

2. Divide 423.4 by 300.

Work:

$$\begin{array}{r} 3\,\underline{)\,4.234} \\ 1.411\tfrac{1}{3} \end{array}$$

3. 7296.45 ÷ 7000 = ?

Work:

$$\begin{array}{r} 7\,\underline{)\,7.29645} \\ 1.04235 \end{array}$$

Remark:

These "short cuts," as they are often called, are very simple and can be used to advantage in the work with decimals.

They are time savers and practical.

Reducing Common Fractions to Decimal Fractions.

The pupil is familiar with division of decimals. If you ask him now to divide 3 by 4 decimally, he writes the work thus:—

$$\begin{array}{r} 4\,\underline{)\,3.00} \\ .75 \end{array}$$

Now ask the pupil to write as a common fraction, 3 ÷ 4. He at one writes $\frac{3}{4}$ from his previous knowledge.

Ask him to write the decimal fraction .75 as a common fraction and reduce it to its lowest terms. He writes:—

$$.75 = \tfrac{75}{100} = \tfrac{3}{4}.$$

Have him treat in a similar manner the following:

1 ÷ 2 3 ÷ 8 2 ÷ 4

3 ÷ 6 7 ÷ 8 7 ÷ 12

From this work he is led to see that:—

To reduce a simple fraction to a decimal fraction,

Divide the numerator by the denominator, treating both as integers, and annex 0's to the dividend until the desired quotient is found.

EXAMPLES.

1. $\frac{7}{8}$ = what decimal fraction?

$$\begin{array}{r} 8\,)\,7.000 \\ \hline .875 \end{array}$$

2. $\frac{1}{2}$ = what decimal fraction?

$$\begin{array}{r} 2\,)\,1.0 \\ \hline .5 \end{array}$$

3. $\frac{7}{12}$ = what decimal fraction?

$$\begin{array}{r} 12\,)\,7.0000 \\ \hline .5833 \end{array}$$

The pupil notices that he cannot reduce $\frac{7}{12}$ to an exact decimal fraction, because he always has the remainder 4.

Give enough examples like the three above to fix the idea firmly with the pupil.

Reducing Mixed Numbers to Decimals.

There is nothing new for the pupil here. He need only be told to take the fractional part of the number and divide as in the preceding case. Then add the decimal fraction found to the whole number — the integer.

EXAMPLE:

Reduce $45\frac{3}{4}$ to a decimal.

$$\frac{3}{4} = .75$$
$$45 + .75 = 45.75.$$

Remember:

$\frac{1}{2} = .5$
$\frac{1}{4} = .25$
$\frac{3}{4} = .75$
$\frac{7}{8} = .875$
$\frac{1}{3}$ = the repetend .333, etc.
$\frac{2}{3}$ = the repetend .666, etc.

CIRCULATING DECIMALS.

The pupil has already discovered that some commom fractions cannot be changed to exact decimal fractions, as—

$\frac{1}{3}$ = .33333 on to infinity.
$\frac{2}{3}$ = .66666 on to infinity.
$\frac{7}{33}$ = .212121, etc.

These decimals are known as **Circulates, Recurring** or **Circulating** decimals.

The part which recurs is called the **Repetend.**

This is marked by putting a dot over the first and last figures of it. For instance, if we write the 21 in the last case above, this way: $\dot{2}\dot{1}$, it indicates that, if written out, the result would be 21212121, etc., on to infinity.

Where a circulating decimal occurs in work, it is best to reduce it to a common fraction. If need be, it may be expressed in the result, as a circulate to any number of decimal places.

PROBLEMS IN DECIMALS.

TALK:

Since problems involving decimals have but one new feature, the use of the decimal point, there is no different reasoning process presented in them from that in problems involving the fundamental operations.

In consequence, the only difficulty that may present itself, if the early work has been properly done, is with reference to the use of the decimal point.

For illustration and practice a few problems are, however, added.

PROBLEM:

A city has parks with these areas: 4.36 A., 7.5 A., 18.625 A., and 7.03 A. What is the total area of its parks?

Work:

$$\begin{array}{r@{.}l l}
4 & 36 & \text{A.} \\
7 & 5 & \\
18 & 625 & \\
7 & 03 & \\
\hline
37 & 515 & \text{A.}
\end{array}$$

Explanation:

The problem is one in addition. The decimal points should form a column. Add.

PROBLEM:

The length of a second's pendulum is 39.1392 inches and a meter is 39.371 inches. What is the difference in their lengths?

Work:

39.371
39.1392
.2318

Explanation:

The problem is one in subtraction. Inspection shows by the first decimal figure that the meter is the longer, so we write it for the minuend. Write with the decimal points in a column, and subtract as in integers.

PROBLEM:

A merchant sold cloth which cost him 3.5¢ a yard, for 5¢. What was his profit on 350 yds?

Work:

5¢ − 3.5¢ = 1.5¢.
1.5¢
3 50
7 50
45
525.0¢ or $5.25.

Solution:

On one yard his profit was the difference between 5¢ and 3.5¢, or, 1.5¢. On 350 yards, his profit was 350 × 1.5¢, or $5.25.

PROBLEM:

In 1900 the population of all Alaska was 63,592. The population of Chicago at the same time was 1,698,575. Find in a decimal fraction to four places, the part the population of Alaska was of the population of Chicago.

$$\frac{63592}{1698575} =$$

.0374 +
1698575) 63592ʻ0000
5095725
12634750
11890025
7447250
6794300
652950

Explanation:

Place the check mark in the dividend. Divide. The quotient is .0374+.

Note.—The sign + after the 4 indicates that there is still a remainder, so the decimal fraction is not exact.

SHORT SUMMARY OF DECIMALS.

A decimal fraction is a fraction whose denominator is ten or some power of ten.

A decimal point is a dot used to locate units.

A decimal is a decimal fraction whose denominator is shown by the number of places it ends to the right of the decimal point. Only the numerator is written.

A complex decimal is a decimal with a common fraction at the right, as, .12½.

A mixed decimal is a whole number with a decimal fraction to its right, as, 34.5.

WRITING DECIMALS.

Write the numerator. From the right, point off as many decimal places as there are 0's in the denominator. Prefix 0's to the numerator, if needed.

READING DECIMALS.

Read the numerator as a simple number and call the whole by the name of the right-hand order.

To reduce decimals to a common denominator,

Annex 0's until their decimal places are equal.

To reduce common fractions to decimals,

Annex decimal 0's to the right of the numerator and divide by the denominator.

To reduce a decimal to a common fraction,

Write the figures of the decimal as a numerator, with 1 and as many 0's as places in the decimal, for the denominator. Reduce to lowest terms.

Principles:

1. Annexing 0's to the right of a decimal, or removing them therefrom, does not change its value.

2. Taking 0's from the left of a decimal increases its value tenfold for each 0 removed.

3. Prefixing 0's to a decimal diminishes its value to one tenth for each 0 prefixed.

PRICE MARKS.

Business men often mark goods in such a way that only themselves and their clerks know the cost and selling price. To do this, they use some word or phrase containing ten letters, each representing a figure, as—

I	m	p	o	r	t	a	n	c	e
1	2	3	4	5	6	7	8	9	0

If the cost of a pair of shoes, for instance, is $2.10, the merchant marks them *mie.* If he wishes to sell them for $3.25, he might mark the selling price *pmr.* For quick comparison they would be written on the card attached to the goods in this way:—

$$\frac{mie}{pmr}.$$

To prevent the use of the same letter twice in succession, an extra letter, or *repeater*, is used. Any letter not found in the key may be used as a repeater.

If the cost price were $4.40, it could be written *ode,* using d as the repeater.

Any word or phrase containing ten letters, no two of which are the same, may be used as a key.

A few examples are:—

Charleston.
Vanderbilt.
Cumberland.

For the Pupil to Do:

1. Write the cost and selling price for several articles, using one of the above as a key.
2. Invent a new key.

3. Notice cost and selling marks on goods bought for the home.

BILLS.

TALK:

The pupil, having learned to use decimals, will have no trouble in performing any of the operations necessary in the work with bills.

Work with bills is practical. Give practice with actual bills presented to you. Let the pupil find if they are correct.

WHAT A BILL IS.

A **bill** is a written statement of goods sold, or services performed, giving the names of the parties, date or dates, items, cost of each, and total cost.

The **debtor** is the one owing.

The **creditor** is the one owed.

Chicago, Ill., December 20, 1902.

Mr. A. L. Grant,

To H. D. Diller, Dr.

1902				
Oct.	*20*	*To 200 lbs. sugar @ 4½¢*	*$ 9*	*00*
"	"	" *60 bbls. flour @ $4.75*	*285*	*00*
Nov.	*12*	" *100 lbs. tea. @ 44¢*	*44*	*00*
"	*14*	" *60 T. coal @ $6.25*	*375*	*00*
			$713	*00*

First, find the value of 200 lbs. of sugar @ 4½¢ a pound and write it opposite, — $9.00. Do so with each item and add up the whole. Write a similar bill and find the amount.

A RECEIPTED BILL.

New York, January 2, 1903.

Roy J. Hutson,

To G. K. Beach & Co., Dr.

1902						
Nov.	*15*	*To 124 sets Students' Work @ $6.50*	*$806*	*00*		
"	*22*	" *72 sets Students' Work @ $6.50*	*468*	*00*		
Dec.	*8*	" *142 copies Zill's Cyclopedia @ $4.37½*	*621*	*25*		
					$1895	*25*
		Cr.				
Dec.	*5*	*By Cash*	*190*	*00*		
"	*30*	" "	*700*		*890*	
					1005	*25*

Received Payment

G. K. Beach & Co.
By L. D.

January 10, 1903.

A **bill** is receipted when paid, by writing "paid" or "received payment" at the bottom or across the face of it, with the signature of the creditor beneath it.

If some person authorized, as a clerk, writes the signature, he writes just below it "per" or "by" and his initials, as shown on last bill here.

Here is a bill to be filled in and receipted by a clerk. Fill it in and receipt it.

Milwaukee, Wis., Jan. 12, 1903.

Lyman Allis,

Bought of Herbert F. John.

1903				
Jan.	*4*	*2500 ft. pine boards @ $14 per M.*		
"	"	*6000 shingles @ $2.50 per M.*		
"	"	*9250 ft. sills @ $23.50 per M*		

NOTE.—Let the child make out bills of various articles and receipt them. Make this work practical.

A CASH ACCOUNT

What It Is:

A **cash account** is a statement or record of money taken in and paid out. It has columns at the left for the date and double money columns at the right. "Cash" is written over the middle.

Over the column for money paid, **Cr.** is written for **creditor.** Over the column for money taken in, **Dr. for debtor,** is written.

Below is a sample *cash account.*

1903		*Cash*	*Dr.*		*Cr.*	
Jan.	*1*	*Cash on hand*	*$400*	*20*		
"	*"*	*Received check*	*50*	*00*		
		Amount on hand			*$450*	*20*
			450	*20*	*450*	*20*
Jan.	*2*	*Balance brought down*	*450*	*20*		
"	*"*	*Received cash*	*7*	*50*		
"	*"*	*Paid tailor's account*			*75*	*50*
"	*"*	*Paid newspaper bill*			*3*	*25*
		Balance on hand			*378*	*95*
"	*3*	*Balance brought down*	*$378*	*95*		

They are simple and easily kept. Teach the child to keep a cash account of his own.

OTHER FORMS.

An **account sales** is a detailed statement which commission men render to merchants. It shows the sales of goods, the charges for attending to same, and the proceeds.

An **account purchase** is a detailed statement sent by a commission man to his principal. It shows the cost of goods bought, and the expense incurred in attending to the purchase.

ACCOUNT SALES.

New York, Feb. 1, 1903.

Sold for account of H. Fetzer Co.,

By L. M. Washburn Co.

1903 *Jan.*	*4*	*100 bbls. Golden Rule @ $4.50*				
"	*10*	*230 bbls. Perfection @ $4.75*				
"	*12*	*300 bbls. Golden Rule @ $4.60*				
"	*15*	*270 bbls. Snow Flake @ $4.90*			***	**
		Charges				
Jan.	*3*	*Transportation 420 bbls. @ 30¢*				
"	*3*	*Cartage 420 bbls. @ 4¢*			***	**

Have the pupil copy this account sales and make the extensions indicated to complete it.

ACCOUNT PURCHASE.

Kansas City, Mo., Jan. 1, 1903.

Purchased by F. I. Pleck & Co.,

For account and risk of F. Ridings & Sons.

	10	*bbls. pork @ $8.10*				
	40	*bu. clover seed @ $7.80*				
	39	*bu. timothy seed @ $1.50*				
	40	*tubs lard @ $3.00*				
	80	*tubs butter @ $12.00*			***	***
		Charges				
		Cartage	*$3*	*25*	**	**

Let the pupil copy and complete this. He may make out a similar one and complete it.

PART SIX.

DENOMINATE NUMBERS.

PRACTICAL ESTIMATES.

"No young lady could have a better safeguard against adversities of fortune, or a better resource in time of need, than a good knowledge of business affairs."

—Harriet Beecher Stowe.

PRACTICAL ARITHMETIC

IN COUNTRY SCHOOLS.

THERE can be no work properly done upon the farm without measuring. Most of this measuring is done by what is called "rule of thumb," or so-called practical judgment. The farmer estimates weight of cattle, hogs, or sheep by sight. He can tell how much cord wood or timber a certain area of forest will produce. In fact, measuring in everything he does is absolutely essential. There is no better way for the teacher to study the processes of measuring, or arithmetic, than to inquire into the everyday demands of farm work, and no better way to teach arithmetic than to bring the measuring necessary for farm work into the schoolroom. The elementary work, and the work that ought to be continued throughout the course, should be largely estimation with eye and hand, of length, of distance, area, volume, bulk, force, and weight; the estimates to be verified by actual measurements. That which a farmer is called upon at every turn to do, should be begun with the children. And here the parent can supplement the teacher at every step.

When developing the mode of judgment, the pupil should be trained to use the chain in measuring areas, the yard-stick in measuring cord wood, forceps in lumber, dry measure for grain, scales for weights, liquid measure for milk, vinegar, or molasses.

The outcome of all raising of crops is commercial value. There should be a system of farm bookkeeping, in which writing and arithmetic play a prominent part. Children could be easily trained to keep books for their parents, and the work of the farm be made to present all the problems and conditions for a complete mastery of all essentials in arithmetic.

— FRANCIS W. PARKER,
In Report of the Committee of Twelve.

DENOMINATE NUMBERS.

TALK:

Pupils need not put much time on the mechanical operations now, if the study of fractions and decimals has been thorough. Their attention should be directed more to close, careful thought of processes.

The use of objects in illustrating points in problems will be very beneficial, and should be constantly used to make every step clear. After one demonstration with objects, many problems should be assigned to give the pupil a thorough test of his ability to grasp that portion of the work. He must build up his knowledge step by step, and reason out each part. The use of the reasoning power should dominate in all the work from now on.

A good plan to follow, and one which will help the pupil to reason, is this:—

State a problem. Ask the pupil to tell the class to which it belongs, give the method of working it, state what is given, and what is to be found.

He will find it tedious to commit tables and definitions, but there is no use in shirking them, for they must be mastered or the work will be a failure.

The size of the units in all the denominate quantities must be known, before the pupil can reduce to a higher or a lower denomination. Hence, he must be well acquainted with the tables before he can do much with this subject. Take a little time each day for drill on tables previously used.

Do not require the pupil to learn all the tables before the addition, subtraction, multiplication and division of denominate numbers are begun. Take the first table given in your plan of work and teach addition, subtraction, multiplication and division of the quantities *involved in that alone. Teach reduction ascending and descending with that table. Then take the second table.*

Many teachers to-day begin with the liquid measure table, as it requires no work in fractions. The order followed makes little difference and is left to the teacher.

When we speak of measures, whether they are of money, extension, time, or weight, we use terms like 5 dollars, 4 yards, 3 hours, or 10 pounds to express the quantity we are talking about. These are called **simple denominate quantities.**

Sometimes we use two or more terms or names to express the measure, as 3 hours, 15 minutes, 10 seconds; 4 gallons, 3 quarts, 1 pint. These are **compound denominate quantities.**

Measures have different denominations to express large or small quantities. Long spaces of time are measured by days, weeks, or years, but short ones by hours, minutes, or seconds.

So it is with all denominate quantities. The different denominations of the same measure may all be reduced to any one denomination by a process called **reduction.**

If changed to a higher denomination, the reduction is **ascending** and if the change is made to a lower denomination, we call it reduction **descending.**

ILLUSTRATIONS.

2 hours = 120 minutes — *reduction descending.*
120 minutes = 2 hours — *reduction ascending.*
24 inches = 2 feet — *reduction ascending.*
2 feet = 24 inches — *reduction descending.*

LONG OR LINEAR MEASURE.

Long or linear measure is used in measuring lines and distances.

There are two systems in use in the United States, the **English System** and the **French System.**

The English System is the one commonly used, while the French System is used only in making scientific measurements. The French System has a decided advantage in being on the scale of 10—the decimal scale.

English Linear Table.

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards, or 16½ feet = 1 rod (rd.)
320 rods = 1 mile (mi.)

The French or Metric System is fully treated in another part of the book.

Surveyors' Linear Table.

7.92 inches = 1 link (l.)
25 links, or 16½ feet = 1 rod (rd.)
4 rods, or 66 feet, or 100 links = 1 chain (ch.)
80 chains = 1 mile (mi.)

Surveyors generally use a chain 66 feet or 100 links long, called a Gunter's chain. Civil engineers use a steel tape 100 feet long.

REDUCTION.

Reduce 10 yds. 8 ft. 10 in. to inches.

Work:

yds.	ft.	in.
10	8	10
3		
38		
12		
456		
10		
466		

Solution:

10 yds. = 10 × 3 ft. = 30 ft.
30 ft. and 8 ft. are 38 ft.
38 ft. = 38 × 12 in., or 456 in. 456 in. + 10 in. = 466 in.

REDUCTION DESCENDING.

Reduce 3 mi. 57 rd. 2 yd. 1 ft. 8 in. to inches.

Work :

320 rd.
3
960
57
1017 rd.
5½
508½
5085
5593½
2
5595½ yd.
3
16786½
1
16787½ ft.
12
201450
8
201458 in.

Solution :

3 mi. = 3 × 320 rd. = 960 rd.
3 mi. 57 rd. = 960 rd. + 57 rd. = 1017 rd.
1017 rd. = 1017 × 5½ yd. = 5593½ yd.
3 mi. 57 rd. 2 yd. = 5593½ + 2 yd. = 5595½ yd.
5595½ yd. = 5595½ × 3 ft. = 16786½ ft.
3 mi. 57 rd. 2 yd. 1 ft. = 16786½ ft. + 1 ft. = 16787½ ft.
16787½ ft. = 16787½ × 12 in. = 201450 in.
3 mi. 57 rd. 2 yd. 1 ft. 8 in. = 201450 in. + 8 in., or 201458 in.

Note.—To prove the above work, use reduction ascending, beginning with the result.

REDUCTION ASCENDING.

Reduce 201458 inches to higher denominations.

Work :

12	201458 in.
3	16788 ft. 2 in.
5½ 2	5596 yd. 2
11	11192 half yd.
320	1017 rd. 5 half yd.
	3 mi. 57 rd. ‖
	2 yd. 1 ft. 6 in.

Solution :

201458 in. = 16788 ft. 2 in.
16788 ft. 2 in. = 5596 yd. 2 in.
5596 yd. 2 in. = 1017 rd. 2 yd. 1 ft. 8 in.
1017 rd. 2 yd. 1 ft. 8 in. = 3 mi. 57 rd. 2 yd. 1 ft. 8 in.

201458 in. = 3 mi. 57 rd. 2 yd. 1 ft. 8 in.

SURFACE MEASURES.

TALK:

Ask the pupil to tell what he means by a surface.

A surface has two dimensions, length and breadth.

If we take a surface and cut it into squares, we will have a number of square units, whether they are 1 inch long and 1 inch wide, 1 mile long and 1 mile wide, larger or smaller. The area of the surface will be the total number of these square units.

Impress the pupil with the fact that the area of a surface is simply the total number of square units in the surface, arranged in rows, with a given number of them in a row.

It is wrong to say that we multiply the length by the breadth, to get the area. Inches multiplied by inches will never give square inches, any more than apples multiplied by apples will give square apples. The pupil has been taught that the product has the same name as the multiplicand and why should he change now, and say that 5 inches times 3 inches are 15 square inches.

To illustrate the above, the following right and wrong ways are given:—

How many square inches in a surface 3 in. long and 5 in. wide?

RIGHT WAY.

SOLUTION:

A surface 1 in. long and 1 in. wide contains 1 sq. in.

A surface 1 in. long and 3 in. wide contains 3 sq. in.

A surface 5 in. long and 3 in. wide contains 5 × 3 sq. in. or 15 sq. in.

WRONG WAY.

WORK:

$$\begin{array}{r} 3\text{ in.} \\ \times 5 \\ \hline 15\text{ sq. in.} \end{array}$$

NOTE.—In practical life, this is the way the work is written. Have him know the correct method, later on in life he will adopt plenty of "short cuts."

English Square Measure Table.

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
10 square chains = 1 A.

The English Square Measures are used in measuring cloth, ceilings, floors, etc.

Surveyors' Square Measure Table.

272¼ square feet = 1 square rod
16 square rods = 1 square chain
{ 160 square rods, or
{ 10 square chains = 1 acre
640 acres = 1 square mile, or section
36 square miles, or 36 sections = 1 township

GOVERNMENT LAND SURVEY.

Before United States lands are offered for sale, they are surveyed and divided into townships, which are blocks of land 6 miles square.

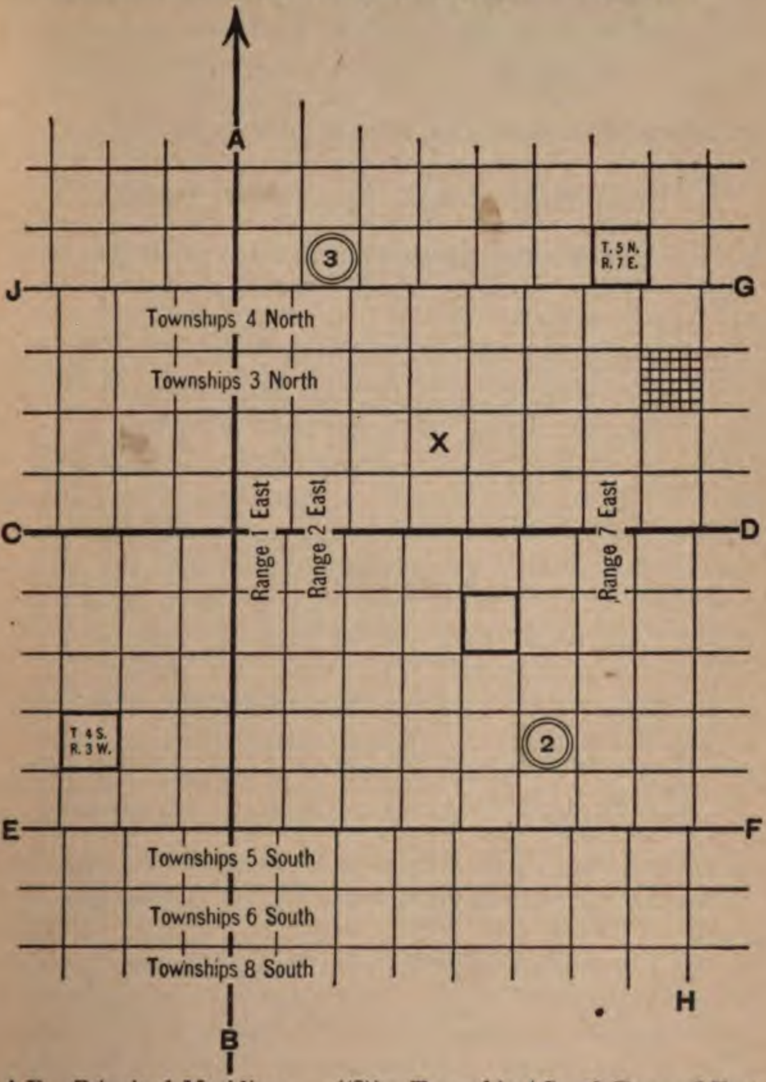
The dividing lines of these townships running east and west are called **parallels,** and those north and south **meridians.**

The parallel which corresponds to some true parallel of latitude is established as a **Base Line** and rows of townships called **ranges** extend north and south of it.

A line of townships running east and west from a meridian designated as a **Principal Meridian,** is called a **tier,** to distinguish from the range, which is a line of townships running north and south.

Any township is located by its tier number north or south of the Base Line, and its range number east or west of the Principal Meridian.

DIAGRAM SHOWING TOWNSHIP SURVEY.



A B = Principal Meridian.
C D = Base Line.
J G and E F = Correction Lines.
G H = Guide Meridian.

((2)) = Township 4 South Range 6 East.
((3)) = Township 5 North Range 2 East.

Thus, township 2 south would be the second township south of the Base Line, and, if it were in the second range east of the Principal Meridian, the location would be township 2 south, range 2 east.

We leave out the name tier, but always give the name range, in locating townships.

In Brief How Public Lands Are Measured.

1. A line is run north and south called the Principal Meridian (P. M.). This is established from some prominent point.
2. Rows of ranges are formed by running lines 6 miles apart, parallel to the Principal Meridian. These ranges are numbered east and west from the P. M.
3. A Base Line is run. It corresponds to some true parallel of latitude.
4. Lines are run parallel to the Base Line 6 miles apart, which divide the ranges into tiers of townships,
5. The townships are located both by their tier number and their range number.
6. Each township is subdivided into 36 sections.
7. Each section is again divided into halves, quarters, half-quarters, and quarter-quarters.

CORRECTION LINES AND GUIDE MERIDIANS.

Correction lines are established at every four townships north, and five townships south, of the Base Line.

They provide for errors resulting from convergence of north and south lines, and prevent changes in the width from becoming so great as to destroy the value of the system.

At every correction line, the townships start with their proper width. Thus, the townships bounded by the correction lines have *double corners*, called the "closing corners," and "standard corners."

Guide Meridians (see diagram, *G H*) are surveyed at intervals of 8 ranges east and west of the Principal Meridian.

United States survey goes no farther than locating section lines. Smaller divisions of land must be surveyed, and marks placed to locate the pieces.

Corner stones are sunk in the ground by the surveyor to be left for future surveys.

Townships and Sections.

TOWNSHIP (X).

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

1 mile.

6 miles.

Townships are six miles on a side, and are divided into 36 one-mile tracts of land, called sections. Each section contains 640 acres. The sections are divided into half-sections, quarter-sections, half-quarter sections, and quarter-quarter sections, or lots. In cities, lots are divided into blocks, and the blocks are subdivided into smaller building lots.

In the newer parts of the United States, section 16 of every township has been set aside for school purposes. This particular section was selected because it is located as near the center of a township as any, and would therefore be most convenient for the location of a school building.

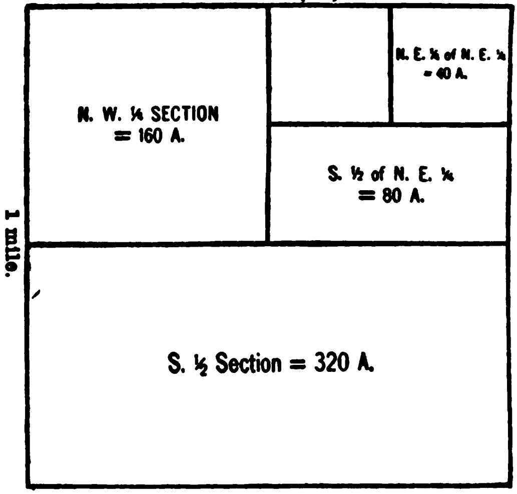
Many communities have sold their section, putting money derived therefrom into the school treasury.

Things for the Pupil to Do:

1. Notice Township X and its location with reference to the Principal Meridian and Base Line.

2. Notice Township X in the diagram by itself. Find Section 15 in Township X.

SECTION (15).



3. Notice Section 15 in the diagram by itself.

4. Locate in writing a half-section of land in Section 12 of Township X.

5. Distinguish between range and township; base line and meridian; principal meridian and parallel of latitude.

6. Draw a diagram of a section and locate the following: (1) N.E. ¼ of S.E. ¼. (2) S.W. ¼ of N.W. ¼, etc.

7. Locate some tract of land belonging to your father or friend.

PROBLEM:

Mr. Gray owns the S.W. ¼ of a section of land. How many acres does he own?

SOLUTION:

One section = 640 A.
¼ section = 160 A.
Mr. Gray owns 160 A.

NOTE.—Read the S.W. ¼ as, "the South West Quarter," or "the South West Fourth."

PROBLEM:

A friend of mine owns the E. ½ of the S. W. ¼ of a section, also the N. W. ¼ of the S. E. ¼ of the same. How many acres has he?

NOTE.—Have the pupil make a diagram to show this.

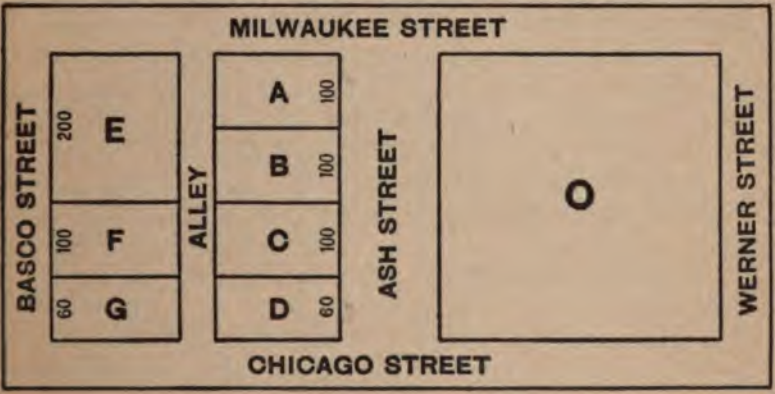
CITY LOTS.

I owned 2 blocks in the city of Milwaukee. They were each 360 feet on a side.

How many acres in the 2 blocks?

I made an alley through the middle of one of the blocks, as shown in the diagram.

Ash St. is the main thoroughfare of my section of the city.



I sold the corner lot *D* for $1250 and *A* for $2000. Lot *A* has a 100 foot frontage, while *D* has only a 60 foot frontage. How much is that per front foot in each case?

Lots *B* and *C* sold for $10.50 per front foot. What did I get for each? How much less is that per front foot than what I received on lot *A*? How does it compare with price on lot *D*?

Lot *E* was sold to the city for $5000. A city building is to be built on it. How will this affect the values of *F* and *G*?

If I paid $1250 for the 2 blocks, what have I cleared so far?

How many acres were taken out for the alley? Why did it pay me to run an alley through?

To the Teacher:

The article on city lots is given a place here to demonstrate a few of the many really practical problems and questions that can be given to the pupil along this line of work.

VOLUME MEASURES.

TALK:

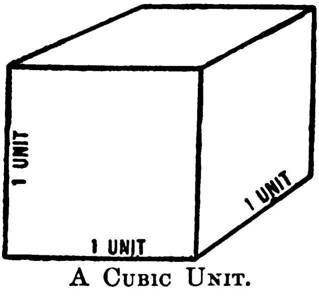
We now have to deal with solids. By means of a block, show that a solid is different from a surface in that it has an additional dimension—thickness.

Every solid must have length, breadth, and thickness.

If you show a solid cut into smaller equal blocks, the pupil will grasp the idea of cubic units. Show him the cubic unit, in this case, is 1 foot long, 1 foot wide, and 1 foot thick. Tell him we call it a cubic foot.

Then the question comes, "Would a solid 1 inch long, 1 inch wide, and 1 inch thick be a cubic unit?" What would you call it?

The pupil should tell what he means by a solid; a cube; cubic inch; cubic yard, etc.

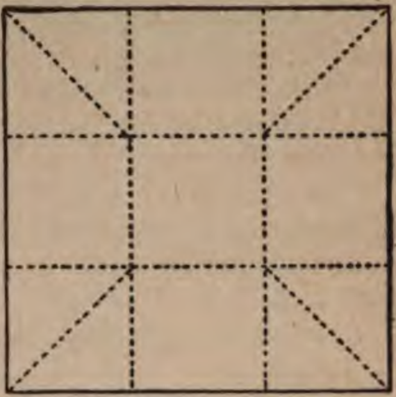


A CUBIC UNIT.

CUBIC MEASURE TABLE.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Something to Do:



Cut a square of heavy paper and mark it as shown here in the drawing. Fold the paper on the dotted lines and you will have a box.

Sand may be measured in this paper box. Try to make one that will hold just a quart; just a pint; a gill, etc.

CAPACITY MEASURES.

Many times we want to know how much space there is in a vessel; we want to know what it will hold.

Measures used in telling the extent of room in vessels are called **measures of capacity.**

There are two kinds of capacity measures, dry measures and liquid measures.

Dry measures are used to measure grain, seeds, and the like.

Liquid measures are used to measure water, milk, oils, etc.

DRY MEASURE TABLE.

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

A common Winchester bushel contains 2150.42 cubic inches.

TALK:

Don't teach that wheat, oats, rye, potatoes, and fruits are bought and sold by dry measure. In nine cases out of ten, they are not. They are bought and sold by weight, so many pounds making a bushel.

Years ago, before weighing scales became common, it was customary to measure grains and vegetables but now all that has been changed. To-day, unscrupulous merchants still use measures, if they think they can get a few extra pounds from an ignorant farmer by so doing.

Teach the custom in your community. That is the one the children are most likely to need.

Below are given weights of commodities as fixed by law in the majority of states. Exceptions are noted to the right:—

WEIGHTS OF COMMODITIES AS FIXED BY LAW.

ARTICLES	POUNDS IN A BUSHEL (Avoirdupois)	EXCEPTIONS
Apples	48	Me., 44; N. J., 50; Ore., 45; Vt., 46; Wis., 57.
Barley	48	Cal., 50; Ga., Ky., Md. and Pa., 47; Wash., 45; Ore. and Vt., 46; La., 32.
Beets	60	Mont., 50; Wash., 50; Wis., 50.
Bran	20	
Carrots	50	Conn., 55.
Coal	80	
Corn on cob	70	Ind., 68.
shelled	56	Ariz., 54; Cal., 52; N. Y., 58; Ill., 52; Ia., 50.
meal	50	Del., 48; Ga., 48; Ill., 48; D. C., 48.
Oats	32	Maine, N.J., N.H. and Pa., 30; Md., 26; Ore. and Wash., 36.
Potatoes	60	Ohio, 58.
Rye	56	Cal., 54; La., 31.
Salt	50	Mass., 70; Mich., 56.
Wheat	60	
Sweet Potatoes	55	

NOTE.—The table on the number of pounds to the bushel, as fixed by law in the various states, is merely for reference.

The pupil should know the table corrected for his own state.

LIQUID MEASURES.

The unit of liquid measure is 1 gallon.

Common Liquid Measure Table.

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

A gallon contains 231 cubic inches.

Apothecaries' Liquid Measure Table.

These measures are used in mixing medicines.

60 minims (m.) = 1 fluid dram (*f* ʒ)
8 fluid drams = 1 fluid ounce (*f* ℥)
16 fluid ounces = 1 pint (*O.*)
8 pints = 1 gallon (*Cong.*)

A minim is about 1 drop.

Note.—It is wrong to have the pupil memorize tables that ill never be used in the ordinary walks of life, as the apothearies' table, etc. Such tables should be used for reference only.

WEIGHT MEASURES.

AVOIRDUPOIS WEIGHT.

This is used in weighing all common articles, as roceries, coal, hay, etc.

Table.

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (t.)

The long ton is used at the U. S. Custom-House, and in weighing coal and iron in wholesale transactions.

TABLE.

112 pounds = 1 long hundredweight
2240 pounds = 1 long ton

TROY WEIGHT.

Troy weight is used in weighing the precious metals and jewels.

TABLE.

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

APOTHECARIES' WEIGHT.

This is used in mixing and selling medicines not liquid. Druggists generally buy by avoirdupois and sell by apothecaries' weight.

TABLE.

20 grains (gr.) = 1 scruple (sc.), or (℈)
3 scruples = 1 dram (dr.), or (ʒ)
8 drams = 1 ounce (oz.), or (℥)
12 ounces = 1 pound (lb.), or (℔)

COMPARISON OF WEIGHTS.

TABLE.

1 pound avoirdupois = 7000 grains
1 ounce avoirdupois = 437½ grains
1 pound Troy, or apothecary = 5760 grains
1 ounce Troy, or apothecary = 480 grains

TIME MEASURES.

TABLE.

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 year (yr.)
366 days = 1 leap year
100 years = 1 century

The unit of time is 1 day.

Time measures are used in figuring interest on notes and bills, in telling the time of day, and in problems in longitude and time.

The **business month** is generally figured as 30 days, and the business year as 360 days.

The business month and year are used in figuring interest.

The **common year** is figured as 365 days, and is the total number of days in the 12 months.

LEAP YEAR.

The average time it takes the earth to revolve once around the sun (1 year) is 365 days, 5 hours, 48 minutes, 47.8 seconds, or *about* 365¼ days.

The Julian Calendar called every year 365¼ days, but instead of counting the ¼ day on each year, it allowed for 3 years of 365 days each, and one year of 366 days (leap year).

The Julian Calendar, in counting 365¼ days as a year, used 11 minutes, 12.2 seconds, too much.

The Gregorian Calendar corrected the mistake by omitting the leap year on centennial years, except on those that were multiples of 400.

All the civilized nations, except Russia, have adopted the Gregorian Calendar. Russia is also about to adopt it.

Briefly,

Leap year comes on every year divisible by 4, and every centennial year divisible by 400.

To help fix the number of days in each month, let the children memorize: —

> *" Thirty days have September,*
> *April, June, and November;*
> *All the rest have thirty-one*
> *Except the second month alone,*
> *Which has but twenty-eight in fine*
> *Till Leap Year gives it twenty-nine."*

Where We Got the Names of the Months.

January, from *Janus,* was the sacred month of the year to the Romans. To them, Janus was the god of the year. During the 18th century, the Europeans started to recognize it as the first month, but previous to this, March was considered the first.

February comes from *februa,* the name of a Roman festival celebrated on the 15th of the second month.

March is from *Mars,* the god of war. March was the first month of the year to the Romans.

April, from the Latin *aperire,* "to open," was probably so called because during this month buds begin to open.

May is from *Maia,* the mother of Mercury. The Romans offered sacrifices to this goddess on the first day of May.

The sixth month in our calendar, *June,* got its name from *Juno,* the wife of Jupiter.

July was so named in honor of Julius Cæsar, who was born in this month.

Emperor Augustus Cæsar commanded that the eighth month be named *August* after him.

September is from the Latin *septem,* meaning seven.

At the time when March was the first month of the year, September was the seventh.

October, November, and *December* were originally the 8th, 9th, and 10th months. *Octo, novem,* and *decem* are Latin numerals for eighth, ninth, and tenth.

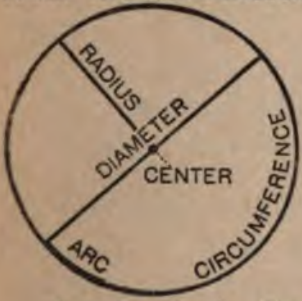
CIRCULAR MEASURES.

Circular or Angular measures are used in surveying, navigation, astronomy, geography, reckoning latitude and longitude, and computing differences in time.

A **Circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within, called the center.

The **Circumference** is the bounding line of a circle.

The **Radius** of a circle is a straight line drawn from the circumference to the center.

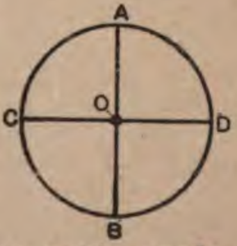


The **Diameter** is a straight line drawn through the center, with the ends terminating in the circumference.

An **Arc** of a circle is any portion of the circumference.

An **Angle** is the difference in direction between two straight lines which meet.

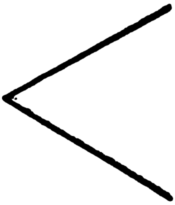
If two diameters divide a circle into four equal parts, these diameters make **right angles** with each other.



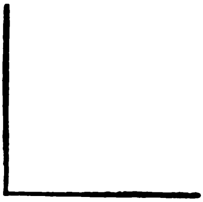
Thus, each of the angles about the center O, in this figure, are right angles.

An angle less than a right angle is an **acute angle.**

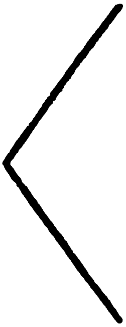
An angle greater than a right angle is an obtuse angle.



Acute Angle.



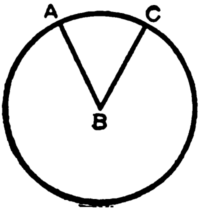
Right Angle.



Obtuse Angle.

The circumference of a circle may be divided into 360 equal parts, called **degrees.** If the circle is large, the degree is large, and if the circle is small, the degree is small, but the degree is always $\frac{1}{360}$ part of the circumference, whatever the size of the circle.

An angle at the center of a circle is measured by the arc which bounds it.



Thus, the angle ABC is measured by the arc AC, which bounds it.

If the angle is a right angle, it is measured by $\frac{1}{4}$ of 360 degrees, or 90 degrees; hence, any angle of 90 degrees is a right angle.

An acute angle is always less than 90 degrees.

An obtuse angle is always more than 90 degrees.

Table of Circular Measure.

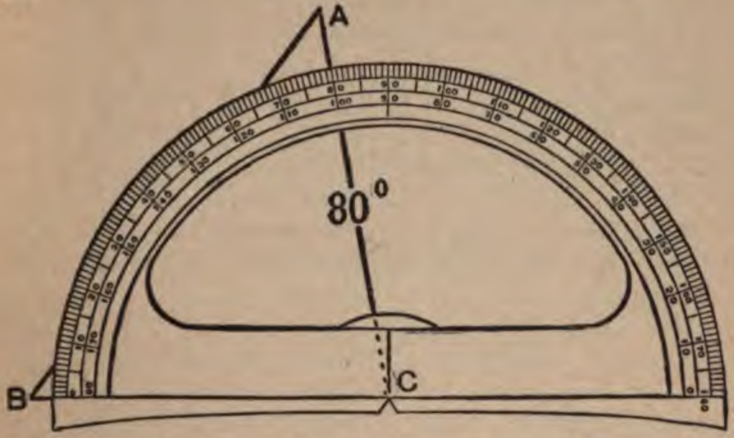
60 seconds ('') = 1 minute (')
60 minutes = 1 degree (°)
360 degrees = 1 circumference (cir.)

Note.—A degree upon any great circle of the earth is 69.16 statute miles, or 60 geographical miles.

A sign is an arc of 30 degrees.

THE PROTRACTOR.

An instrument, called a protractor, has been made for measuring and constructing angles of any given size.



The pupil will readily learn to use a protractor.

Things to Do with a Protractor:

1. Draw a triangle, making one side 3 inches long and two of the angles 45° each.

2. Estimate the number of degrees in different angles shown you, and measure with your protractor, to verify your estimates.

REMARK:

Every pupil should make himself a protractor of stiff paper or cardboard. Notice the one given above and copy.

MEASURES OF VALUE.

TALK:

Don't spend any time with foreign money, unless the pupils are apt to need it. It is a waste of time.

The values, in our money, of a mark, rouble, kopek, franc, shilling, pound, are all that is needed.

The table on values of foreign coins is merely for reference.

TABLE SHOWING VALUES OF FOREIGN COINS.

COUNTRY	STANDARD UNIT	VALUE IN U. S. MONEY
Austria	Florin	$0.401
Belgium	Franc *	.193
Brazil	Milreis	.546
Chili	Peso	.912
Cuba	Peso	.932
Denmark	Krone	.268
Egypt	Piaster	.049
France	Franc *	.193
Great Britain	Pound	4.8665
Greece	Drachma*	.193
German Empire	Mark	.238
Hawaiian Islands	Dollar	1.00
India	Rupee	.386
Italy	Lira*	.193
Japan	Yen	.876
Mexico	Dollar	.882
Netherlands	Florin	.402
Norway	Krone	.268
Portugal	Milreis	1.08
Russia	Rouble	.65
Spain	Peseta*	.193
Sweden	Krone	.268
Switzerland	Franc *	.193
Turkey	Piaster	.044
United States	Dollar	1.00

Notice that several coins of different countries have the same value, 19.3¢. They are marked with a * in the table.

Federal Money is money of the United States. It has been discussed before in this book.

Sterling Money is currency of Great Britain and Ireland.

TABLE OF STERLING MONEY.

4 farthings (far.) = 1 penny (d.)
12 pence (not pennies) = 1 shilling (s.)
20 shillings = 1 pound (£), or sovereign
5 shillings = 1 crown
21 shillings = 1 guinea

The standard unit of Sterling Money is 1 pound or sovereign, whose value in our money is $4.8665.

The standard unit of French Money is 1 franc, equal to 19.3¢ in our money.

Germany's standard unit is 1 mark, or 23.85¢.

The foregoing table gives the values of some of the foreign coins.

FOREIGN MONEY PROBLEMS.

TALK:

In schools where foreign money is taught, the following problems and solutions may be helpful:—

PROBLEM:

When in France, I bought goods as follows:—

3 books @ 2 francs,
½ doz. pipes @ 1 franc,
2 pictures @ 4 francs.

What was the cost in U. S. money?

WORK:

3 books @ 2 francs cost....... 6 francs.
½ doz. pipes @ 1 franc cost.... 6 francs.
2 pictures @ 4 francs cost..... 8 francs.

Cost of all................20 francs.
20 francs = 20 × 19.3¢, or $3.86.

PROBLEM:

I have £5 Sterling. What is the value in U. S. money?

SOLUTION:

The value is 5 × $4.8665, or $24.33.

PROBLEM:

While in Europe, I collected the following coins:— A krone in Norway, 2 drachmas in Greece, 3 florins in Austria, 4 liras in Italy, 2 roubles in Russia, and 3 pesetas in Spain.

What is the equivalent of all in our money?

WORK:

1 krone	=	$ .268
2 drachmas	=	.386
3 florins	=	1.203
4 liras	=	.772
2 roubles	=	1.30
3 pesetas	=	.579
Value of all	=	$4.508

MISCELLANEOUS MEASURES.

PAPER MEASURE TABLE.

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

NUMBER TABLE.

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

ADDITION OF DENOMINATE QUANTITIES.

Add 2 bu. 3 pk. 4 qt. 1 pt. and 6 bu. 1 pk. 2 qt.

WORK:

bu.	pk.	qt.	pt.
2	3	4	1
6	1	2	0
8	4	6	1

In Adding Denominate Numbers:

FIRST.— Write the names of the different units to be used in the addition, placing them in a horizontal row, the largest to the left.

NEXT.— Write the numbers of each unit to be added, below the names of the units, each in its proper place.

THEN.— Add as before taught, placing each sum below the column added.

Add 7 hr. 15 min. 30 sec., 9 hr. 30 min. 40 sec., and 11 hr. 40 min. 32 sec.

WORK:

hr.	min.	sec.
7	15	30
9	30	40
11	40	32
28	26	42

EXPLANATION:

32 sec. + 40 sec. + 30 sec. = 102 sec.

But, 102 sec. = 1 min. 42 sec. Write the 42 below and carry the 1 min.

1 min. (carried) + 15 min. + 30 min. + 40 min. = 86 min.

But, 86 min. = 1 hr. 26 min. Write the 26 and carry the 1 hr.

1 hr. + 11 hr. + 9 hr. + 7 hr. = 28 hr.

Result = 28 hr. 26 min. 42 sec.

SUBTRACTION OF DENOMINATE QUANTITIES.

Subtract 6 T. 12 cwt. 9 lb. 10 oz. from 15 T. 7 cwt. 13 lb. 9 oz.

WORK:

T.	cwt.	lb.	oz.
15	7	13	9
6	12	9	10
8	15	3	15

EXPLANATION:

I. Place as in addition of denominate quantities. 10 oz. cannot be taken from 9 oz., so we must take 1 lb. from the 13 lb. and add it to the 9 oz. 16 oz. + 9 oz. = 25 oz. 25 − 10 = 15. Write the 15 below.

II. Now there are only 12 lb. left to take the 9 from. 12 − 9 = 3. Write the 3 below.

III. 12 is larger than 7. 1 T. + 7 cwt. = 27 cwt. 27 − 12 = 15. Write the 15 below.

IV. 14 − 6 = 8. Write the 8 below.

V. Result = 8 T. 15 cwt. 3 lb. 15 oz.

MULTIPLICATION OF DENOMINATE QUANTITIES.

Multiply 21 yd. 2 ft. 11 in. by 6.

Work:

yd.	ft.	in.
21	2	11
		6
131	2	6

Explanation:

I. 6 × 11 in. = 66 in. = 5 ft. 6 in. Write the 6 below and carry the 5.

II. 6 × 2 ft. = 12 ft. 12 ft. + 5 ft. (carried) = 17 ft., or 5 yd. 2 ft. Write the 2 below and carry the 5.

III. 6 × 21 yd. = 126 yd. 126 yd. + 5 yd. = 131 yd.

IV. Result = 131 yd. 2 ft. 6 in.

DIVISION OF DENOMINATE QUANTITIES.

Divide 3 yr. 9 mo. 4 da. by 12.

Work:

	yr.	mo.	da.	hr.
12)	3	9	4	0
	0	3	22	20

Explanation:

I. We cannot divide 3 by 12, so we reduce 3 yr. to months. 3 yr. = 36 mo. 36 mo. + 9 mo. = 45 mo. 45 ÷ 12 = 3, and a remainder 9. Write the 3 and carry the remainder 9.

II. 9 mo. (carried) = 270 da. 270 da. + 4 da. = 274 da. 274 ÷ 12 = 22, and a remainder 10. Write the 22 and carry the 10.

III. 10 da. = 240 hr. 240 ÷ 12 = 20. Write the 20.

IV. Result = 3 mo. 22 da. 20 hr.

Divide 150 da. 2 hr. 5 min. by 18.

Work:

8 da. 8 hr. 6 min. $56\frac{2}{3}$ sec.
18) 150 da. 2 hr. 5 min.
6 da. (remainder)
144 hr.
2
146 hr.
2 hr. (remainder)
120 min.
5
125 min.
17 min. (remainder)
1020 sec.

Explanation:

This example illustrates a feature of work that should be required—omitting every detail that can be worked without the use of pencil.

The pupil is getting old enough now to appreciate short methods, and he should be given them. This does not apply to all "short cuts," but only to those that have to do with well understood processes.

DENOMINATE FRACTIONS.

A **denominate fraction** is one denoting a denomination.

As, $\frac{5}{8}$ of a mile, $\frac{3}{7}$ bu., .57 acres, etc.

REDUCTION ASCENDING WITH DENOMINATE FRACTIONS.

1. Reduce 1 da. 14 hr. 24 min. to a fraction of a month.

Solution:

24 min. ÷ 60 = $\frac{24}{60}$ hr. $\frac{24}{60}$ hr. + 14 hr. = $14\frac{24}{60}$ hr. = $\frac{72}{5}$ hr.
$\frac{72}{5}$ hr. + 24 hr. = $24\frac{72}{5}$ hr. = $\frac{192}{5}$ hr.
$\frac{192}{5}$ hr. ÷ $(24 \times 30) = \frac{4}{75}$.
1 da. 14 hr. 24 min. = $\frac{4}{75}$ mo.

Work:

1 da. 14 hr. 24 min. = $\frac{\frac{192}{5}}{24}$ da.

$$\frac{\frac{192}{5}}{24} \text{ da.} = \frac{\frac{\frac{192}{5}}{24}}{30} \text{ mo}$$

$$\frac{\overset{4}{\cancel{\overset{8}{\cancel{192}}}}}{5} \times \frac{1}{24} \times \frac{1}{\underset{15}{\cancel{30}}} = \frac{4}{75} \text{ mo.}$$

2. Reduce 1 da. 14 hr. 24 min. to the decimal of a month.

WORK:

60)24 min.
24)14.4 hr.
30)1.6 da.
.053$\frac{1}{3}$ mo.

NOTE.—Here some of the written work has been eliminated.

REDUCTION DESCENDING WITH DENOMINATE FRACTIONS.

Reduce $\frac{5}{9}$ of a mile to rods, yards, feet, etc.

SOLUTION:

$\frac{5}{9}$ of 320 rd. = $\frac{1600}{9}$ rd. = 177$\frac{7}{9}$ rd.
$\frac{7}{9}$ of $\frac{11}{2}$ yd. = $\frac{77}{18}$ yd. = 4$\frac{5}{18}$ yd.
$\frac{5}{18}$ of 3 ft. = $\frac{15}{18}$ ft. = $\frac{5}{6}$ ft.
$\frac{5}{6}$ of 12 in. = $\frac{60}{6}$ in. = 10 in.
$\frac{5}{9}$ of a mile = 177 rd. 4 yd. 0 ft. 10 in.

Proceed with decimals in the same way in Reduction Descending.

LONGITUDE AND TIME.

TALK:

The subject of longitude and time should be taken up in connection with geography and map study.

Show the pupil the lines drawn through the maps. Tell him these lines represent imaginary lines running from pole to pole, and east and west parallel to the equator, on the surface of the earth.

The name of the imaginary lines running north and south on the earth's surface is **meridians**; those east and west, **parallels.**

One meridian, called the **prime meridian,** has been selected, and all the other meridians are reckoned as east or west of it. Find Greenwich, England, a small town near London, on the map. The prime meridian runs through **Greenwich.**

Longitude is distance east or west of the prime meridian. When we say that the longitude of Paris is 2° 20′ East, we mean that the meridian running through Paris is 2° 20′ east of the prime meridian that runs through Greenwich, England. The longitude of Washington, D. C., is 77°.7′ West. That means that the meridian which passes through Washington is 77° 7′ west of the prime meridian.

For the Pupil to Do:

1. Find the longitude of your home town.
2. Find the longitude of Chicago.
3. Find the longitude of Denver.
4. Find the longitude of Salt Lake City.
5. Find the longitude of San Francisco.

The longitude of a place tells in degrees, minutes, and seconds, the distance it is east or west of the prime meridian.

Philadelphia is 75° 9′ W. longitude and Portland, Me., is 70° 15′ W. longitude.

The difference in longitude is 4° 54′. We subtract to get the difference in longitude.

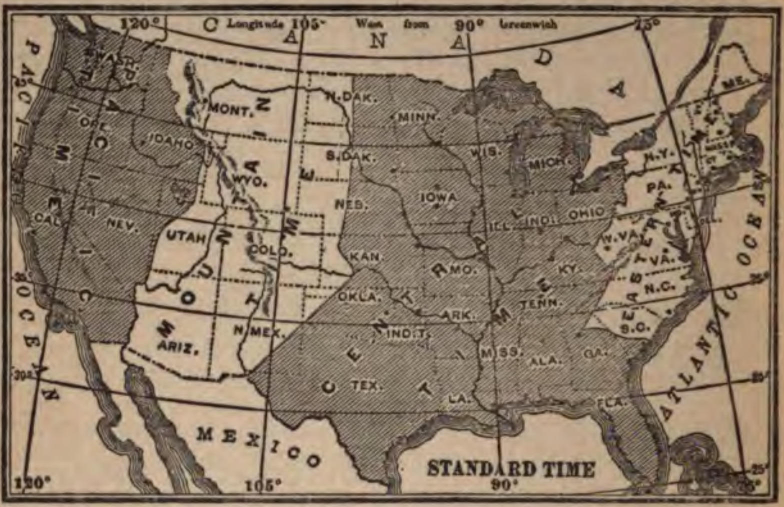
Paris is 2° 20′ E. longitude and Philadelphia is 75° 9′ W. longitude.

The difference in longitude is 77° 29′. Here we must add to get the difference in longitude.

If the pupil does not understand why we add in the first case, and subtract in the second, explain by marking off inches on a globe. Ask him how many inches two places are apart, when both are on the same side of the prime meridian. When on opposite sides.

Take the map of the United States and show the pupil the sections of country in **Eastern Time, Central Time, Mountain Time,** and **Pacific Time.**

The pupil should know the number of degrees between the meridian lines that bound these **Time Belts.**



New York people are in the Eastern Time Belt. If they rise at 6 o'clock in the morning, they will be up a whole hour before Chicago people, who get up at the same hour.

Thus, each day begins an hour sooner in New York than in Chicago, two hours before Denver, and three hours before San Francisco.

Ask the pupil to tell what time belt he lives in. If he has any friends or relatives in any of the other time belts of U. S., have him tell what time they have at the present moment.

WHERE THE DAY BEGINS.

The day begins earlier as you go east from New York, until you meet the 180th meridian. This is where the day begins. Starting here, it travels westward, giving the whole world a new day. The 180th meridian is called the **International Date Line (I. D. L.)** but in reality, the date line is a crooked line which zigzags across the 180th meridian.

From the time the day starts at the I. D. L., until the sun again reaches that line, the same day is in progress the world over.

Location of the International Date Line.

The *Pathfinder* in 1899 showed a diagram of the I. D. L., as its course was then marked. Since then it has been changed.

As marked *now*, it passes southward through Behring Sea, then westerly, then returns to the 180th meridian at about 40° north. It then follows the 180th meridian to 10° south, where it swerves east but returns again to the 180th meridian at about 50° south. It then follows that meridian. The date line is not so crooked as before the change.

The sun *appears* to move in a circle around the earth, passing through 360° in 24 hours:—

Then,

In 24 hr., the sun passes through 360°
In 1 hr., the sun passes through $\frac{1}{24}$ of 360° = 15°
In 1 min., the sun passes through $\frac{1}{60}$ of 15° = 15′
In 1 sec., the sun passes through $\frac{1}{60}$ of 15′ = 15″

Every 15° that one passes through, going either east or west, makes a difference of one hour in his time.

To reduce longitude in time to longitude in degrees, multiply by 15; to reduce longitude in degrees to longitude in time, divide by 15.

PROBLEMS.

TALK:

Two kinds of problems are generally given in longitude and time, along with those in which one is asked *to merely find differences in longitude and time. They are (1) Given the longitude of two places and the time at one of them, to find the time at the other; and*

(2) Given the times of two places and the longitude of one, to find the longitude of the other. All will be explained here.

PROBLEM:

Find the difference in time between Brooklyn 73° 58′ W. and Omaha 95° 28′ W.

WORK:

$$\begin{array}{r} 95°\ 28' \\ 73\ \ 58 \\ \hline 15)\ 21°\ 30' \\ \hline 1\text{ hr. }26\text{ min.} \end{array}$$

21° 30′ difference in longitude.

1 hr. 26 min. difference in time.

PROBLEM:

What is the difference in longitude between two places whose difference in time is 2 hr. 33 min. 12 sec.?

WORK:

hr.	min.	sec.	
2	33	12	difference in time.
		15	
30°	495′	180″	
38°	18′	0″	difference in longitude.

PROBLEM:

It is noon Thursday, 130° W. (just west of San Francisco). What time is it 160° E. (in Siberia)?

WORK:

$$\begin{array}{r} 130° \\ 160 \\ \hline 15)\ 290° \\ \hline 19\tfrac{1}{3} \end{array}$$

$19\frac{1}{3}$ hr. = 19 hr. 20 min., the difference in time.

Day comes earlier 160° E.

Therefore, it is 7.20 A. M. Friday, in Siberia.

PROBLEM:

When it is noon at Greenwich, what time is it at Philadelphia?

WORK:

Philadelphia is 75° W.
Greenwich is 0°

$$15\,\overline{)75^\circ}\ \text{difference in longitude.}$$
$$5\ \text{hr. difference in time.}$$

It is 5 hr. *earlier* in Philadelphia, or 7 A. M.

For the Pupil to Do:

1. When it is 3 o'clock P. M. at Philadelphia, where is it 2 P. M.? where is it 1 P. M.? where is it noon?

2. When it is 10 A. M. at Greenwich, what time is it at Boston? at Chicago? at San Francisco?

PARTICULAR MEASURES.

NOTE.— These are not in frequent use. Neither are the measures always definitely known.

A **hand,** used in measuring the height of horses, is 4 inches.

A **stone** of lead or iron is 14 pounds.

A **span** is 9 inches.

A **square,** for measuring floors, etc., is 100 square feet.

A **degree** of longitude at the equator is 69.16 miles.

A degree of longitude on the 40th parallel is about 53 miles.

A **fathom,** used in measuring depth at sea, is 6 feet.

A **knot,** or geographical mile, or nautical mile, is 1.15 common miles.

A **carat,** used in weighing diamonds, weighs 3⅕ gr. Troy.

In gold and silver coins, $\frac{1}{10}$ is copper and $\frac{9}{10}$ pure gold or silver.

A **solar year** is 365 days, 5 hours, 48 minutes, 49.7 seconds.

A bushel equals 2150.42 cubic inches.
A gallon equals 231 cubic inches.
A square 208.71 feet on a side contains an acre.

PRACTICAL COST METHODS.

TALK:

When goods are sold by the hundred, thousand, or ton, the methods given here will be found very practical.

They are simple and will save much time.

1. When goods are sold by the hundred.

PROBLEM:

Find the cost of 454 lb. of sugar @ $4 per cwt.

WORK:

$$\begin{array}{r} 4.54 \\ \$4 \\ \hline \$18.16 \end{array}$$

@ @ @

RULE:

Point off two decimal places in the number of pounds and multiply.

In other words, divide the weight by 100, and multiply the quotient by the cost per cwt.

Dividing by 100 gives the number of cwt. as a decimal.

2. When goods are sold by the thousand.

PROBLEM:

What is the cost of 4250 shingles @ $2.25 per M.?

NOTE.—M. is often used to denote thousand.

WORK:

$$\begin{array}{r} 4.25\\ \$2.25\\ \hline 2125\\ 850\\ 850\\ \hline \$9.5625\end{array}$$

SHORTER WAY:

$$\begin{array}{rr} & 4.25\\ & \$2\frac{1}{4}\\ \hline \frac{1}{4}\text{ of }4.25 & 1.06\frac{1}{4}\\ & 8.50\\ \hline & \$9.56\frac{1}{4}\end{array}$$

EXPLANATION:

Point off three decimal places in the number of shingles, and multiply.

In other words, divide the number of shingles by 1000 and multiply the quotient by the cost per M.

Dividing by 1000 gives the number of M. in decimal form.

3. When goods are sold by the ton.

PROBLEM:

Find the cost of 2925 lb. of hay @ $16 per ton.

WORK:

$$\$16 \div 2 = \$8$$
$$2.925 \times \$8 = \$23.40$$

EXPLANATION:

1. Find the cost of 1000 lb. by dividing the cost of a ton by 2.
2. Point off three decimal places in the number of pounds and multiply by the cost of 1000 lb.

TALK:

Let the pupil make and solve many problems involving articles bought and sold by the hundred. Do the same with articles bought and sold by the thousand, or by the ton.

PAGE FROM A CREAMERY BOOK.

TALK:

A page from the book kept by a small creamery is shown here. It shows the number of pounds of milk *brought each day by its patrons and the totals for the week.*

It also shows the value of the milk computed at 90c. per hundred pounds.

*Make a page similar to the one given and let the pupil fill in the totals. He may prove his work by finding the sum of the totals below and in the column at the right. The sums in both cases should be the same. The total amount received during the week is marked with a *.*

Let him compute for other values than 90c. per hundred pounds.

Have him make without aid pages similar to the one given.

Take pages from books used in the community where the pupil lives, and use them in the same way.

PAGE FROM A CREAMERY BOOK.

NAME	MON.	TUES.	WED.	THURS.	FRI.	SAT.	TOTAL FOR WEEK	VALUE
Rufus Hall	314	296	304	301	297	314	1826	$16.43
John Collins	216	204	210	217	215	211	1273	11.46
Thos. Goss	416	398	399	412	412	403	2440	21.96
James Riding	394	399	401	391	396	398	2379	21.41
Ignatz Janish	127	123	130	125	131	129	765	6.89
Geo. Myer	224	220	213	221	217	223	1318	11.86
John Kehoe	165	164	149	171	164	164	977	8.79
Henry Oldenburg	314	312	303	311	309	306	1855	16.70
Carl Seifert	181	196	201	187	—	203	968	8.71
Abner Calhoun	217	220	198	206	211	199	1251	11.26
Totals	2568	2532	2508	2542	2352	2550	15052 *	135.47

TIME BOOKS.

A page from the time book used in a small mill is given here. All parts have been filled in.

In the page given, 10 hours constitute a day's work. Have the pupil make a page of a time book where a day's work consists of 8 hours. Let him include more or less men than the number given on the page here. Also have him change the wage schedule.

Give him practice in such work. It is practical. Also have him find the totals. This gives him the total sum paid out in wages each week, and enables him to find the average wages per hour or day.

Tell the pupil of the time books used in large mills and factories, where hundreds and, in many cases, thousands of people work.

PAGE FROM A TIME BOOK.

NAME	DAILY WAGES	MON.	TUES.	WED.	THU.	FRI.	SAT.	TOT'L HRS.	D'YS.	AM'T E'RN'D
John Carlson	$2.00	10	9	9	11	12	12	63	6.3	$12.60
R. E. Mann	2.00	9	11	11	10	9	11	61	6.1	12.20
Gilbert Hanson	2.50	11	12	11	10	11	12	67	6.7	16.75
Robt. Bell	3.00	9	8	9	0	4	10	40	4	12.00
Henry Hall	2.00	9	8	11	10	11	12	61	6.1	12.20
Sam. Rogers	2.50	11	9	8	10	11	9	58	5.8	14.50
Ralph Gromann	3.00	9	8	10	9	9	8	53	5.3	15.90
E. B. Cochems	2.50	9	8	11	12	9	10	59	5.9	14.75
Carl Jobson	3.00	12	10	11	11	10	9	63	6.3	18.90
A. F. Beule	2.00	10	11	12	10	9	8	60	6	12.00
		99	94	103	93	95	101	585	58.5	141.80

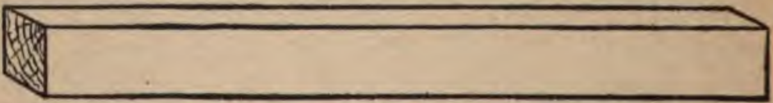
PRACTICAL ESTIMATES.

TALK:

A man builds a house. Before it is completed and ready for occupancy, many little experiences creep in to demonstrate that practical knowledge of measures means a good deal.

This chapter gives some idea of what a builder or repairer has to contend against.

Estimating means thinking, for every problem differs from every other, certain allowances have to be made for many reasons, and what is true in one case may not be in another. Hence, the pupil must derive much good from this line of work. It is now time to stop thinking that problems should work out even to be correct. Do not allow him to work merely for results. Reasons should be given for every step.



LUMBER.

Facts That Should be Known about Lumber:

1. The unit of measure for boards, planks, timbers, and joists is a **board foot.**

A board foot is 1 foot long, 1 foot wide, and 1 inch thick.

2. Lumber is always considered 1 inch thick, unless otherwise stated. If a board is 2 inches thick. it contains twice as many board feet as one of the same length and width but 1 inch thick.

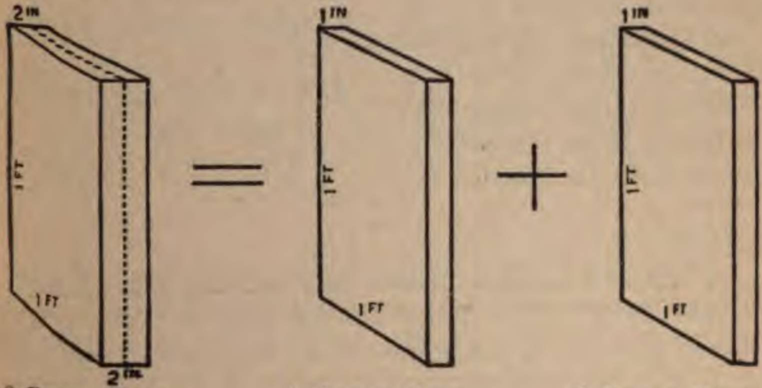
3. Lumber less than 1 inch thick is considered an inch thick in measuring; over 1 in. to 1¼ in., as 1¼ in.; over 1¼ to 1½, as 1½ in., etc.

Fractions of an inch in the width should be omitted and the nearest integer taken.

4. If a board or plank is wider at one end, the average width is taken as the width of the whole.

From the definition of a board foot, the pupil will readily see that the number of board feet in a timber is the same as the number of square feet in one face, multiplied by the thickness in inches.

TO ILLUSTRATE:



2 Board Feet. 1 Board Foot. 1 Board Foot.

If the board containing 2 board feet (shown above) be cut or sawed where the dotted lines are shown, there will be 2 boards, each containing 1 board foot.

Take a piece of wood and show the actual work done. The pupil must understand just what a board foot is.

To find the number of feet of lumber in any timber,

Multiply the area of one face, in square feet, by the thickness in inches.

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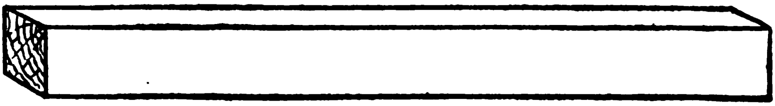

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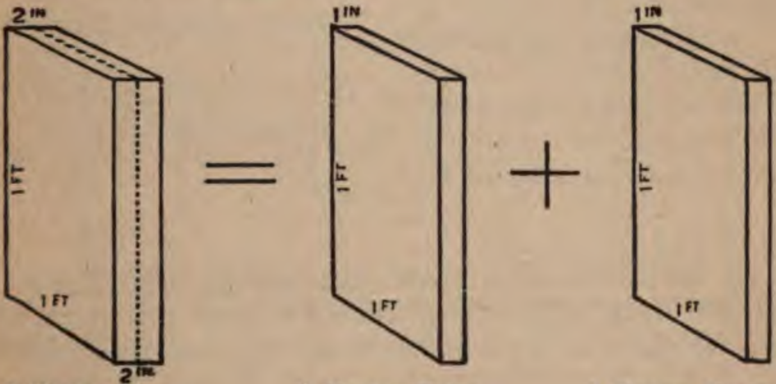

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To find the number of feet of lumber in any timber,

Multiply the area of one face, in square feet, by the thickness in inches.

PROBLEM:

How much lumber is needed for a bin 6 ft. deep, 10 ft. long, and 8 ft. wide?

SOLUTION:

2 sides each 6 ft. × 10 ft. have........120 sq. ft.
2 sides each 8 ft. × 6 ft. have......... 96 sq. ft.
Bottom 8 ft. × 10 ft. has.............. 80 sq. ft.
Total surface = 296 sq. ft.

Hence, the bin will take 296 ft. of lumber.

PROBLEM:

What will the above lumber cost at $15 per M.?

SOLUTION:

1000 ft. cost $15.
296 ft. cost .296 × $15, or $4.44.

TALK:

Many problems may be given in finding how much lumber is needed for various buildings.

They should be made practical, and to that end, it is best to take actual buildings in the vicinity. Be careful not to make the work too complex at first.

Talk such work over with a builder or carpenter.

Give considerable oral drill before proceeding to the written work.

Be sure that the pupil knows what a board foot is, then proceed like this:—

"How many board feet in each of these boards, all being one inch thick—

1 in. wide and 12 ft. long?
2 in. wide and 12 ft. long?
9 in. wide and 12 ft. long?
13 in. wide and 12 ft. long?
12 in. wide and 6 ft. long?
12 in. wide and 3 ft. long? etc."

The following lumber measure reference table may be found helpful:—

LUMBER MEASURE, JOIST, SCANTLING AND TIMBERS.

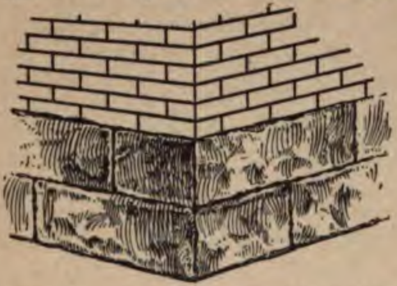
SIZE IN INCHES.	LENGTH OF TIMBERS IN FEET.									
	12	14	16	18	20	22	24	26	28	30
	NUMBER OF BOARD FEET IN PIECE.									
2x 4	8	9	11	12	13	15	16	17	19	20
2x 6	12	14	16	18	20	22	24	26	28	30
2x 8	16	19	21	24	27	29	32	35	37	40
2x10	20	23	27	30	33	37	40	43	47	50
2x12	24	28	32	36	40	44	48	52	56	60
3x 6	18	21	24	27	30	33	36	39	42	45
3x 8	24	28	32	36	40	44	48	52	56	60
3x10	30	35	40	45	50	55	60	65	70	75
3x12	36	42	48	54	60	66	72	78	84	90
3x14	42	49	56	63	70	77	84	91	98	105
4x 4	16	19	21	24	27	29	32	35	37	40
4x 6	24	28	32	36	40	44	48	52	56	60
4x 8	32	37	43	48	53	59	64	69	75	80
6x 6	36	42	48	54	60	66	72	78	84	90
6x 8	48	56	64	72	80	88	96	104	112	120
6x10	60	70	80	90	100	110	120	130	140	150
8x 8	64	75	85	96	107	117	128	139	149	160
8x10	80	93	107	120	133	147	160	173	187	200
8x12	96	112	128	144	160	176	192	208	224	240
10x10	100	117	133	150	167	183	200	217	233	250
10x12	120	140	160	180	200	220	240	260	280	300
12x12	144	168	192	216	240	264	288	312	336	360
12x14	168	196	224	252	280	308	336	364	392	420
14x14	196	229	261	294	327	359	392	425	457	490

NOTE.—Let the pupil prove that the above table is correct, by finding the board feet in many of the timbers indicated. Let him measure a few actual timbers, if possible.

BRICK AND STONE.

TALK:

This work in practical estimates furnishes excellent opportunity to develop the pupil's reasoning power. As far as possible, have him imagine the conditions of new problems, when he has become familiar with those containing like conditions, which have been made clear to him by means of illustrations.



Have many estimates on simple problems made without the use of pencil and paper. Again, let us say, give much oral work as the pupil goes along.

When he once understands the why in the work of any class of problems, let the oral work come in to fix it permanently and clear up any haze that is apt to linger on the borders of his mental pictures.

Facts That Should be Known about Brick and Stone.

1. Bricks vary in size but generally are 8 in. × 4 in. × 2 in.

2. We estimate about 22 bricks with mortar to a cubic foot. Allow $\frac{1}{6}$ of the space for mortar.

3. Bricklayers are paid by the day, by the job, or by the number of thousand laid.

4. Stone work is estimated by the perch. Ordinarily a perch is considered to be 25 cubic feet.

5. In estimating labor on stone work, the general custom is to measure the corners twice and to deduct one half of the openings.

6. One cord of stone makes about 100 cubic feet of wall.

PROBLEM:

Estimate the number of bricks needed to lay a wall 160 ft. long, 9 ft. high, and 20 in. thick.

SOLUTION:

$$20 \text{ in.} = 1\tfrac{2}{3} \text{ ft.}$$

Number of cu. ft. in wall $= 1\frac{2}{3} \times 9 \times 160 = 2400$.

If 22 bricks are needed per cubic foot, for 2400 cubic feet of wall, 2400 × 22 bricks, or 52,800 bricks, are needed.

PROBLEM:

How many cords of stone are needed for the walls of a building 40 ft. × 60 ft., if the walls are 2 ft. thick and 10 ft. high?

NOTE.—Draw a picture of the above.

SOLUTION:

2 walls, each 60 ft. × 10 ft. × 2 ft. contain 2400 cu. ft.

2 walls, allowing for corners, contain 2 × 36 × 10 × 2 cu. ft., or 1440 cu. ft.

2400 cu. ft. + 1440 cu. ft. = 3840 cu. ft.

Since a cord of stone makes about 100 cubic feet of wall, there are needed about 39 cords of stone.

WORK:

$$\begin{array}{r} 2 \times 60 \times 10 \times 2 = 2400 \\ 2 \times 36 \times 10 \times 2 \times \underline{1440} \\ 3840 \end{array}$$

3840 ÷ 100 = 38.4 = (about) 39.

NOTE.—Make clear how you figure two walls only 36 feet long. The sign × is to be read "by" in some cases here.

LATHS AND SHINGLES.

TALK:

Tell the pupils what is meant by laying shingles "4 inches to the weather."

Show shingles wider than four inches and some less than four inches in width.

Explain that they are estimated four inches wide in *figuring the number to a bunch, and in finding the required number for a given surface.*

Facts That Should be Known about Laths and Shingles.

1. Laths are 4 ft. long, 1½ inches wide, and are usually laid ⅜ of an inch apart at the sides.

2. 50 laths make a bunch or bundle.

3. Allowing for waste, a bunch of laths will cover about 3 square yards.

4. Lathing is estimated by the square yard. About one half of the surface of openings is deducted. Plastering also is estimated by the square yard and only one half of the surface of openings is charged.

5. A bunch or bundle of shingles contains 250.

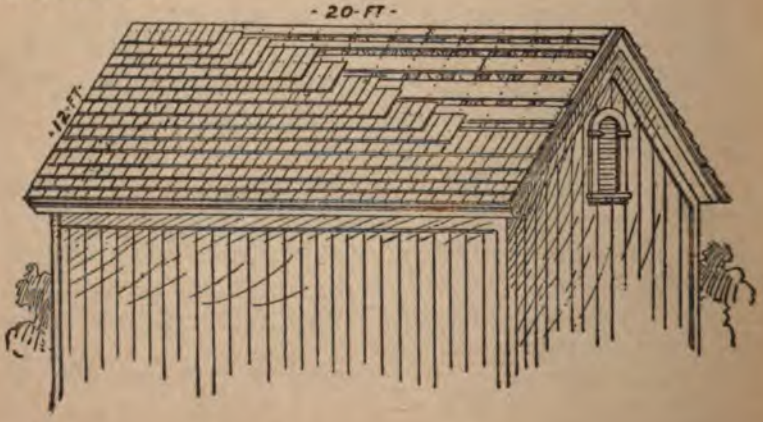
6. Shingles are 16 inches long, are considered 4 inches wide in estimates, and are laid 3 or 4 inches to the weather.

7. 1000 shingles are called a square of shingles, and will lay about 100 square feet, 4 inches to the weather.

8. Lumber dealers will not sell a part of a bunch.

PROBLEM:

The sides of a gable roof are 12 ft. × 20 ft. How many shingles laid 4 in. to the weather are needed to cover it?



Solution:

One side.

How many in a row?

Each shingle is estimated 4 in. wide. There will be 3 shingles to each foot, or 60 shingles in each row.

How many rows?

Each row covers 4 in. of exposed surface. Then there are 3 rows to a foot, or 36 rows on a side.

How many shingles?

If one row contains 60 shingles, 36 rows contain 36 × 60 shingles, or 2160 shingles.

Both sides.

Since there are 2 sides to the roof, it will take 2 × 2160 shingles, or 4320 shingles.

Work:

$3 \times 20 = 60.$

$3 \times 12 \times 60 \times 2 = 4320.$

Another Solution (equally as good and simpler):

1 sq. ft. of roof surface requires 3 shingles in width and 3 in length, or 9 shingles.

On each side of roof there are 12 × 20 sq. ft., or 240 sq. ft. of roof surface.

On both sides there are 2 × 240 sq. ft., or 480 sq. ft. of roof surface.

If we estimate 9 shingles per square foot, it will need 480 × 9 shingles = 4320 shingles.

To estimate the number of shingles required for a roof, when laid 4 in. to the weather,

Multiply the number of square feet of roof surface by 9.

It is easy to see why the above is true. Each shingle is 4 in. wide and 4 *in. only* of its length are left exposed, hence it covers 16 sq. in., or $\frac{1}{9}$ of a square foot. 9 shingles will cover a square foot.

If laid 3 in. to the weather, 12 shingles will be required to cover a square foot, so, in that case, multiply the number of square feet of roof surface by 12.

PROBLEM:

How many shingles are needed for a gable roof 15 ft. by 30 ft., shingles to be laid 4 in. to the weather?

SOLUTION:

2 × 15 × 30 sq. ft. = 900 sq. ft. roof surface.
900 × 9 = 8100 shingles required.

Another way to estimate easily and quickly the number of shingles required for a roof, when laid 4 in. to the weather, is,

Allow 1000 shingles to each 100 square feet of roof surface.

PROBLEM:

How many shingles laid 4 in. to the weather are needed for both sides of a roof 20 ft. by 40 ft.?

SOLUTION:

2 × 20 × 40 sq. ft. = 1600 sq. ft. of roof surface.
16 × 1000 = 16000 = 16 M. shingles.

PROBLEM:

When one bundle of shingles is allowed for 25 sq. ft., how many bundles will be required for a gable roof 22 ft. by 16 ft., and what will be the cost at $2.75 per M.?

SOLUTION:

2 × 22 × 16 sq. ft. = 704 sq. ft. on both sides.

704 ÷ 25 = 28$\frac{4}{25}$, or 29 bundles of shingles required.

250 × 29 = 7250, the number of shingles required, or 7.25 M. shingles.

7.25 × $2.75 = $19.94, cost of shingles.

PAPERING.

TALK:

The one thing to emphasize here is the fact that wall paper is sold in rolls and no part of a roll can be bought, so exact estimates are out of place.

Be sure to buy enough paper, even if you buy several rolls more than are needed, for unbroken rolls can be returned.

Don't go into details about the subject of papering, as it is seldom that the pupil exercises his knowledge of it. The paper-hanger does the estimating and is not very particular about close estimates either. Notice how quickly he estimates how many rolls will be required.

Facts That Should be Known about Wall Paper.

1. Wall paper in this country is $\frac{1}{2}$ yd. wide, and comes in rolls 8 yd. long, or in double rolls, 16 yd. long.
2. It is sold by the roll only.
3. Bordering is sold by the linear yard.
4. Make liberal allowances for waste in matching figures.
5. If the border is wide, the strips need not extend to the ceiling.

The usual way of reckoning amount of paper required,

1. **Measure the distance around the room in feet.**
2. **Deduct the width of doors and windows.**
3. **Divide the difference by $1\frac{1}{2}$, and the quotient will be the number of strips needed.**
4. **Multiply the number of strips by the number of yards in a strip, and the product is the NUMBER OF YARDS NEEDED, approximately.**
5. **Divide the number of yards by 8, and the result is the NUMBER OF SINGLE ROLLS NEEDED.**

For rooms of ordinary height,

1. Find the distance around the room in feet.

2. Deduct width of doors and windows.

3. Divide the difference by 7, and this will give the NUMBER OF DOUBLE ROLLS REQUIRED.

PROBLEM:

A room of ordinary height, 16 ft. by 24 ft., has 3 windows and 2 doors, each 4 ft. wide. How many rolls of paper are needed to paper the sides?

SOLUTION:

Distance around the room = 80 ft.
Width of doors and windows = 20 ft.
After deducting for doors and windows 60 ft.
$60 \div 7 = 8\frac{4}{7}$, or 9 double rolls.

PROBLEM:

How much paper will be required for the ceiling?

SOLUTION:

Each strip is 18 in., or $1\frac{1}{2}$ ft. wide, and as many strips will be needed as $1\frac{1}{2}$ is contained times in 24, or 16.

Each strip is 16 ft., or $5\frac{1}{3}$ yd. long, and since there are 16 strips, there will be needed $16 \times 5\frac{1}{3}$ yd. $= 85\frac{1}{3}$ yd., or 86 yd.

Each single roll contains 8 yd., so there will be as many rolls as 8 is contained times in 86, or $10\frac{3}{4}$.

Hence, 11 single rolls will be required.

PLASTERING AND PAINTING.

Facts That Should be Known about Plastering and Painting.

1. Plastering or painting is usually done by the square yard.

2. Allowance is made for one half the area of the openings.

3. Allowance is made for base-boards and wainscoting.

NOTE.—Ask the pupil why plasterers and painters are unwilling to deduct the total area of openings.

PROBLEM:

Find the cost of plastering the walls and ceiling of a room 30 ft. by 25 ft. and 9 ft. high, at 20¢ per square yard, allowing 300 sq. ft. for openings.

SOLUTION:

The number of square feet on the two sides of the room is 30 × 9 × 2, or 540.

The number of square feet on the two ends is 25 × 9 × 2, or 450.

The number of square feet on the ceiling is 30 × 25, or 750.

The total number of square feet in the room = 540 + 450 + 750, or 1740.

Since 300 sq. ft. are deducted for openings, the actual surface to be plastered is 1740 sq. ft. − 300 sq. ft., or 1440 sq. ft.

1440 sq. ft. = 160 sq. yd.

If 1 square yard cost 20¢, 160 sq. yd. will cost 160 × 20¢, or $32.

CARPETING.

TALK:

Most carpet houses employ men for visiting homes to take measurements for carpets. The carpets are cut, sewed, and matched before they are delivered.

Notwithstanding this, it is valuable for us all to understand something of carpeting.

Facts That Should be Known about Carpeting.

1. Carpeting is usually $\frac{3}{4}$ yd. wide and is sold by the linear yard.

2. Always draw a diagram of the floor or stairs to be covered.

3. The number of yards required depends on which way the strips run — whether lengthwise or across the room. Sometimes by running the strips lengthwise, there is less waste in matching the pattern.

4. The part cut off in matching patterns is charged to the purchaser.

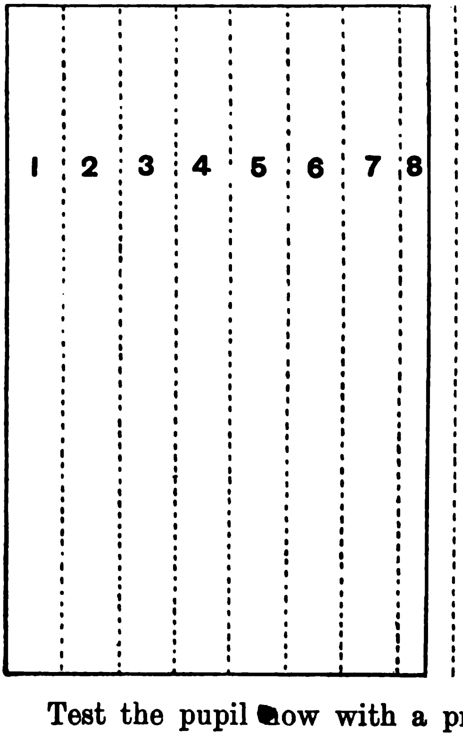
The required number of yards will be,

The number of yards in a strip (including the waste for matching), multiplied by the number of strips.

PROBLEM:

What is the cost of carpeting a room 16 ft. × 24 ft., at 85¢ per yard? The carpet is 2¼ ft. wide and the strips run lengthwise.

Diagram of the Room.



Solution:

16 ÷ 2¼ = 7⅑. Hence, I must buy 8 strips.

24 ÷ 3 = 8, which is the number of yards in a strip.

8 × 8 yd. = 64 yd.

64 yd. will cost 64 × 85¢, or $54.40.

To this must be added the cost of sewing, the laying of the carpet, and the waste in matching the pattern.

Test the pupil now with a problem like this: — Which way is it best to lay a carpet 1 yd. wide on a floor 24 ft. by 17 ft., so that no strip must be cut off, or turned under, at the sides? How many yards will be required, no allowance being made for matching the pattern?

NOTE.—Be sure to draw a diagram.

PROBLEM:

How many yards of carpet must I buy for a room 24 ft. × 30 ft., carpet 27 in. wide, strips to run lengthwise?

SOLUTION:

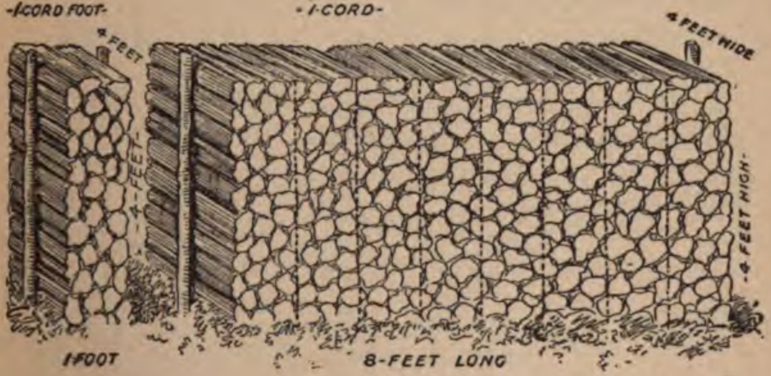
27 in. = $2\frac{1}{4}$ ft.

$24 \div 2\frac{1}{4} = 10\frac{2}{3}$. Hence, I must get 11 strips.

$30 \div 3 = 10$, which is the number of yards in a strip.

11 × 10 yd. = 110 yd.

WOOD BY THE CORD.



This picture shows a pile of wood that is 4 ft. high, 8 ft. long, and made up of sticks 4 ft. long.

Such a pile is called a cord.

To the left is seen a pile of sticks 4 ft. long, 4 ft. high, and 1 ft. wide.

Such a pile is called a cord foot.

A cord and cord foot may be illustrated by placing 8 books of the same size side by side. The whole pile represents a cord and each book a cord foot.

From the above, it is readily seen that a cord contains 8 × 4 × 4 cu. ft., or 128 cubic feet. Hence, a cord foot is $\frac{1}{8}$ of a cord.

Wood is sometimes bought by the load, but in most cases it is bought and sold by the cord. Sticks of wood 4 ft. long constitute what is generally known as cord wood.

Facts That Should be Known about Cord Wood.

1. A pile of wood 8 ft. long, 4 ft. wide. and 4 ft. high makes a cord. 1 cord = 128 cu. ft.
2. A pile of wood 4 ft. high, 4 ft. wide, and 1 ft. long makes a cord ft. 8 cord ft. = 1 cord.

PROBLEM:

A pile of wood is 32 ft. long, 4 ft. high, and 4 ft. wide. How many cords in it?

SOLUTION:

The pile is 1 cord high and 1 cord wide. Since it is 32 ft. long, it is just 4 cords long. Hence, the pile contains 4 cords.

PROBLEM:

A wood pile is 8 ft. high and 40 ft. long. The sticks are 4 ft. long. How many cords in it?

SOLUTION:

Being 8 ft. high, it is 2 cords high. 40 ft. in length equal 5 cords in length. Hence, the pile contains 2 × 5 cords, or 10 cords.

PROBLEM:

How many cords of wood can be piled in a shed 30 ft. long, 20 ft. wide, and 11 ft. high, inside measurements?

Solution:

The number of cubic feet in the shed is 30 × 20 × 11, or 6600.

As many cords can be put into the shed as 128 cu. ft. is contained times in 6600 cu. ft., or $51\frac{9}{16}$. About $51\frac{1}{2}$ cords can be piled in the shed.

Work:

$$\overset{15}{30} \times \overset{5}{20} \times 11 \times \frac{1}{\underset{\underset{16}{32}}{128}} = \frac{825}{16} = 51\frac{9}{16}.$$

PROBLEM:

At $3.75 a cord, what will be the cost of a pile of stone 16 ft. by 5 ft. by 9 ft.?

Solution:

The number of cubic feet in the rock pile is 16 × 5 × 9, or 720.

There are as many cords as 128 cu. ft. is contained times in 720 cu. ft., or $5\frac{5}{8}$.

If one cord costs $3.75, $5\frac{5}{8}$ cords cost $5\frac{5}{8}$ × $3.75, or $21.09.

ESTIMATES ON THE FARM OR RANCH.

I. Finding the Capacity of a Bin in Bushels.

By actual measurements, it has been found that a bushel, dry measure, contains about $1\frac{1}{4}$ cubic feet. This makes it easy to estimate about how many bushels any bin will hold.

To estimate a bin,

1. **Find the number of cubic feet in the bin.**
2. **Divide the number of cubic feet by $1\frac{1}{4}$.**
3. **The result is the number of bushels.**

PROBLEM:

How many bushels will a bin hold, if its inside measurements are, length 20 ft., width 12 ft., depth 8 ft.?

SOLUTION:

The number of cubic feet in the bin is 8 × 12 × 20, or 1920.

If 1 bushel contains $1\frac{1}{4}$ cu. ft., in 1920 cu. ft. there are as many bushels as $1\frac{1}{4}$ is contained times in 1920, or 1536.

WORK:

$$8 \times 12 \times 20 = 1920$$
$$1920 \div 1\tfrac{1}{4} = 1536.$$

The work may be indicated in this way as well:—

$$8 \times 12 \times \overset{4}{20} \times \tfrac{4}{5} = 1536.$$

REMARK:

For bins whose dimensions in feet are even numbers and do not exceed 15, the problems may be done orally. It is not difficult to divide by $1\frac{1}{4}$. It merely means taking $\frac{4}{5}$ of a number.

II. Finding the Capacity of a Tank in Gallons.

Actual measurement shows that about $7\frac{1}{2}$ gallons occupy a cubic foot of space.

Hence, to find the number of gallons which any tank, cistern, or vessel will hold,

1. **Find the number of cubic feet in the vessel.**
2. **Multiply the number of cubic feet by $7\frac{1}{2}$.**

PROBLEM:

I have a tank 6 ft. long, 3 ft. wide, and 3 ft. high. How many gallons will it hold?

SOLUTION:

The number of cubic feet in the tank is 6 × 3 × 3, or 54.

If 1 cubic foot holds $7\frac{1}{2}$ gallons, 54 cubic feet will hold $54 \times 7\frac{1}{2}$ gal., or 405 gal.

WORK:

$$6 \times 3 \times 3 \times 7\tfrac{1}{2} = 405.$$

PROBLEM:

My windmill filled a tank 6 ft. long, 4 ft. wide, and 3 ft. deep, with water in 1 hour. How many gallons did it pump in 1 minute?

Solution:

The number of cubic feet in the tank is 6 × 4 × 3, or 72.

In 72 cubic feet of space there are 72 × 7½ gal., or 540 gal.

Since the windmill pumped 540 gal. in 1 hour, or 60 min., in 1 minute it pumped $\frac{1}{60}$ of 540 gal., or 9 gal.

Work:

$$\overset{3}{6} \times 4 \times 3 \times \frac{15}{2} \times \frac{1}{\underset{4}{60}} = 9$$

III. Finding the Number of Tons of Hay in a Barn or Stack.

1. Divide the number of cubic feet in the barn or stack by 500, if the hay is packed closely.

2. The result is the number of tons.

3. If clover, divide by 550.

PROBLEM:

A barn is 40 ft. by 80 ft., and 15 ft. high. How many tons of hay will it hold?

Solution:

The number of cubic feet in the barn is 40 × 80 × 15, or 48000.

If 500 cu. ft. hold 1 ton, 48000 cu. ft. hold as many tons as 500 cu. ft. is contained times in 48000 cu. ft., or 96.

Work:

$$40 \times 80 \times \overset{3}{15} \times \frac{1}{500} = 96.$$

IV. Finding the Number of Bushels of Corn in a Crib.

To get the number of heaped bushels of corn in the ear in a crib,

1. Multiply the length of the crib in inches by the width in inches.

2. Multiply the product obtained, by the height of the corn in the crib in inches.

3. Divide the result by 2748.

NOTE.—2 heaped bushels will make 1 bushel of shelled corn.

If the crib is flaring at the sides, measure the width at the top of the corn and also at the bottom. Divide their sum by 2, which gives the average width.

PROBLEM:

How much corn in the ear can I put into a crib 12 ft. wide, 20 ft. long, and 10 ft. deep?

SOLUTION:

The number of cubic inches in the crib is 144 × 240 × 120, or 4,147,200.

Since 2748 cu. in. hold 1 bu., 4,147,200 cu. in. hold as many bushels as 2748 is contained times in 4,147,200, or 1509+ bu.

WORK:

$$\frac{144 \times 240 \times 120}{2748} = 1509+.$$

PART SEVEN

PERCENTAGE.

INTEREST. DISCOUNT.
EXCHANGE. INSURANCE.
TAXES AND DUTIES.
STOCKS AND BONDS.

"If the power to do hard work is not a talent, it is the best possible substitute for it."

—James A. Garfield.

PERCENTAGE.

In some books on arithmetic percentage is treated as if it were a special process involving certain distinctive principles, and therefore entitled to rank as a separate department. This elaborate treatment seems to be a mistake on both the theoretical and the practical side. . . . In the growth of number as measurement, percentage presents nothing new. — *The Psychology of Number.*

INTEREST.

The pupil having learned the meaning and use of the term *per cent*, should find very little difficulty in the subject of interest. However, in the problems of interest and kindred commercial work, pupils frequently fail; but if the cause of the failure is examined into, it will nearly always be found to be, not so much an inability to meet the mathematics of the problems as a want of accurate knowledge of the terms used, and of acquaintance with the business forms and operations involved. On this account, in taking up the applications of arithmetic to commercial work, the teacher should be at great pains to ensure that every pupil understands well, and sees clearly through all such forms and operations.

— *The Psychology of Number,*
By McLellan and Dewey.

PERCENTAGE.

TALK:

Percentage, like many other things in arithmetic, is not difficult if the child is given a natural and simple introduction to it. Informally, it may be connected with the child's work in fractions in such a way that at no stage of the study will he fear it and call it "hard."

The manner of presenting given here is one way. The original teacher may have as good, or even a better way.

DEVELOPING THE IDEA.

Present in this way:—If anything is divided into a hundred parts, each part is called *one hundredth*. Three parts are called three hundredths; four parts are called ——? six parts ——? twenty parts ——? How many hundredths in all of a thing?

(Illustrate all of the above, if the child's previous work does not enable him to grasp it readily.)

Well, we are going to give *all of a thing* a new name and call it *100 per cent.*

1 hundredth of it, we will call 1 per cent.
4 hundredths of it, we will call 4 per cent.
10 hundredths of it, we will call 10 per cent, etc.
25 hundredths are —— per cent?
50 hundredths are —— per cent?

Remember:

PER CENT means HUNDREDTHS.
100 per cent of a number is all of it.

Since per cent means hundredths,

50 per cent of a number is $\frac{50}{100}$ of it. $\frac{50}{100} = \frac{1}{2}$
25 per cent of a number is $\frac{25}{100}$ of it. $\frac{25}{100} = \frac{1}{4}$
75 per cent of a number is $\frac{75}{100}$ of it. $\frac{75}{100} = \frac{3}{4}$
10 per cent of a number is $\frac{10}{100}$ of it. $\frac{10}{100} = \frac{1}{10}$

So any per cent of a number may be written as hundredths and often can be reduced to a fraction with a small denominator.

Since per cent means hundredths, we may write any fraction whose denominator is 100 as so many *per cent*, as,

$\frac{6}{100}$ = 6 per cent.
$\frac{10}{100}$ = 10 per cent.
$\frac{17}{100}$ = 17 per cent.
$\frac{25}{100}$ = 25 per cent.
$\frac{50}{100}$ = 50 per cent.
$\frac{66\frac{2}{3}}{100} = 66\frac{2}{3}$ per cent.

To the Pupil:

Write other fractions whose denominators are 100 as per cent.

THE SIGN %.

When we wish to write *per cent* quickly, we use this sign, %, which is always read "*per cent.*" 7% is read "*7 per cent.*" 10% is read "*10 per cent.*"

We will use this sign hereafter in most cases.

TO REMOVE THE SIGN %.

Remember:

To write a rate per cent as a common fraction,

1. Write the number of per cent for the numerator and 100 for the denominator.

2. Reduce the fraction to lowest terms.

Observe this illustrated in the table that follows:

Table of Values.

The pupil knows how to write any fraction, whose denominator is 10 or a multiple of 10, as a decimal fraction. Consequently, he has no trouble with table:—

SYMBOL.		DECIMAL.		COMMON FRACTION.
1%	=	.01	=	$\frac{1}{100}$
2%	=	.02	=	$\frac{2}{100} = \frac{1}{50}$
3%	=	.03	=	$\frac{3}{100}$
4%	=	.04	=	$\frac{4}{100} = \frac{1}{25}$
5%	=	.05	=	$\frac{5}{100} = \frac{1}{20}$
6%	=	.06	=	$\frac{6}{100} = \frac{3}{50}$
7%	=	.07	=	$\frac{7}{100}$
8%	=	.08	=	$\frac{8}{100} = \frac{2}{25}$
9%	=	.09	=	$\frac{9}{100}$
10%	=	.10	=	$\frac{10}{100} = \frac{1}{10}$
20%	=	.20	=	$\frac{20}{100} = \frac{1}{5}$
25%	=	.25	=	$\frac{25}{100} = \frac{1}{4}$
50%	=	.50	=	$\frac{50}{100} = \frac{1}{2}$
100%	=	1.0	=	$\frac{100}{100} = 1$

The pupil should know the above thoroughly. Here are a few others that should be learned:—

$6\frac{1}{4}\% = \frac{1}{16}$ of 100%.	$16\frac{2}{3}\% = \frac{1}{6}$ of 100%.
$8\frac{1}{3}\% = \frac{1}{12}$ of 100%.	$33\frac{1}{3}\% = \frac{1}{3}$ of 100%.
$12\frac{1}{2}\% = \frac{1}{8}$ of 100%.	$66\frac{2}{3}\% = \frac{2}{3}$ of 100%.

To write a decimal as per cent:—

This is very simple. Write the decimal as hundredths, and the number expressing the number of hundredths is the per cent.

EXAMPLES.

$$.4 = .40 = \frac{40}{100} = 40\%.$$

$$.8 = .80 = \frac{80}{100} = 80\%.$$

$$.25 = \frac{25}{100} = 25\%.$$

$$.33\tfrac{1}{3} = \frac{33\frac{1}{3}}{100} = 33\tfrac{1}{3}\%.$$

$$.50 = \frac{50}{100} = 50\%.$$

$$.87\tfrac{1}{2} = \frac{87\frac{1}{2}}{100} = 87\tfrac{1}{2}\%.$$

If the decimal has more than two decimal places, the figures after the second one are written as a fraction of a per cent, as,—

$$.255 = \frac{25\frac{1}{2}}{100} = 25\tfrac{1}{2}\%$$

$$.163 = \frac{16\frac{3}{10}}{100} = 16\tfrac{3}{10}\%.$$

To write a common fraction as per cent:—

The pupil knows how to reduce a common fraction to a decimal. Then

To change a common fraction to per cent,

1. **Change the fraction to a decimal.**
2. **Express the decimal as hundredths.**
3. **The result is the per cent desired.**

EXAMPLES.

$\frac{1}{2} = .5 = .50 = 50\%.$

$\frac{3}{4} = .75 = 75\%.$

$\frac{2}{3} = .66\frac{2}{3} = 66\frac{2}{3}\%.$

$\frac{9}{10} = .90 = 90\%.$

$\frac{8}{9} = .88\frac{8}{9} = 88\frac{8}{9}\%.$

$\frac{7}{8} = .87\frac{1}{2} = 87\frac{1}{2}\%.$

$\frac{25}{26} = .96\frac{2}{13} = 96\frac{2}{13}\%.$

The pupil may write them this way also:—

$$\tfrac{3}{4} = \tfrac{3}{4} \text{ of } \frac{100}{100} = \frac{75}{100} = 75\%.$$

$$\tfrac{2}{3} = \tfrac{2}{3} \text{ of } \frac{100}{100} = \frac{66\frac{2}{3}}{100} = 66\tfrac{2}{3}\%.$$

$$\tfrac{1}{2} = \tfrac{1}{2} \text{ of } \frac{100}{100} = \frac{50}{100} = 50\%.$$

TERMS USED.

The **base** is the number of which a certain per cent is taken.

The **rate** is the proportional part of 100 per cent of the base that is taken.

The **rate per cent** is a term used in many arithmetics, but is never met in actual business life. The distinction between it and rate is *not* important, and much confusion will be avoided by refusing to attempt it with school pupils.

The **percentage** of a number is the result of finding a certain per cent of it. 7% of 200 is 14. 14 is the percentage.

To the Teacher:

In all work in percentage, remember that—

The shorter the method, the better it is.

CLASSES OF PERCENTAGE PROBLEMS.

There are three general classes of problems in percentage.

1. To find a percentage of a number.

This embraces problems where a given number of % of a number, more or less than the number itself, is to be found.

2. To find a number, when a percentage of the number and the rate are given.

3. To find the rate, when the number and a percentage of it are given.

This includes finding how many % one number is of another, how many % it is less than another, or how many % it is more than another.

1. To find a percentage of a number:—

This is sometimes written in the form: *Rate and base given, to find percentage.*

PROBLEM:

Find 10% of 330.

1st Way:	2d Way:
$10\% = \frac{1}{10}$	$10\% = .10$
$\frac{1}{10}$ of $330 = 33$	$.10 \times 330 = 33$

PROBLEM:

Find 25% less than $60.

1st Way:	2d Way:
$25\% = \frac{1}{4}$	$100\% - 25\% = 75\%$
$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$	$75\% = .75$
$\frac{3}{4}$ of \$60 = \$45	$.75 \times \$60 = \$45$

PROBLEM:

What is 50% more than 30 days?

1st Way:	2d Way:
$50\% = \frac{1}{2}$	$100\% + 50\% = 150\%$
$\frac{2}{2} + \frac{1}{2} = \frac{3}{2}$	$150\% = 1.5$
$\frac{3}{2}$ of 30 days = 45 days	1.5×30 days = 45 days

PROBLEM:

I sold my farm at a gain of 20%. It cost me $800. What did it sell for?

1st Way:	2d Way:
$20\% = \frac{1}{5}$	$20\% = .20$
$\frac{1}{5}$ of \$800 = \$160 gain.	$.20 \times \$800 = \$160$
\$800 cost + \$160 gain = \$960, selling price.	\$800 cost + \$160 gain = \$960, selling price.

PROBLEM:

Washing 80 yd. of flannel causes it to shrink 25%. How many yd. does it measure after washing?

1st Way:	2d Way:
$25\% = \frac{1}{4}$	$25\% = .25$
$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$	$1 - .25 = .75$
$\frac{3}{4}$ of 80 yd. = 60 yd.	$.75 \times 80$ yd. = 60 yd.

PROBLEM:

Last year I had 60 sheep. Now I have $33\frac{1}{3}\%$ more. How many sheep have I now?

1st Way:	2d Way:
$33\frac{1}{3}\% = \frac{1}{3}$	$33\frac{1}{3}\% = .33\frac{1}{3}$
$\frac{3}{3} + \frac{1}{3} = \frac{4}{3}$	$1 + .33\frac{1}{3} = 1.33\frac{1}{3}$
$\frac{4}{3}$ of 60 sheep = 80 sheep.	$1.33\frac{1}{3} \times 60$ sheep = 80 sheep.

NOTE.—In such problems, the common fraction method is to be preferred.

2. To find a number, when the rate and a
centage of it are given:—

Sometimes written: *Rate per cent and percen*
given, to find base.

PROBLEM:

25% of a number is 50. What is the numbe

1st Way:

25% of the number = $\frac{1}{4}$ of the number.

$\frac{1}{4}$ of the No. = 50.

$\frac{4}{4}$ of the No. = 4 × 50, or 200.

2d Way:

25% of the No. =
of it.

.25 of the No. =

$\frac{1}{100}$ of the No. =
of 50, or 2.

$\frac{100}{100}$ of the No. = 100
2, or 200.

PROBLEM:

\$60 is 40% less than what sum of money?

1st Way:

100% of the No. = the number.

100% of the No. − 40% of the No. = 60% of the No.

60% of the No. = \$60.

1% of the No. = $\frac{1}{60}$ of \$60, or \$1.

100% of the No. = 100 × \$1, or \$100.

2d Way:

40% of No. = $\frac{2}{5}$ of No.

$\frac{5}{5}$ of No. − $\frac{2}{5}$ of No. = $\frac{3}{5}$ of No.

$\frac{3}{5}$ of No. = \$60.

$\frac{1}{5}$ of No. = $\frac{1}{3}$ of \$60, or \$20.

$\frac{5}{5}$ of No. = 5 × \$20, or \$100.

PROBLEM:

240 bu. are 20% more than what No. of bu.?

1st Way:

100% of No. of bu. + 20% of No. of bu. = 120% of No. of bu.

120% of No. of bu. = 240 bu.

1% of No. of bu. = $\frac{1}{120}$ of 240 bu., or 2 bu.
100% of No. of bu. = 100 × 2 bu., or 200 bu.

2D WAY:

20% of No. of bu. = $\frac{1}{5}$ of No. of bu.
$\frac{5}{5}$ of No. of bu. + $\frac{1}{5}$ of No. of bu. = $\frac{6}{5}$ of No. of bu.
$\frac{6}{5}$ of No. of bu. = 240 bu.
$\frac{1}{5}$ of No. of bu. = $\frac{1}{6}$ of 240 bu., or 40 bu.
$\frac{5}{5}$ of No. of bu. = 5 × 40 bu., or 200 bu.

NOTE.—Use the way of solution most convenient

PROBLEM:

I sold a consignment of goods for $480 and gained 60%. What was the cost?

SOLUTION:

100% of cost + 60% of cost = 160% of cost.
160% of cost = $\frac{16}{10}$ of cost = $\frac{8}{5}$ of cost.
$\frac{8}{5}$ of cost = $480.
$\frac{1}{5}$ of cost = $\frac{1}{8}$ of $480, or $60.
$\frac{5}{5}$ of cost = 5 × $60, or $300.

PROBLEM:

My agent sold goods for $90, and by so doing, lost 40% on them. What was their cost?

SOLUTION:

100% of cost − 40% of cost = 60% of cost.
60% of cost = $\frac{3}{5}$ of cost.
$\frac{3}{5}$ of cost = $90.
$\frac{1}{5}$ of cost = $\frac{1}{3}$ of $90, or $30.
$\frac{5}{5}$ of cost = 5 × $30, or $150.

PROBLEM:

Twenty per cent of my land is under timber. There are 160 acres under timber. How much land have I?

SOLUTION:

20% of my land = $\frac{1}{5}$ of my land.
$\frac{1}{5}$ of my land = 160 acres.
$\frac{5}{5}$ of my land = 5 × 160 acres, or 800 acres.

PROBLEM:

34.5% of an army are 2500 men. How many men in the army?

SOLUTION:

$$1\% \text{ of army} = \frac{1}{34.5} \text{ of } 2500 \text{ men.}$$

$$100\% \text{ of army} = 100 \times \frac{1}{34.5} \text{ of } 2500 \text{ men, or } 7246 \text{ men.}$$

NOTE.— Where the per cent cannot be reduced easily to a common fraction, use in the way here indicated.

3. To find the rate, when the number and a percentage of it are given:—

Sometimes written: *Base and percentage given, to find rate per cent.*

PROBLEM:

3 is what % of 4?

SOLUTION:

$4 = 100\%$ of itself.
3 is $\frac{3}{4}$ of 4.
$\frac{3}{4}$ of $100\% = 75\%$.

PROBLEM:

6 is what % of 5?

SOLUTION:

$5 = 100\%$ of itself.
$6 = \frac{6}{5}$ of 5.
$\frac{6}{5}$ of $100\% = 120\%$.

PROBLEM:

25 is how many % less than 30?

SOLUTION:

$30 = 100\%$ of itself
$25 = \frac{25}{30}$, or $\frac{5}{6}$ of 100%
$\frac{5}{6}$ of $100\% = 83\frac{1}{3}\%$
$100\% - 83\frac{1}{3}\% = 16\frac{2}{3}\%$

PROBLEM:

My neighbor has a farm of 600 acres. I have 1000 acres. His is what per cent of mine?

SOLUTION:

1000 acres = 100% of mine.

$$600 \text{ acres} = \frac{600}{1000}, \text{ or } \frac{3}{5} \text{ of } 100\% \text{ of mine.}$$

$\frac{3}{5}$ of 100% of mine = 60% of mine.

Mine is what % of his?

Solution:

$600\text{ A.} = 100\%\text{ of his.}$

$1000\text{ A.} = \frac{1000}{600}\text{, or }\frac{5}{3}\text{ of his.}$

$\frac{5}{3}\text{ of }100\%\text{ of his} = 166\frac{2}{3}\%\text{ of his.}$

VARIOUS PROBLEMS IN PERCENTAGE.

PROBLEM:

I sold land which cost me $10 an acre at $14 an acre. What was my per cent of gain?

Solution:

$\$14 - \$10 = \$4\text{, gain.}$

$\$4 = \frac{4}{10}\text{, or }\frac{2}{5}\text{ of }\$10.$

$\frac{2}{5}\text{ of }100\% = 40\%.$

PROBLEM:

If I am sent $14 for collecting a debt of $350, what % commission am I paid?

Solution:

$\$14 = \frac{14}{350}\text{ or }\frac{7}{175}\text{ of }\$350.$

$\frac{7}{175}\text{ of }100\% = 4\%.$

PROBLEM:

I bought a sewing machine for $40. After using it a year, I sold it for $32. What per cent did I lose?

Solution No. 1:

I lost $8.

$\$8 = \frac{8}{40}\text{ of cost, or }\frac{1}{5}\text{ of }100\%\text{ of cost.}$

$\frac{1}{5}\text{ of }100\%\text{ of cost} = 20\%\text{ of cost.}$

Solution No. 2.

$\$40 = 100\%\text{ of cost.}$

$\$32 = \frac{32}{40}\text{, or }\frac{4}{5}\text{ of }\$40.$

$\frac{4}{5}\text{ of }100\%\text{ of cost} = 80\%\text{ of cost.}$

$100\%\text{ of cost} - 80\%\text{ of cost} = 20\%\text{ of cost.}$

PROBLEM:

A piece of cloth 40 yards long shrinks 12% when wet. How long is it after shrinking?

SOLUTION No. 1:

100% of length − 12% of length = 88% of length.
88% of 40 yd. = 35.2 yd., length after shrinkage.

SOLUTION No. 2:

12% = .12.
.12 of 40 yd. = 4.8 yd.
40 yd. − 4.8 yd. = 35.2 yd., the length after shrinkage.

PROBLEM:

After a fire, goods were sold at a discount of 15%. What do I pay for a suit that was marked $30 before the fire?

SOLUTION:

100% − 15% = 85%
85% = .85
I pay .85 × $30 = $25.50.

PROBLEM:

A city with a population of 502000 gains 214033 in ten years. What is the per cent of gain?

SOLUTION:

$$214033 = \frac{214033}{502000} \text{ of } 100\% = 42.63+\%.$$

PROBLEM:

In 1880 the population of Chicago was 503304. In 1890 it was 1099850. What was the per cent of increase?

SOLUTION:

The actual increase was 1099850 − 503304, or 596546.
Query? 596546 is what % of 503304?

$$596546 = \frac{596546}{503304} \text{ of } 100\% = 118.5+\%.$$

118.5% − 100% = 18.5%, increase.

PROBLEM:

An agent, after taking out his commission of 5%, sent his principal $190. What did he collect in all?

SOLUTION:

100% of collection − 5% of collection = 95% of collection.
95% of collection = $190.
100% of collection = $\frac{100}{95}$ of $190 = $200.

WORK:

$$100\% - 5\% = 95\%.$$
$$\$190 \times \frac{100}{95} = \$200.$$

PROBLEM:

I sold my piano at an advance of 20% and received $480 for it. What was the cost of it?

SOLUTION:

100% of cost + 20% of cost = 120% of cost.
120% of cost = $480.
100% of cost = $\frac{100}{120}$ of $480, or $400.

PROBLEM:

In a land deal I made 40% clear on an investment of $6000. I invested 30% of my gain in a farm. What did my farm cost?

SOLUTION:

40% of $6000 = $2400, gain on investment.
30% of $2400 = $720, amt. invested in farm.

NOTE.—Problems like the following are found in arithmetics. They are not practical, but possess some disciplinary value :—

PROBLEM:

75% of Jollif's money equals 30% of Mark's. How many per cent of Jollif's is all of Mark's?

Solution:

30% of Mark's = 75% of Jollif's.

1% of Mark's = $\frac{1}{30}$ of 75% of Jollif's = $\frac{75}{30}$% of Jollif's.

100% of Mark's = 100 × $\frac{75}{30}$% of Jollif's = 250% of Jollif's.

.˙. Mark's money = 250% of Jollif's.

PROBLEM:

15% of $60 is what per cent of 20% of $50?

Solution:

15% of $60 = .15 × $60, or $9.
20% of $50 = .20 × $50, or $10.
Query? $9 is what % of $10?
$9 = $\frac{9}{10}$ of $10.
$\frac{9}{10}$ of 100% = 90%.
.˙. 15% of $60 is 90% of 20% of $50.

PROBLEM:

A man and his daughter and son have $500. The daughter has 30% less than the father, and the son has 20% less than the father. How much has each?

Solution:

The man has 100% of his own.

The daughter has 30% less, or 70% of man's.

The son has 20% less, or 80% of man's.

They all have 100% of man's + 70% of man's + 80% of man's, or 250% of man's.

250% of man's = $500.

1% of man's = $\frac{1}{250}$ of $500 = $2.

100% of man's = 100 × $2 = $200.

70% of man's, or the daughter's = 70 × $2 = $140.

80% of man's or the son's = 80 × $2 = $160.

FOR QUICK PERCENTAGE RESULTS.

TALK:

The pupil may become familiar with the processes and analysis of percentage problems by the use of very small numbers. Much of the work may be oral. Let him make many problems with small numbers and give solutions to them.

Anyone who has much computation to do in percentage should make use of the rules which may be deduced from the work already done. To aid in this, there follows a summary of rules with the work on one or more problems under each.

1. To find a percentage of a number.

Remember:

Multiply the number by the rate in the form of a decimal or common fraction.

PROBLEM:

Find 20% of $50.

WORK:

$ 50
.20
$10.00

PROBLEM:

I pay a man 8% to collect a debt of $400. What is his commission?

WORK:

$400
.08
$32.00

PROBLEM:

A merchant told me that 60% of a shipment of 4250 bbls. flour were spoiled. How many barrels were spoiled?

WORK:

4250 bbls.
.60
2550.$\cancel{0}\cancel{0}$ bbls.

2. To find a number, when the rate and a percentage of the number are known.

Remember:

Divide the percentage by the rate and multiply by 100.

PROBLEM:

7% of a number is 21. What is the number?

Work:

$$\begin{array}{r} 7\,\underline{)\,21} \\ 3 \times 100 = 300. \end{array}$$

Note.—The pupil at once sees that the quotient is 1% of the number, and 100% will be 100 times the quotient.

PROBLEM:

I invested $200 and found it was 16% of my money. How much had I?

Work:

$$\begin{array}{lll} 16\,)\,\$200.0\,(& \$12.5 \\ \quad\ \ \underline{16} & \underline{\quad 100} \\ \quad\ \ \ 40 & \$1250. \\ \quad\ \ \ \underline{32} \\ \quad\ \ \ \ 80 \\ \quad\ \ \ \ \underline{80} \end{array}$$

3. To find the rate, when the number and a percentage of it are given.

Remember:

Divide the percentage by the number and reduce to hundredths.

PROBLEM:

3 is what per cent of 4?

Work:

$3 \div 4 = .75 = 75\%.$

PROBLEM:

260 is what per cent of 5330?

Work:

$260 \div 5330 = .048+ = 4\frac{4}{5}+\%.$

PROBLEM:

14 is what per cent of 10?

Work:

$14 \div 10 = 1.40 = 140\%.$

PROFIT AND LOSS.

This is sometimes spoken of **as gain and loss.**

The cost is taken as the basis for reckoning gain or loss.

The **profit** or **gain** is what the selling price exceeds the buying price.

The **loss** is what the selling price is less than the cost.

NOTE.—That which costs nothing and is sold for money cannot be classed here.

PROBLEMS.

PROBLEM:

I buy wheat at 60¢ and sell it for 75¢. What per cent do I gain?

SOLUTION:

I gain the difference between 75¢ and 60¢, or 15¢. 15¢ is 25% of the cost. Hence, I gain 25%.

WORK:

75¢ − 60¢ = 15¢.
15¢ ÷ 60¢ = .25, or 25%.

PROBLEM:

I bought flour at $3.50 per barrel. For what must I sell it to gain 20%?

SOLUTION:

I must sell it for 100% of the cost plus 20% of the cost, or 120% of the cost.

120% of $3.50 = $4.20.
∴ I must sell it at $4.20.

PROBLEM:

I bought a 54 gal. barrel of syrup at 9¢ a gal. 10 gal. were wasted. At what must I sell the balance to gain 30%?

SOLUTION:

The cost was 54 × 9¢ = $4.86.

The selling price is 100% of the cost + 30% of the cost = 130% of the cost.

130% of $4.86 = $6.32.

Syrup on hand is 54 gal. − 10 gal. = 44 gal.

The selling price of 1 gal. = $\frac{1}{44}$ of $6.32, or $14\frac{1}{3}$¢.

NOTE.—By selling at $14\frac{1}{3}$¢, my profit would be slightly more than 25%; but for practical purposes, $14\frac{1}{3}$¢ a gal. is correct.

PROBLEM:

A manufacturer of tables makes one kind at a cost of $2.00. He sells them at a profit of 15% to the wholesale dealer. The wholesaler sells at a profit of 20% to the retailer. The retailer sells the table to his customer at a profit of 10%. What does the table cost the customer?

SOLUTION:

The manufacturer sells it for 115% of $2.00, or $2.30.

The wholesaler sells it for 120% of $2.30, or $2.76.

The retailer sells it for 110% of $2.76, or $3.04.

∴ the retailer's customer pays $3.04.

NOTE.—The *cost* to each party is the base on which he reckons his selling price.

PROBLEM:

I sold my carriage for 80% of its cost and received $90 for it. What was the cost?

SOLUTION:

1% of the cost is $\frac{1}{80}$ of $90, or $1.125.

100% of the cost = 100 × $1.125, or $112.50.

PROBLEM:

A merchant bought 90 pieces of cloth at an average cost of $3. Damage by water compelled him to sell the whole at a loss of $27. The selling price was what per cent of the cost?

SOLUTION:

The cost = 90 × \$3, or \$270.
The selling price was \$270 − \$27, or \$243.
\$270 = 100% of cost.
\$1 = $\frac{1}{270}$ of 100% of cost, or $\frac{10}{27}$% of cost.
\$243 = 243 × $\frac{10}{27}$% of cost, or 90% of cost.

NOTE.—This may be given in the way here: "The selling price is as many per cent of the cost as \$243 ÷ \$270, or 90."

PROBLEM:

A jeweler sold two watches at \$20 each. On one he lost 25% and on the other he gained 25%. What did each cost?

SOLUTION:

On the first 75% of cost = \$20.
The cost was $\frac{4}{3}$ of \$20, or \$26$\frac{2}{3}$.
On the second 125% of cost = \$20.
The cost was $\frac{4}{5}$ of \$20, or \$16.

COMMISSION.

Sometimes one man sells goods owned by another, and for his work receives a part of what the goods bring. In such cases, he is said to sell on commission.

Commission is the sum paid an agent for transacting or doing business. If goods are bought, commission is a percentage of the cost of the goods. If goods are sold, commission is a percentage of the money received for the goods. If a collection is made, commission is a percentage of the amount collected.

A **commission merchant** is one who sells goods for another on commission. He is sometimes called a **factor**. He usually has temporary possession of the goods, over which he is bound to exercise a reasonable amount of care while they are in his possession.

An **agent** is a person authorized to transact business for another, who is called the **principal.**

A **broker** negotiates sales of property, but, as a rule, never has possession of it.

A **collector** is one employed to collect debts.

All of the people named here, except a principal, work, in most cases, on commission. After they have been paid, what is left is called the **net proceeds.**

NOTE.—Children should understand clearly all terms connected with this work. Most difficulties in such subjects arise from inadequate understanding of terms used.

Principles:

1. The commission is some number of per cent of the price of what is bought or sold.

2. The proceeds equal the selling price minus the commission.

3. The amount equals the selling price plus the commission.

Commission presents two classes of problems. One of these classes may be called "*buying problems.*" The other may be called "*selling problems.*"

BUYING PROBLEMS.

PROBLEM:

I sent my agent $1977.60 to buy wild farm lands in northern Wisconsin, at $3 per acre. He was to receive 3% for his work. How many acres did he buy?

NOTE.—The point to be made clear to the pupil here is that each time the agent paid out $1 for the land, he took out 3% of that amount, or 3¢, for himself. For each acre he took out $3 to pay for the land and 9¢ as his commission, or in all $3.09 for each acre.

Work and Explanation

3% of $3 = $.09.

Cost to me of 1 acre is $3 + $.09 = $3.09.

For $1977.60, he buys as many acres as $3.09 is contained times in $1977.60, or 640. Hence, he buys 640 acres.

PROBLEM:

How many barrels of flour can an agent buy with $828.75, if flour is worth $3.25 a barrel and his commission is 2%?

Work and Explanation:

Commission on each barrel is 2% of $3 25 = $.065.

Cost of each barrel to principal is $3.25 + $.065 = $3.315.

Agent buys as many barrels as $3.315 is contained times in $828.75, or 250.

SELLING PROBLEMS.

PROBLEM:

I sell 40 cows at $40 each on a commission of 10%. What is my commission? What does my principal receive?

Work and Explanation:

Cows sell for 40 × $40, or $1600.

My commission = 10% of $1600, or $160.

Principal receives $1600 − $160, or $1440.

PROBLEM:

A real estate agent sold land so that his commission at 5% was $140. What did the land sell for? What did the principal receive?

Work and Explanation:

5% of selling price = $140.

1% of selling price = $\frac{1}{5}$ of $140, or $28.

100% of selling price = 100 × $28, or $2800.

Principal received $2800 − $140, or $2660.

PROBLEM:

My agent sells 360 lb. of butter for me at 20¢. He pays $4.20 freight charges and $9.60 for storage. His commission is 5%. What does he send me?

WORK AND EXPLANATION:

360 lb. @ 20¢ = $72.00
Freight is $4.20
Storage is 9.60
Commission is 5% of $72, or 3.60
Total charges = $17.40
He sends me the difference, or $54.60

PROBLEM:

A real estate agent wrote his principal: "I have sold 200 acres of your land at 30% advance on its cost." What did he get for it, if the cost was $10 per acre?

WORK AND EXPLANATION:

The cost was 200 × $10 = $2000
Gain was 30% of $2000 = $600
Selling price was $2600.

OR

Selling price = 130% of cost, or $2600.

NOTE.— In this problem you may find the selling price of one acre, and work to the whole from that, as,

Selling price of 1 A. = 100% of $10 + 30% of $10, or 130% of $10.

130% of $10 = $13.00.

Selling price of 200 A. = 200 × $13, or $2600.

PROBLEM:

If, in the preceding problem, the agent receives a commission of 5% on the first $1000 and 3% on the rest of the sale, what would his whole commission be?

WORK AND EXPLANATION:

He receives 5% commission on $1000 =	$50
He receives 3% commission on $1600 =	$48
His total commission is	$98

TRADE DISCOUNT.

This is sometimes called commercial discount. When the large dealers send out prices to the smaller business houses, they usually quote their retail prices. Then they offer the small dealer a net price by giving so many per cent off, as "50% off," or, "20 and 10 off."

The prices sent in the first way are called **list prices.** After the discount has been taken out, the amount left is called the **net price.**

From this, we may say that **trade discount** is a reduction from the list price of an article, from a bill of goods, or from a debt.

The amount of the discount sometimes depends on the size of the order, sometimes on the terms of payment.

Two or more discounts are often quoted. The first is a discount off the list price; the second is a discount on what is left when the first discount has been made; the third is a discount on what is left, etc.

PROBLEMS.

PROBLEM:

I receive a bill of goods amounting to $100, 20% off. What is the net cost?

1ST WAY:	2D WAY:
20% of $100 = $20	100% − 20% = 80%
$100 − $20 = $80	80% of $100 = $80

PROBLEM:

I bought a watch for $150. The net price on the bill was "20 and 10" off. What did I pay cash?

1st Way:	2d Way:
20% of $150 = $30	100% − 20% = 80%
$150 − $30 = $120	80% of $150 = $120
10% of $120 = $12	100% − 10% = 90%
$120 − $12 = $108	90% of $120 = $108

Note.—Observe that where two or more discounts are given, the base changes with each successive discount. For that reason, the discounts cannot be added and taken out at once. Each must be taken by itself.

PROBLEM:

A thresher is sent to a farmer with a bill quoting "$\frac{1}{3}$ and 10% off" for cash. What should he pay, if the bill calls for $1500?

1st Way:	2d Way:
$\frac{1}{3}$ of $1500 = $500.	$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$.
$1500 − $500 = $1000.	$\frac{2}{3}$ of $1500 = $1000.
10% of $1000 = $100.	100% − 10% = 90%.
$1000 − $100 = $900.	90% of $1000 = $900.

Note.—Any bright pupil will be able to work such problems orally. The actual work for such a problem may be written like this:—

$$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}.$$

$$\begin{array}{lr} & \tfrac{2}{3} \text{ of } \$1500 = \$1000 \\ 100\% - 10\% = 90\% & \underline{\times .90} \\ & \$900.00 \end{array}$$

The Wanamaker Method.

This plan is followed in the Wanamaker stores. It is most convenient and practical. The plan is to find what per cent of the whole will be left when the discounts are taken out, and from that to find the actual amount.

PROBLEM:

A bill for $237.43 comes to me marked "20 and 15 off." What is the net bill?

Work and Explanation:

$$\begin{array}{l} 100\% - 20\% = 80\%. \\ 100\% - 15\% = 85\%. \\ 85\% \text{ of } 80\% = 68\%. \\ 68\% \text{ of } \$237.43 = \$161.45. \end{array}$$

NOTE.—The odd number of cents in the bill might necessitate much writing of figures in making the computation in the ordinary way. By the Wanamaker plan, that is not necessary.

PROBLEM:

An automobile was listed at $800 with discounts of 20, 10, and 5 off. What is the net price?

Work and Explanation:

On the same plan as in the preceding problem:—

$$\begin{array}{rrr} 100\% & 100\% & 100\% \\ -\ 20\% & -\ 10\% & -\ 5\% \\ \hline 80\% & 90\% & 95\% \end{array}$$

$$\begin{array}{l} .80 \times .90 \times .95 = .684 \\ .684 \text{ of } \$800 = \$547.20. \end{array}$$

PROBLEM:

A merchant receives two bills of $200 each. On one there is a discount of 25%; on the other, 15% and 10%. What must he pay on each, net?

1st Bill:

$$\begin{array}{l} 100\% - 25\% = 75\%, \text{ or } \frac{3}{4} \\ \frac{3}{4} \text{ of } \$200 = \$150. \end{array}$$

2d Bill:

$$\begin{array}{l} 100\% - 15\% = 85\%. \\ 100\% - 10\% = 90\ \% \\ 90\% \text{ of } 85\% = 76.5\%. \\ .765 \times \$200 = \$153. \end{array}$$

INTEREST.

TALK:

Thus far in percentage, the pupil has not considered time in his calculations. In the study of interest, he will find that it becomes an important factor.

He will also be confronted by new expressions and terms. Do not at any time proceed with the work unless the child knows thoroughly the meaning of every term used. It has been proved time and again that where there has been trouble with interest, it was due in most cases to a lack of knowledge of the language used, or of business forms involved.

Different conditions call for certain ways of reckoning interest, so our text-books of to-day give a number of methods. Many of them are good but instead of teaching several, it is generally best to adopt and follow only one.

Business men generally use what is known as the 60-day method. The 6-per cent method is much used. But rather than teach three or four methods, teach but one.

Present the subject of interest this way: No man has everything he wants. He must buy or borrow of others. If he borrows, he pays for the use of the other's property.

If a horse is borrowed, the money paid for its use is called *hire;* if a house is used, *rent;* and if money, **interest.**

Suppose a man pays 6¢ a year on each of $100 for the use of the money. The interest will be $6. The principal is $100.

Remember:

INTEREST is money paid for the use of money.

The PRINCIPAL is the money for which the interest is paid.

At the end of the year, if the total is paid, the man who lends the money will receive principal plus interest, or $106. This total is called the **Amount.**

Because the man who borrows the money pays 6¢ on a dollar for a year's use, he is said to pay 6 per cent (6%) of the principal.

Remember:

The AMOUNT is the total of the principal and the interest.

The RATE of interest is the per cent of the principal paid for the use of money for a given time.

COMMON INTEREST METHOD.

The rate of interest in all the problems is "per annum," or by the year, so the time is reckoned in years and parts of years.

Interest at 5% means 5% of the principal for 1 year.

If we were to find the interest on a sum of money for 3 yr. 4 mo. 5 da., we would find the interest for 1 yr., then for 1 mo. ($\frac{1}{12}$ of a year), then for 1 da. ($\frac{1}{360}$ of a year). Having the interest for 1 yr. 1 mo. 1 da., it is a simple matter of multiplication to get it for 3 yr. 4 mo. 5 da.

PROBLEM:

What is the interest on $520 for 1 yr. 3 mo. at 6%?

WORK:

1 yr. 3 mo. = 1¼ yr.
$520 Principal.
.06
4) $31.20 Int. 1 yr.
7.80 Int. ¼ yr.
$39.00 Int. 1¼ yr.

When days are included in the time, it is best to find the interest for 1 month, next for 1 day, and then for the number of days, *unless the number of days is a reducible fraction of a month.* 20 days is a reducible fraction of a month, since it equals $\frac{20}{30}$, or $\frac{2}{3}$ of a month. Get the interest for $\frac{2}{3}$ of a month at once, rather than to get it for 1 day and multiply by 20.

PROBLEM:

What is the interest on $360 for 100 days at 7%?

WORK:

100 da. = 3 mo. 10 da.
= $3\frac{1}{3}$ mo.
$360 Principal.
.07
12) $25.20 Int. 1 yr.
3) $2.10 Int. 1 mo.
.70 Int. 10 da.
6.30 Int. 3 mo.
$7.00 Int. 3 mo. 10 da.

NOTE.—Another way would be to find the interest for 3 mo. ($\frac{1}{4}$ of $25.20) and for 10 da. ($\frac{1}{9}$ of $6.30).

Same problem, using Cancellation Method.

WORK:

$$3\tfrac{1}{3}\text{ mo.} = \tfrac{10}{3}\text{ mo.} = \tfrac{10}{36}\text{ yr.}$$

$$\frac{\overset{10}{\cancel{\$360}} \times 10 \times 7}{\cancel{36} \times \underset{10}{\cancel{100}}} = \$7$$

When the time can be reduced to a fraction of a year, use cancellation as above. The rule is

PRINCIPAL X RATE X TIME = INTEREST.

PROBLEM:

Find the amount of $400 for 1 yr. 4 mo. at 6%.

WORK:

$400 Principal.
.06
3) $24.00 Int. 1 yr.
8.00 Int. 4 mo.
$ 32.00 Int. 1 yr. 4 mo.
400
$432 Amount.

Same problem, using Cancellation Method.

WORK:

Principal = \$400
Rate = 6% = $\frac{6}{100}$
Time in years = $\frac{4}{3}$ yr.

$$\text{Principal} \times \text{Rate} \times \text{Time} = \frac{\overset{4}{\cancel{400}} \times \overset{2}{\cancel{6}} \times 4}{\cancel{100} \times \cancel{3}} = \$32 \text{ Int. 1 yr. 4 mo.}$$

$$\underline{\quad 400}$$

$$\$432 \text{ Amount.}$$

THE SIX PER CENT METHOD.

By the 6% method, we find first the interest for \$1, then multiply it by the number of dollars in the principal.

If 10¢ is the interest on \$1 for a given time, the interest on \$20 will be 20 × 10¢, or \$2.

At 6%

the interest for 1 yr. = .06 of the principal
for 1 mo. = $\frac{1}{12}$ of .06, or .005 of the principal
for 1 da. = $\frac{1}{30}$ of .005, or .000$\frac{1}{6}$ of the principal.

Thus, At 6%

the interest on \$1 for 1 yr. = \$.06
on \$1 for 1 mo. = \$.005
on \$1 for 1 da. = \$.000$\frac{1}{6}$.

Also, the interest on \$2 for 1 yr. = 2 × \$.06 = \$.12
on \$2 for 1 mo. = 2 × \$.005 = \$.010
on \$2 for 1 da. = 2 × \$.000$\frac{1}{6}$ = \$.000$\frac{1}{3}$.

And the interest on \$1 for 2 yr. = 2 × \$.06 = \$.12.
on \$1 for 8 mo. = 8 × \$.005 = \$.04.
on \$1 for 6 da. = 6 × \$.000$\frac{1}{6}$ = \$.001.

Remember:

To get the interest at 6%,

Multiply \$.06, \$.005, and \$.000$\frac{1}{6}$ by the number of years, months, and days, respectively. Add the results and multiply by the principal.

PROBLEM:

What is the interest on $320 for 7 mo. 20 da. at 6%.

SOLUTION:

The interest on $1 for 7 mo. = 7 × $.005 = $.035.
for 20 da. = 20 × $.000⅙ = .003⅓.
for 7 mo. 20 da. = $.038⅓.
The interest on $320 is 320 × $.038⅓ = $12.27−.

In the majority of states, the legal rate of interest is 6%, so it is important that the pupil understands this method. There is another reason, too, why he should know the 6% method. He can use it to advantage with other rates, by dividing the interest for the given time at 6% by 6, and then multiplying by the given rate per cent.

PROBLEM:

What is the interest on $360 for 2 yr. 2 mo. 5 da. at 7%? Use the 6% method.

SOLUTION:

The interest on $1 for 2 yr. = 2 × $.06 = $.12.
for 2 mo. = 2 × $.005 = .01.
for 5 da. = 5 × $.000⅙ = .000⅚.
The interest on $1 for 2 yr. 2 mo. 5 da. = $.130⅚.
The interest on $360 is 360 × $.130⅚ at 6% = $47.10.
at 1% = 7.85.
at 7% = $54.95.

Remember:

To get the interest at any rate, using the 6% method,

Divide the interest at 6% for the given time by 6, and multiply by the given rate per cent.

60-DAY METHOD.

The 60-day method is used quite extensively by business men. It is the 6% method improved with a few short cuts.

By it the interest is ascertained first for 2 mo., by getting $\frac{1}{100}$ of the principal. Call the given times in days, months, and years, *fractional parts of 2 mo.*, and the interest is then easily deduced from the interest for 60 days.

PROBLEM:

Find the interest on $500 for 1 yr. 4 mo. 20 da. at 6%.

WORK:

$500.00	Principal.
5.00	Int. for 2 mo. ($\frac{1}{100}$ of Prin.).
$40.00	Int. for 16 mo. (8 × $5).
1.67	Int. for 20 da. ($\frac{1}{3}$ of 60 da.).
$41.67	Int. for 16 mo. 20 da.

Give the pupil many problems to be worked by the 60-day method. Not all are so easy as the one given above. The thing desired is skill in finding the desired fractional parts of 2 mo. or 60 da.

If another rate than 6% is given, find the interest for the given time at 6%, then for the given rate, as before explained.

METHODS OF RECKONING TIME.

1. The Common Method:

When the time is long, generally 30 days are considered a month.

2. The Exact Method:

When the time is short, the exact number of days is generally counted but we sometimes find the exact number of days also when the time is long.

3. The Banker's Method:

Bankers get the exact number of days between two dates, but each day is reckoned as $\frac{1}{360}$ of a year.

1. Finding the difference in time when the time is long:

PROBLEM:

Find the time between April 12, 1895, and Sept. 22, 1899.

BEST METHOD.

From April 12, 1895 to April 12, 1899 is *4 yr.*
From April 12, 1899 to Sept. 12, 1899 is *5 mo.*
From Sept. 12, 1899 to Sept. 22, 1899 is *10 da.*
Time between dates = 4 yr. 5 mo. 10 da.

ANOTHER METHOD.

1899	9	22
1895	4	12
4	5	10

NOTE.—If the rate and principal are given, it is a simple matter to find the interest, now that we have the time.

EXPLANATION:

The results by these two methods do not always agree, but they never vary by more than 2 days. The difference lies in the fact that in the first method we find first the whole number of years, then whole number of months, then days; while in the latter method, we call *every month* 30 days, when in fact some have more, some less.

In actual work, the first method is carried out in the head, with little or no written work.

2. Finding the difference in time when the time is short:

PROBLEM:

Find the difference in time between April 12 and July 15, 1902.

WORK:

Number of days left in April = 18.
in May = 31.
in June = 30.
in July = 15.
Total number of days = 94.

NOTE.—If the rate and principal are given now, it is a simple matter to find the interest.

Although exact time is generally found only when the time is short, yet sometimes we find the exact number of days when the time is long.

The following table gives the exact time in days between two dates. If February of a leap year is included, count an additional day. The day of the date of any note is not included.

EXACT-TIME TABLE

JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
1	32	60	91	121	152	182	213	244	274	305	335
2	33	61	92	122	153	183	214	245	275	306	336
3	34	62	93	123	154	184	215	246	276	307	337
4	35	63	94	124	155	185	216	247	277	308	338
5	36	64	95	125	156	186	217	248	278	309	339
6	37	65	96	126	157	187	218	249	279	310	340
7	38	66	97	127	158	188	219	250	280	311	341
8	39	67	98	128	159	189	220	251	281	312	342
9	40	68	99	129	160	190	221	252	282	313	343
10	41	69	100	130	161	191	222	253	283	314	344
11	42	70	101	131	162	192	223	254	284	315	345
12	43	71	102	132	163	193	224	255	285	316	346
13	44	72	103	133	164	194	225	256	286	317	347
14	45	73	104	134	165	195	226	257	287	318	348
15	46	74	105	135	166	196	227	258	288	319	349
16	47	75	106	136	167	197	228	259	289	320	350
17	48	76	107	137	168	198	229	260	290	321	351
18	49	77	108	138	169	199	230	261	291	322	352
19	50	78	109	139	170	200	231	262	292	323	353
20	51	79	110	140	171	201	232	263	293	324	354
21	52	80	111	141	172	202	233	264	294	325	355
22	53	81	112	142	173	203	234	265	295	326	356
23	54	82	113	143	174	204	235	266	296	327	357
24	55	83	114	144	175	205	236	267	297	328	358
25	56	84	115	145	176	206	237	268	298	329	359
26	57	85	116	146	177	207	238	269	299	330	360
27	58	86	117	147	178	208	239	270	300	331	361
28	59	87	118	148	179	209	240	271	301	332	362
29	—	88	119	149	180	210	241	272	302	333	363
30	—	89	120	150	181	211	242	273	303	334	364
31	—	90	—	151	—	212	243	—	304	—	365

HOW TO USE THE EXACT-TIME TABLE.

Suppose we wish to get the exact time between the two dates given in the last problem, that is, from April 12 to July 15, 1902.

April 12 is numbered 102. July 15 is numbered 196. 196 − 102 = 94. 94 includes the last day but not the first. Result, 94 da.

Practice using the table.

THE THREE CLASSES OF INTEREST PROBLEMS.

There are only three classes of problems in interest. Only one illustration of the work necessary will be given under each class, but many others may be taken from the pupil's text-book.

I. To find the Rate, when Principal, Interest, and Time are given.

PROBLEM:

What will be the rate on \$300 to gain \$20.25 in 1 yr. 6 mo.?

WORK:

\$300 Principal.
.01
2)\$3.00 Int. for 1 yr. at 1%.
\$1.50 Int. for 6 mo. at 1%.
\$4.50 Int. for 1 yr. 6 mo. at 1%.
20.25 ÷ 4.50 = 4½.
Rate = 4½%.

RULE:

Divide the interest by the interest on the principal for the given time at 1%.

II. To find the Principal, when Amount, Rate, and Time are given.

PROBLEM:

What principal at 5% will amount to \$319 in 1 yr. 3 mo. 6 da.?

Work:

$ .05 Int. on $1 for 1 yr.
.0125 Int. on $1 for 3 mo.
.0008$\frac{1}{3}$ Int. on $1 for 6 da.

$ .0633$\frac{1}{3}$ Int. on $1 for the entire term.
1.00

1.0633\frac{1}{3}$ Am't of $1 for the term.

$$\begin{array}{r|l} 1.0633\frac{1}{3} & 319.00 \\ 3 & 3 \\ \hline 3.19\cancel{0}\cancel{0} & 957.00(\$300 \text{ Principal.} \\ & 957 \\ \hline & 00 \end{array}$$

RULE:

Divide the amount by the amount of $1 for the given time and rate.

III. To find the Time, when Principal, Interest, and Rate are given.

PROBLEM:

How long will it take $600 to yield $76.50 at 6%?

Work:

$36 = Int. on $600 for 1 yr.
76.50 ÷ 36 = 2.125.
= 2.125 yr.
= 2 yr. 1 mo. 15 da.
Time = 2 yr. 1 mo. 15 da.

RULE:

Divide the given interest by the interest on the principal for 1 year.

NOTES.

When a person borrows money, he is asked to give a signed promise to pay it back, usually with interest. He is said to be "giving his note."

In the **promissory note,** or **note** (as it is generally called), he has promised to pay back the money in a specified time, with a stated interest added.

Suppose A. C. Brown wishes to borrow $100 from Geo. H. Gray for 1 year at 6%. The note found on this page is similar to one that would be made out by Brown and given to Gray.

Geo. H. Gray is the **Payee**, or the one to whom the money is to be paid.

> *$100.^{00}*
> PORTLAND, ORE., *July 25, 1903.*
> One year after date I promise to pay
> to *George H. Gray* or order,
> *One Hundred and* $\frac{no}{100}$ Dollars,
> with interest at 6 per cent. Value Received.
> Payable at Portland, Ore.
> *A. C. Brown.*

A. C. Brown is the **Maker,** or the one who signs the note.

$100, or the sum of money named in the note, is the **Face** of the note.

This note is a **Negotiable Note,** because it can be sold to another.

The words **"or order"** after Mr. Gray's name, gives him the right to order Mr. Brown to pay the money, when due, to some third party. If Mr. Gray needs money before the note is due, he may sell it.

Suppose he sells it to F. S. Wing. Then Mr. Wing becomes the Holder.

The **Holder** is the one who lawfully holds the note.

When Mr. Gray sells the note, he writes on the back of it, "Pay to F. S. Wing or order" and signs his name below. This constitutes an **"indorsement in full."** But, if Mr. Gray signs only his name on the back of the note, such an indorsement is called a **"short indorsement,"** or an **"indorsement in blank."**

INDORSEMENT IN FULL.

Pay to F. S. Wing,
or order.
Geo. H. Gray.

[BACK OF NOTE.]

SHORT INDORSEMENT.

Geo. H. Gray.

[BACK OF NOTE.]

Any holder can collect when the short indorsement is written upon the back of the note.

Writing one's name on the back of notes is called Indorsing.

The name written is the name of the Indorser.

The person to whom the note is indorsed is called the Indorsee.

VALUABLE FACTS ABOUT NOTES.

1. All notes are not written exactly like the ones given here, but so long as they are distinct promises to pay, they need not follow a set form.

2. The words "For value received" may be omitted, although it is advisable to have the stipulation

in the note that the maker has received the money, or its equivalent.

3. The face of the note need not be expressed in both words and figures, but it is best to have both.

4. A note may be written on any paper with either pencil or ink, or a printed form may be filled in.

5. A note made out on Sunday is void.

6. If a note does not state that interest is to be paid, it does not bear interest until after it is due.

7. If any one obtains a note by fraud or from an intoxicated person, he cannot collect.

8. The indorser is responsible for the payment of the note, unless he writes "without recourse" above his signature.

9. The holder has a claim against every person who indorses the note.

10. The dates and amounts of partial payments on a note, before it is finally paid in full, are placed on the back.

11. The day of maturity is the day on which the note falls due.

12. The place of payment, if not mentioned, is at the maker's place of business or residence, during reasonable business hours.

13. Three days over time are generally allowed beyond the specified time to pay, and the holder gets interest for these *three days of grace.* Days of grace are only allowed on notes and bills and not on all debts bearing interest.

DAYS OF GRACE.

Tell the pupil to allow 3 days of grace if the wording of the note is given, in those states where days of grace are allowed. But, if the problem reads "What is due Jan. 9, 1901?" or "it runs for 6 mo.," use the specified time and no more.

The states in which days of grace are not allowed are — Connecticut, California, Dist. of Columbia, Illinois, Idaho, Maryland, Massachusetts, Montana, New York, New Jersey, North Dakota, Ohio, Oregon, Pennsylvania, Utah, Vermont and Wisconsin.

OTHER FORMS OF NOTES.

$ *500.00* CHICAGO, ILL., *July 1,* 189*5.*

Six months after date, I promise to pay to the order of ~~~~~ *Albert F. Green* ~~~~~

~~~~~ *Five Hundred Dollars,* ~~~~~

with interest at the rate of *seven* per cent per annum.

G. N. Whitney.

500\frac{no}{100}$ **SHEBOYGAN, WIS.,** *Jan. 15,* **189** *7.*

For Value Received

I promise to pay C. E. Oliver, or order, Five Hundred and $\frac{no}{100}$ *Dollars, in two years from this date.*

M. E. Wilson.

BOSTON, MASS., *April 15, 1901.*

$161.75

On demand, *I* promise to pay *Cyrus Noble,* or order, *One hundred sixty-one and* $\frac{75}{100}$ Dollars, with interest at 6%. VALUE RECEIVED.

H. K. Polk.
~~~~~

Demand Notes are due whenever payment is demanded. The preceding note shows the form of a demand note.

There are three principal kinds of notes — **Time Notes, Joint Notes,** and **Joint and Several Notes.**

A **Time Note** must be paid in a specified time.

A **Joint Note** is one signed by two or more persons who are jointly liable for its payment.

A **Joint and Several Note** is a note signed by two or more persons who are both jointly and individually liable for its payment. Each man who signs the note is as much responsible for the payment of the whole sum, as if he had signed alone.

A JOINT NOTE.

$200.42 PORTLAND, ME., *July 9, 1903.*

Three months after date, we jointly promise to pay *F. S. Rau,* or order, *Two Hundred and* $\frac{42}{100}$ Dollars, with interest at *7%.*

Value Received.

T. J. Cole

Ray E. Matthews.

A JOINT AND SEVERAL NOTE.

$40.00 CHICAGO, ILL., *Aug. 15,* 190*3.*

One Year after date, we jointly and severally promise to pay *R. N. Mc Dill,* or order, *Forty and* $\frac{00}{100}$ Dollars at *6%* interest.

Value received.

Tom Bacon

Rastus S. Shaw.

NOTE.—Each maker of a joint note is only responsible for his part of the note, but in the joint and several note each maker can be individually sued for the whole amount of the note. This makes quite a distinction between the two.

PARTIAL PAYMENTS ON NOTES.

Many times a man wishes to pay a note in parts or instalments. A **partial payment** is a part-payment on a note before it is paid up in full.

Two methods are used to find the value of notes at maturity. The first method is used in short time notes of 1 year or less, while the second method is only used in computing on long time notes of a year or more.

The first method for short time notes is governed by the **Merchant's Rule.**

For long time notes, we make use of the **United States Rule.**

MERCHANT'S RULE:

(Use the Exact Method in reckoning the time.)

1. Find the amount of the note from the time it begins to draw interest to the time of settlement, paying no attention to payments.

2. Find the amount of each payment from its date to the time of settlement.

3. Subtract the sum of the amounts of the payments from the amount of the note. The remainder is what is due.

PROBLEM:

I borrow $1000 on a note for 1 year at 6% and pay $300 in 120 da. How much do I owe at the end of the year?

WORK AND SOLUTION:

Int. on $1000 for 1 yr. at 6% = $60.

$1000 + $60 = $1060, Amount of note at date of settlement, if no payments were made.

From date of 1st payment to time of settlement there are 240 da. = $\frac{2}{3}$ yr.

Int. on \$300 for $\frac{2}{3}$ yr. at 6% = \$12.

\$300 + \$12 = \$312 Amount of 1st payment.

\$1060 − \$312 = \$748 Amount I owe at the end of the year.

To the Teacher:

A great number of problems should be given in partial payments, because it will take considerable practice before the pupil works with facility. Interest must be thoroughly taught, for the whole thing is practical in business life.

PROBLEM:

A note for \$1000 was given for 10 months with interest at 6%. At the end of three months \$250 was paid on it, and after 7 months, \$400. What was due at the end of the ten months?

Work and Solution:

\$1000 Principal
50 Int. for 10 mo. at 6%.
\$1050 Amount at date of settlement, if no payments were made
\$250.00 1st payment
8.75 Int. for 7 mo. at 6%.
400.00 2d payment
6.00 Int. for 3 mo. at 6%.
\$ 664.75 Amount of partial payments.
\$ 385.25 Balance due.

Note.—These problems in partial payments ought to be readily understood by the pupil.

UNITED STATES RULE:

1. Find the amount of the principal to the time when the payment or payments are as great or greater than the interest then due.

2. Subtract the payment, or sum of payments, from this amount.

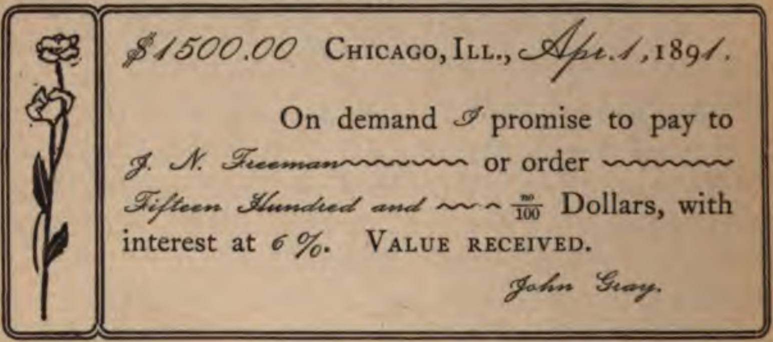
3. Consider the remainder as a new principal and continue this process.

4. The last amount is the sum due.

NOTE.—The above rule is based on the supposition that interest is due whenever a payment is made.

PROBLEM:

Find what is due Sept. 9, 1893, on the following note:—



$1500.00 CHICAGO, ILL., Apr. 1, 1891.

On demand I promise to pay to J. N. Freeman or order Fifteen Hundred and $\frac{no}{100}$ Dollars, with interest at 6 %. VALUE RECEIVED.

John Gray.

[Back of Note Showing Indorsements.]

Sept. 16, 1891 - - - - - *$250*

May 6, 1892, - - - - - - *$375*

June 24, 1893 - - - - - *$675*

EXPLANATION:

When payments are made by instalments upon a note before the final settlement is made, these partial payments together with the dates on which they were made, are placed on the back of the note as shown on this page.

These records on the back of the note constitute an indorsement, but they are money indorsements, not signature indorsements.

A special receipt is given for such payments.

On page 328 is given another plan for the arrangement of work in partial payments.

Work and Solution:

Dates.			
1893	9	9	
1893	6	24	
1892	5	6	
1891	9	16	
1891	4	1	*Payme*
	5	15	........$25
	7	20	 375
1	1		 675
	2		

Int. on $1500 for 5 mo. . at 6% = $41.25.
$1500 + $41.25 − 250 = New Principal.
Int. on $1291.25 for 7 mo. 20 da. at 6% = $49.497.
$1291.25 + $49.497 − 37 = $965.747 2d New Principal.
Int. on $965.747 for 1 yr. 1 mo. 18 da. at 6% = $65.67.
$965.747 + $65.67 − 675 = $356.417 3d New Principal.
Int. on $356.417 for 2 mo. 15 da. at 6% = $4.455.
$356.417 + $4.455 = $360.872 Amount due Sept. 9, 1893.

WHAT TO DO:

First—

Arrange the dates in order, as given above, and subtract each date from the next above.

Next—

Find the different new principals, as given in the rule.

The last amount is the sum due.

In the preceding problems, the payment in no case was less than the interest. In the problem given next, we have one case where the interest is more than the payment, the interest being $23.10 and the payment only $15. Here it is hard to tell whether the interest exceeds the payment without first finding what the interest is for the given time.

PROBLEM:

$600.00 BALTIMORE, MD., *July 3*, 1900.

For Value Received, *I* promise to pay *Henry George*, or order, *three years* from date, *Six Hundred Dollars*, with interest at *5* per cent. Payable at New Orleans.

Herman Borgstadt.

On this note, payments as follows were made: Nov. 23, 1900, $100; Oct. 18, 1901, $15; April 13, 1903, $75. What was due July 3, 1903?

WORK AND SOLUTION:

	PAYMENTS.	
$600.00 Principal. 11.67 Interest to November 23, 1900. $611.67 Amount due November 23, 1900. 100 $511.67	$100	1900—11—23 1900— 7— 3 4 20
~~$511.67 New Principal.~~ ~~23.10 Interest to October, 18, 1901.~~	~~$15~~ ~~Interest exceeds Payment.~~	~~1901—10—18~~ ~~1900—11—23~~
$511.67 New Principal. 38.02 Interest on $511.67 to April 13, '03. $549.69 Amount. 90.00 $459.67	$75 plus $15 $90	1903— 4—13 1901—10—18 1 5 25
$459.67 New Principal. 5.11 Interest to maturity. $464.78 Amount due at maturity.	Paid up.	1903— 7— 3 1903— 4—13 2 20

TALK:

Explain clearly to the pupil that because the secon payment in the foregoing problem is less than the interest, it is set aside and the interest is found from th time of the second payment to the time of the thir *payment. This interest, with the previous interest, is added to the principal. The sum of the two payments, being greater than the interest, is taken out of the amount.*

This is the only new feature of these problems. Give the pupil as many of them as he needs to make him familiar with the principle.

Above all, be sure he understands what the work means. Bring into the class room real notes having partial payments indorsed upon them and let him work them out.

ANNUAL INTEREST.

TALK:

Annual interest is interest on interest as well as on the principal.

If a person fails to pay his interest at the end of the first year, the interest then due will bear simple interest to the day of settlement. At the end of each year in succession, when the interest is due, it bears simple interest to the day of settlement.

Show the pupil the difference between annual interest and compound interest. Make it clear to him that annual interest never becomes a part of the principal.

PROBLEM:

Find the annual interest on \$700 for 4 yr. 4 mo. at 5%, interest payable annually.

WORK AND SOLUTION:

\$35 = Simple interest due each year.
\$35, due the 1st yr., draws interest for 3 yr. 4 mo.
\$35, due the 2d yr., draws interest for 2 yr. 4 mo.
\$35, due the 3d yr., draws interest for 1 yr. 4 mo.
\$35, due the 4th yr., draws interest for 0 yr. 4 mo.
Total Int. on Int. = Int. on \$35 for 7 yr. 4 mo.
$= \$12.83\frac{1}{3}$.
Simple Int. on \$700 for the whole time $= \$151.66\frac{2}{3}$.
Total Int. $= \$12.83\frac{1}{3} + 151.66\frac{2}{3} = \$164.50$.

COMPOUND INTEREST.

The rules of many banks state that interest on money placed in the bank not called for when due, shall be immediately added to the principal and shall draw interest just as if it were a new deposit. In other words, *the interest as fast as it becomes due, becomes a part of the principal.*

The total interest obtained in this manner is called **Compound Interest.**

Compound interest can seldom be collected at law, unless it is so written in the note.

PROBLEM:

I deposited $400 in a New York city bank April 1, 1898, and left it there 3 years before drawing out any of it. The bank allowed 4% interest paid April 1 of each year, but only on such sums as had been in the bank a whole year. It also allowed the interest to be compounded annually. How much could I draw Apr. 1, 1901? What was the total compound interest?

Work:

$400
.04
$16.00 Int. for 1st year.
400
$416.00 Amount to my credit Apr. 1, 1899.
.04
$16.64 Int. for the 2d year.
416
$432.64 Amount to my credit Apr. 1, 1900.
.04
$17.3056 Int. for the 3d year.
432.64
$449.95 Amt. to my credit Apr. 1, 1901, or what I drew.
400
$49.95 Compound Int.

Let the pupil find the *common* interest 3 yr. and note how that sum differs fr pound interest.

PROBLEM:

Suppose the bank spoken of in the pı lem allowed 4% a year, interest compo annually instead of annually, if not call due. What would the total compound in e

WORK:

2% each half-year is the same as 4% a year.

$400
.02

$8.00 Int. for 1st half-year.
400

$408 Amount to my credit Oct. 1,
.02

$8.16 Int. for 2d half-year.
408.

$416.16 Amount to my credit Apr. 1, 1899.
.02

$8.3232 Int. for 3d half-year.
416.16

$424.4832 Amount to my credit Oct. 1, 1899.
.02

$8.489664 Int. for 4th half-year.
424.4832

$432.97 Amount to my credit Apr. 1, 1900.
.02

$8.6594 Int. for 5th half-year.
432.97

$441.6294 Amount to my credit Oct. 1, 1900.
.02

$8.832588 Int. for 6th half-year.
441.6294

$450.46 Amount to my credit Apr. 1, 1901.
400

$50.46 Compound Interest.

EXPLANATION:

We found in the previous problem that if compounded annually, the interest for the given time is $49.95. If compounded semi-annually, we find the interest for the same time is $50.46. You can see that after a few years this would begin to count fast. Hence, it makes a deal of difference how often interest is compounded. Compare the examples carefully and see where they differ.

In the two problems given, the work is arranged in such a way that it combines a solution. It may also be arranged similar to the partial payment problems. That is a matter of choice. So long as the form is neat and correct in expression, it matters little which one is followed.

Bankers and business men would find it very tedious if they had to stop and figure out compound interest.

This has all been worked out for them and printed in books.

Interest is compounded annually, semi-annually, or quarterly, according to agreement. The interest is understood to be compounded annually, unless otherwise stated.

For the Pupil:

1. What do you mean by simple interest?

2. How does compound interest differ from annual interest?

3. In computing by the common interest method, how do you reckon the time?

4. Define interest, rate, principal, amount.

On the opposite page is given a table showing the amounts of $1, at 3%, 4%, 5%, 6%, 7%, and 8% for from 1 to 25 yr., interest compounded annually.

COMPOUND INTEREST TABLE.

Year.	3%	4%	5%	6%	7%	8%
1	1.03	1.04	1.05	1.06	1.07	1.08
2	1.060	1.0816	1.1025	1.1236	1.1449	1.1664
3	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.125509	1.169859	1.215506	1.262477	1.310796	1.360488
5	1.159274	1.216653	1.276282	1.338226	1.402551	1.469328
6	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.229874	1.315932	1.407100	1.503630	1.605781	1.713824
8	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.304773	1.423312	1.551328	1.689479	1.838459	1.999004
10	1.343916	1.480244	1.628895	1.790848	1.967151	2.158924
11	1.384234	1.539454	1.710339	1.898299	2.104851	2.331638
12	1.425761	1.601032	1.795856	2.012196	2.252191	2.518170
13	1.468534	1.665074	1.885649	2.132928	2.409845	2.719623
14	1.512590	1.731676	1.979932	2.260904	2.578534	2.937193
15	1.557967	1.800944	2.078928	2.396558	2.759031	3.172169
16	1.604706	1.872981	2.182875	2.540352	2.952163	3.425942
17	1.652848	1.947900	2.292018	2.692773	3.158815	3.700018
18	1.702433	2.025817	2.406619	2.854339	3.379932	3.996019
19	1.753506	2.106849	2.526950	3.025600	3.616527	4.315701
20	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957
21	1.860295	2.278768	2.785963	3.399564	4.140562	5.033833
22	1.916103	2.369919	2.925261	3.603537	4.430401	5.436540
23	1.973587	2.464716	3.071524	3.819750	4.740529	5.871463
24	2.032794	2.563304	3.225100	4.048935	5.072366	6.341180
25	2.093778	2.665836	3.386355	4.291871	5.427432	6.848475

The following facts will show the rapidity with which compound interest accumulates, if left on deposit.

	Amounts.
At 1%, $1 in 100 years will yield $	2.75
" 2%, " " " " " "	7.25
" 3%, " " " " " "	19.25
" 4%, " " " " " "	50.50
" 5%, " " " " " "	131.50
" 6%, " " " " " "	340.00
" 7%, " " " " " "	868.00
" 8%, " " " " " "	2,203.00
" 9%, " " " " " "	5,543.00
" 10%, " " " " " "	13,809.00
" 24%, " " " " " "	2,551,799,404.00

Families with great fortunes can have large incomes even with money let out at a low rate of interest.

The following table shows rates of interest allowed in various states. First is given the legal rate, then the highest rate allowed only when a written contract is made between the parties. The cross (+) means any rate that can be made by contract.

LEGAL INTEREST RATE TABLE.

States	Rate per cent		States	Rate per cent		States	Rate per cent	
Alabama	[illegible]	[illegible]	Louisiana	[illegible]	[illegible]	Ohio	6	8
Arizona	[illegible]	—	Maine	[illegible]	—	Oklahoma	7	12
Arkansas	[illegible]	[illegible]	Maryland	[illegible]	[illegible]	Oregon	6	10
California	[illegible]	—	Massachusetts	[illegible]	—	Pennsylvania	6	6
Colorado	[illegible]	—	Michigan	[illegible]	[illegible]	Rhode Island	6	+
Connecticut	[illegible]		Minnesota	[illegible]	[illegible]	South Carolina	7	8
Delaware	[illegible]	[illegible]	Mississippi	[illegible]	[illegible]	South Dakota	7	12
Dist. of Columbia	[illegible]	[illegible]	Missouri	[illegible]	[illegible]	Tennessee	6	6
Florida	[illegible]	[illegible]	Montana	[illegible]	—	Texas	6	10
Georgia	[illegible]	[illegible]	Nebraska	[illegible]	[illegible]	Utah	8	+
Idaho	[illegible]	[illegible]	Nevada	—	—	Vermont	6	6
Illinois	[illegible]	[illegible]	New Hampshire	[illegible]	[illegible]	Virginia	6	6
Indiana	[illegible]	[illegible]	New Jersey	[illegible]	[illegible]	Washington	8	+
Iowa	[illegible]	[illegible]	New Mexico	[illegible]	[illegible]	West Virginia	6	6
Kansas	[illegible]	[illegible]	New York	[illegible]	[illegible]	Wisconsin	7	10
Kentucky	[illegible]	[illegible]	North Carolina	[illegible]	[illegible]	Wyoming	12	—
			North Dakota	[illegible]	[illegible]			

BANK DISCOUNT.

TALK:

Before taking up Bank Discount it is important that the pupils know something of banks and banking.

Read the subject in any up-to-date encyclopedia, that you may be informed yourself. Talk it over with a practical banker, if possible.

Speak of the kinds of banks and the advantage of a bank of deposit.

Procure a five-dollar bill issued by a local bank and talk about it, telling the pupil who made it, where it was made, etc. Speak of the reason why the failure of a bank does not make its notes void.

Have the pupil write out a check.

If a man borrows money of another and gives his note for the same, he pays both the principal and the interest when due.

But, if one pays a debt by giving a note and the one who gets the note wants to realize money on it at the bank, he gets the full sum minus the interest for the given time, or the proceeds.

Mr. Brown owes Mr. Gray $240. He has no ready money, so he gives his note to Mr. Gray for 2 months at 10%. Suppose Mr. Gray wants the ready money on the same day. He takes it to the bank and *discounts it*, that is, he sells the note and gets the $240 minus the interest, or $236.

The date he presents it for discount is the date of discount. On the date of discount, Mr. Gray gets $236 from the bank.

But, suppose Mr. Gray waited 30 days before he needed the money. Then the bank would have discounted only for 1 month instead of 2, and Mr. Gray would have received $238.

The **Term of Discount** is the time between the date of discount and the date of maturity. It includes the date of maturity.

The **Date of Maturity** is the date on which the note is legally due. It is then, the last day of grace.

The **Date of Discount** is the date on which the note is sold.

The **Proceeds** are the value of the note minus the bank discount.

Bank Discount is simple interest reckoned on the face of a note from the date of discount to the date of maturity.

Fix these facts in mind:

1. The face of the note is the amount to be discounted.

2. The face of the note is the amount due at maturity. If the note bears interest, it is the amount of principal and interest to the date of maturity. If the note does not bear interest, the principal is the face of the note.

3. The bank discount is simple interest on the face of the note.

4. The face of the note minus the bank discount gives the proceeds.

THE DATE OF MATURITY.

To find the date of maturity:

a. If a certain date in a month is given, then count by months, figuring 1 month as the time to the same date in the next month.

b. If the time is expressed in days, *count the exact number of days to maturity.

c. In some states the day of discounting is added into the term of discount as well as the date of maturity.

*Use the table on *Exact Time.*

PROBLEMS.

1st Case — Note not bearing interest.

PROBLEM:

Find the bank discount, time to run, and proceeds of the following: —

A note of $200, dated Jan. 1, 1900, time 4 mo., discounted March 5, 1900, at 10%.

WORK:

I. The *date of maturity* is 4 mo. 3 da. from Jan. 1, or May 4.

II. The *time to run* or *term of discount* is from Mar. 5 to May 4, or $26 + 30 + 4 = 60$ da. $= \frac{1}{6}$ yr.

III. The *discount* is $\frac{1}{6}$ of $20 = $3.33\frac{1}{3}$. ($20 is the discount for 1 yr.)

IV. The *proceeds* are \$200 − \$3.33$\frac{1}{3}$ = \$196.66$\frac{2}{3}$.

Note.—In problems where interest plays no part, there are four steps in the solution, as shown above. The steps are:—

I. The Date of Maturity.
II. The Time to run.
III. The Discount for the given time.
IV. The Proceeds.

2d Case—Note bearing interest.

PROBLEM:

Find the bank discount, time to run, and proceeds of the following:—

A note for \$300, dated June 1, 1891, bearing 6% interest, discounted Sept. 1, 1891, at 10%. The note was due 10 mo. from date.

Work:

I. The *date of maturity* is 10 mo. 3 da. from June 1, 1891, or Apr. 4, 1892.

II. The *interest* on \$300 for 10 mo. 3 da. at 6% = \$15.15.

III. The *face of the note* = \$300 + \$15.15 = \$315.15.

IV. The *time to run* or *term of discount* is from Sept. 1, 1891, to Apr. 4, 1892, or 216 da. = $\frac{216}{360}$ yr. = $\frac{3}{5}$ yr.

V. The *discount* = $\frac{3}{5}$ of \$31.515 = \$18.92. (\$31.515 is the discount for 1 yr.)

VI. The *proceeds* = \$315.15 − \$18.92 = \$296.23.

Note.—In problems where interest plays a part, there are six steps, instead of four, in the solution.

The steps are:—

I. The Date of Maturity.
II. The Interest.
III. The Face of Note.
IV. The Time to run.
V. The Discount.
VI. The Proceeds.

PRESENT WORTH AND TRUE DISCOUNT.

The **Present Worth** of any debt is a sum, which, if put at interest, will amount to that debt in the given time.

The **True Discount** is the difference between the debt at maturity and its present worth.

Remember:

1. To allow 3 days of grace, if the debt discounted is a note.

2. To add the interest due at maturity to the principal, before discounting, if the note bears interest.

PROBLEMS.

Case I.—Note not bearing interest.

PROBLEM:

What is the present worth and true discount on a note of $200, if paid 6 mo. before due, the discount being 6%.

SOLUTION:

Amount of $1 for 6 mo. at 6% = $1.03. If $1.03 = amount of $1, $200 is the amount of as many dollars as $\frac{200}{1.03}$, or $194.17 –.

$194.17 is the present worth. $200 – $194.17 = $5.83 true discount.

The following rule can be deduced from the foregoing solution:—

RULE:

1. To find the present worth, divide the debt by the amount of $1 for the given time.

2. To find the true discount, subtract the present worth from the

Case II. — Note bearing interest.

PROBLEM:

What is the present worth of a note of $300, bearing 6% interest, due in 2 yr. 4 mo., if money is worth 10%?

SOLUTION:

Int. on $300 for 2 yr. 4 mo. at 6% = $42.
$300 + $42 = $342. Amount due at maturity.
Amount of $1 for 2 yr. 4 mo. at 10% = $1.23⅓.
If $1.23⅓ = amount of $1, then $342 is the amount of $\$\frac{342}{1.23\frac{1}{3}}$, or $277.29.
$277.29 = present worth.

EXCHANGE.

TALK:

A man in Chicago owes a man in New York city a sum of money. He can send it to him in one of four ways: —

1. *By Check*
2. *By Post-office Order*
3. *By Express Order*
4. *By Bill of Exchange.*

The pupil understands something of the first three ways, but does he know about the bill of exchange?

SENDING BILLS OF EXCHANGE.

If, for every little business transaction, money had to be sent from one business center to another, much *needless* inconvenience and expense would be incurred.

Suppose Mr. Green of Chicago owes Mr. Brown of Milwaukee $200 for groceries and Mr. Allen of Milwaukee owes Mr. Robinson of Chicago $200 for rent. Wouldn't it save expense and trouble if Mr. Green should go to Mr. Robinson and Mr. Allen to Mr. Brown? Thereby two debts are cancelled by two city transactions and no money need be sent from one city to another.

This is all there is to Exchange, only in business life, *banks* instead of individuals, take it upon themselves to transact the business.

Exchange is no more than a method of making payments between distant places without transmitting the money each time.

Only a small percentage of the money really passes from one city to another.

Exchange in the United States is carried on mostly by banks located in the large cities which charge a small fee for transacting the business.

A **Bill of Exchange** or **Draft** is a written order for the payment of a certain sum of money by one person on another.

A **Bank Draft** is one made out by one bank on another but may be cashed at nearly any banking house.

The **Drawer** of the draft is the signer.

The **Drawee** is the one to whom it is addressed.

The **Payee** is the one to whom it is payable.

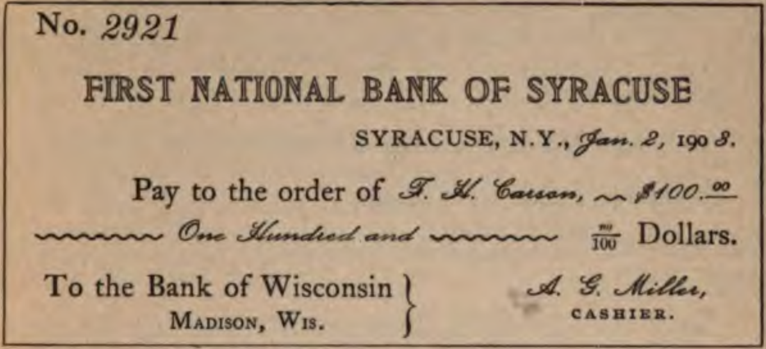
The **Holder** is the one who has possession of the draft.

If the drawee accepts the draft, he writes across the face of it "**Accepted**" with the date and his signature. This is called an **Acceptance.**

Once accepted, the draft becomes a note with the same laws regulating it. If the draft is not accepted, it is not binding and we say that it has been "**dishonored.**"

A bill of exchange is entitled to days of grace, if it is payable in a state where grace is allowed, unless a particular day is named in the draft. In most states, no grace is allowed on sight drafts.

A BANK DRAFT.



No. *2921*

FIRST NATIONAL BANK OF SYRACUSE

SYRACUSE, N.Y., *Jan. 2,* 190*8*.

Pay to the order of *T. H. Carson,* *$100.00*

One Hundred and $\frac{no}{100}$ Dollars.

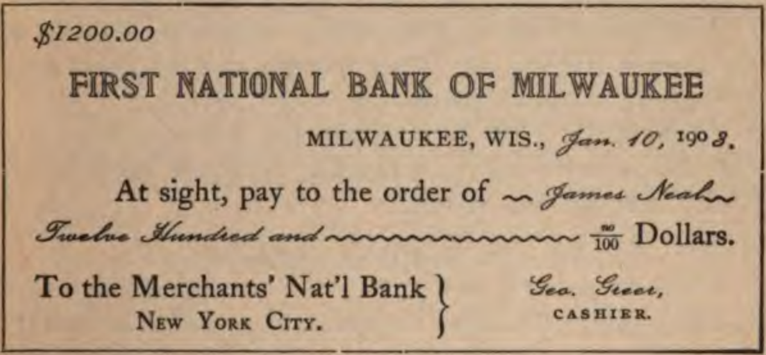
To the Bank of Wisconsin
MADISON, WIS.

A. G. Miller,
CASHIER.

There are two kinds of bank drafts, **Sight Drafts** and **Time Drafts.**

A sight draft must be paid on presentation.

A SIGHT DRAFT.



$1200.00

FIRST NATIONAL BANK OF MILWAUKEE

MILWAUKEE, WIS., *Jan. 10,* 190*8*.

At sight, pay to the order of *James Neal*

Twelve Hundred and $\frac{no}{100}$ Dollars.

To the Merchants' Nat'l Bank
NEW YORK CITY.

Geo. Greer,
CASHIER.

EXPLANATION:

James Neal of Milwaukee owes James McCarthy of New York $1200. He goes into the First Nat'l Bank of Milwaukee and buys the above draft. On the back he writes "Pay to the order of Jas. McCarthy" and then mails it to Jas. McCarthy. Mr. McCarthy calls at his bank and receives the money.

A TIME DRAFT.

FIRST NATIONAL BANK OF MILWAUKEE

$1200.00 **Milwaukee, Wis.,** *Feb. 6,* **190***2.*

At *15* **days sight, pay to the order of**

Jas. McCarthy *Twelve Hundred Dollars.*

Value received, and charge the same to the account of

To the Merchants' Nat'l Bank
NEW YORK.

Geo. Greer,
Cashier.

EXPLANATION:

Here Mr. Neal of Milwaukee sends the above to Jas. McCarthy. Mr. McCarthy indorses it and presents it at the bank. If the cashier writes on it "accepted," his bank becomes liable for its payment 15 days after presentation.

Drafts of another character are used extensively in the collection of debts.

Suppose Smith & Gray, retail clothiers at Oakland, Cal., owe French & Sons, wholesalers at San Francisco, six hundred dollars for clothing.

If Smith & Gray fail to pay the amount when due, the following draft is a sample of those used in the collection of such debts:—

A COMMERCIAL DRAFT.

No. *13987* SAN FRANCISCO, CAL., *Feb. 23,* 190*8.*

At sight, pay to the order of ……………

THE FIRST NATIONAL BANK, San Francisco, *$600.00*

Six Hundred and $\frac{00}{100}$ Dollars.

To Smith and Gray
OAKLAND, CAL.

French & Sons.

EXPLANATION:

The above draft will be sent by the San Francisco bank to some bank in Oakland for collection.

Smith & Gray will be seen by a messenger from the Oakland bank and if they accept the draft, they will write "accepted" and their signature, on the face.

The draft is again presented when due, if days of grace are allowed, and collected.

If the draft is not accepted by Smith & Gray, it goes back to French & Sons, who must take other means for collection.

DOMESTIC EXCHANGE.

Domestic Exchange is exchange between different parts of the same country.

A draft is **at par,** when it sells for its face value. If sold for less than its face value, a draft is **at a discount**; if for more than its face, it is **at a premium.** The rate charged depends upon the demand for drafts.

TO ILLUSTRATE:

Suppose $1,000,000 worth of drafts were made out in N. Y. city during the last few days on Chicago, and only half that amount of drafts were made out in Chicago on N. Y. city. Drafts in Chicago would be at a discount, while in N. Y. city they would be at a premium.

The reason a premium is charged in N. Y. is because they will have to *actually* send money to Chicago, or pay interest on the balance they hold. Hence, they charge less and make better terms.

FOREIGN EXCHANGE.

Foreign Exchange is exchange between different countries, differing very little from domestic exchange and subject to about the same regulations.

We go abroad. We write home for money. Suppose it were sent in the form of a draft, we would designate it as a **Bill of Exchange** to distinguish from a domestic draft.

Two copies of the bill of exchange are made out and sent by different mails to guard against loss in carriage.

FIRST COPY OF SET OF EXCHANGE.

£ *15* NEW YORK, *Jan. 2*, 190*3*. No. *52,891*

At sight of this first of exchange (second of the same tenor and date unpaid) pay to the order of *M. Daugherty*, *Fifteen pounds sterling* value received, and charge to the account of

To Grenham Bros.
LONDON, ENG.

R. E. Cannon.

SECOND COPY OF SET OF EXCHANGE.

£ *15* NEW YORK, *Jan. 2*, 190*3*. No. *52,891*

At sight of this second of exchange (first of the same tenor and date unpaid) pay to the order of *M. Daugherty*, *Fifteen pounds sterling* value received, and charge to the account of

To Grenham Bros.
LONDON, ENG.

R. E. Cannon.

When one is paid it makes the duplicate void.

Foreign bills of exchange are made payable in 3 or 60 days after sight.

If made for 60 days after sight, a lower rate is quoted on account of the discount.

Tourists of to-day patronize express companies for **Foreign Money Orders.** These are made out similar to regular express money orders and may be cashed in any of the larger cities of all foreign countries. They take the place, to a large extent, of **Letters of Credit,** which are letters from banking houses in one country to those in another, allowing sums to be drawn not to exceed a total named in the letter.

PROBLEMS.

PROBLEM:

What will be the cost of a Milwaukee draft of $500 at ½% premium?

WORK:

$1.00
.005
$1.005 Cost of $1
500
$\$502.50\cancel{0}$ Cost of draft.

EXPLANATION:

Since the exchange is at a premium in Milwaukee, each dollar of the draft will cost $1.005. The draft will cost 500 × $1.005, or $502.50.

PROBLEM:

What will be the cost of the same Milwaukee draft at ½% discount?

WORK:

$1.00
.005
$ .995 Cost of $1
500
$\$497.50\cancel{0}$ Cost of draft.

EXPLANATION:

The exchange is at a discount and each dollar of the draft will cost only $.995.

A $500 draft will cost 500 × $.995, or $497.50.

PROBLEM:

I live in New York. I buy a draft on Chicago for $500, payable 30 days after sight, at ½% premium. What must be paid in Chicago for the draft?

WORK:

$1.00
.005

$1.005 Cost of $1 on sight.
.00458 Discount on $1 for 33 da.

$1.00042 Cost of $1
500

$500.21 Cost of $500 draft.

EXPLANATION:

Each dollar of the draft would cost me $1.005 on sight.

As the draft is not payable until 33 da. after sight, it will be discounted at the Illinois rate of 5% for that time. Each dollar will cost $1.005 − $.00458, or $1.00042.

The cost of the draft will be 500 × $1.00042, or $500.21.

PROBLEM:

How large a sight draft can be bought in Chicago for $502.50, exchange being at ½% premium?

WORK:

$1.00
.005

$1.005 Cost of $1.

$502.50 ÷ 1.005 = $500.

EXPLANATION:

Every dollar of the draft will cost $1.005 and as many dollars can be bought for $502.50 as 1.005 is contained times in 502.50, or 500. A $500 draft can be bought.

PROBLEM:

How large a time draft payable 30 days after sight, can be bought for $500.22, exchange being at ½ premium?

WORK:

$1.00	
.005	
$1.005	Cost of $1 at sight.
.00458	Discount on $1 for 33 da.
$1.00042	Cost of $1.

$500.22 ÷ 1.00042 = $500, Cost of draft.

EXPLANATION:

$1.05 would be the cost of every $1 of the draft *at sight*. *But*, the draft is discounted for 33 days at the Illinois rate of 5%. Thus 1.005 − .00458, or $1.00042 is the cost of each dollar of the draft.

$500.22 ÷ 1.00042 = $500, cost of the whole draft.

EXPRESS ORDERS.

Express Orders are sold at the following rates to any part of the U. S. or Canada:—

For order not over $	2.50	3¢.
	5.00	5¢.
	10.00	8¢.
	20.00	10¢.
	30.00	12¢.
	40.00	15¢.
	60.00	18¢.
	75.00	23¢.
	100.00	28¢.

Single express orders are not made out for more than $50, but for larger amounts of money separate orders may be issued.

POST-OFFICE ORDERS.

Post-office Money Orders are sold at the following rates to any part of the U. S., Porto Rico, or the Philippines.

Have the pupil note when it is cheaper to buy post-office orders. Compare rates with those of express companies.

For orders not over $ 50	Same as express rates.
60	20¢.
75	25¢.
100	30¢.

Note:—Single post-office orders exceeding $100 are not issued.

POSTAL RATES.

Domestic—

Postal Cards 1¢ each.

1st Class (letters) 2¢ an oz.

2d Class (newspapers and other periodicals) 1¢ for 4 oz.

3d Class (printed matter) 1¢ for 2 oz.

4th Class (merchandise and photos) 1¢ per oz.

Foreign—

These rates are to those countries subject to the Postal Union.

Postal Cards 2¢ each.

1st Class (letters) 5¢ per ½ oz.

2d Class (newspapers and other periodicals) 1¢ per 2 oz.

DIRECT TAXES.

TALK:

Every property holder usually pays taxes. In many places the adult male citizen pays a poll tax. These things combine to make at least some elementary knowledge of taxes, the way they are proportioned, levied, and collected, of practical value to those who in *a few years will experience the duty of paying taxes themselves.*

Poll taxes are falling into disrepute, as they are not considered the most equitable.

When we need money to cover the expenses of the town, city, county, or state government, a tax is levied on the property. The town, in many cases, also levies a tax on all the men in it who are 21 years old or over.

The tax levied on property is called **a property tax.**

The tax levied on persons is called **a poll tax.** This is sometimes called **a capitation** (by the head) **tax.**

Sometimes a man's income is taxed. This is an **income tax.**

After the amount of money to be raised by tax is decided upon, a man, called the **assessor**, examines each piece of taxable personal property and real estate, and places a value upon it. This is taken as a basis for proportioning the tax among the property owners.

A **tax collector** is one who collects the taxes. He is sometimes paid a salary. Sometimes he gets only a percentage of the money he collects.

The **treasurer** receives and takes care of the money collected by the tax collector. He is paid a salary.

Some Uses for Taxes:

1. To pay for state government.
2. To pay for county government.
3. To pay for town or city government.

Among these expenses are,

1. Building and repairing roads, bridges, etc.
2. Building and repairing public buildings; as schools, city halls, libraries, etc.

3. Paying the salaries or wages of public officers, teachers, etc.

USE OF THE MILL.

The denomination of our money system called the *mill* has practically its only use in the levy of taxes. When a tax is apportioned, it is usually found, that if a few mills are paid on each dollar's worth of property in the district, the aggregate amount is equal to the whole sum of tax needed. Consequently, you often hear of tax levies of so many mills on the dollar, as, 2 mills on the dollar, 5 mills on the dollar, etc.

Assessors make use of a table like the one given here. This table is based on a tax levy of 9 mills on the dollar.

Property Value	Tax	Property Value	Tax	Property Value	Tax	Property Value	Tax
\$1	\$0.009	\$10	\$0.09	\$100	\$0.90	\$1,000	\$ 9.00
2	0.018	20	0.18	200	1.80	2,000	18.00
3	0.027	30	0.27	300	2.70	3,000	27.00
4	0.036	40	0.36	400	3.60	4,000	36.00
5	0.045	50	0.45	500	4.50	5,000	45.00
6	0.054	60	0.54	600	5.40	6,000	54.00
7	0.063	70	0.63	700	6.30	7,000	63.00
8	0.072	80	0.72	800	7.20	8,000	72.00
9	0.081	90	0.81	900	8.10	9,000	81.00

EXPLANATION OF THE TABLE.

The second column shows the tax at nine mills on the dollar, for values of \$1 to \$10. The fourth column shows the tax for values of \$10 and multiples of ten, to \$100. The sixth column shows the tax for values of $100 and multiples of one hundred to $1,000. The last column shows the tax for values of $1,000 to $10,000.

PROBLEM ON THE TABLE:

Wayne's property is assessed at $18436. What is his tax at 9 mills on the dollar?

SOLUTION:

Inspection shows him to have twice $9000, and $436. Then, from the table:—

Tax on $18000 is 2 × $81 =	$162.
“ “ $400 is	3.60
“ “ $30 is	.27
“ “ $6 is	.05
Total tax	$165.92.

NOTE.—Have the pupil make tables where the rate is 2, 3, 4, 5, 7, etc., mills on the dollar. Then let him work problems, using the tables made.

Finding the Rate of Tax.

Sometimes the rate is fixed by law or by vote of the citizens. More often the lump sum to be raised is named, and the assessor determines the rate.

When the assessor is to determine the rate, he proceeds in this way: First, he assesses each piece of property, usually not at its full market value. Then he determines the total value of all the property in his district. Next, he divides the total tax to be raised by the total value of the property in his district. The result is the rate of tax on the dollar.

PROBLEM:

The town of Grant is to raise $4725 in tax. The property in the town has an assessed valuation of $395140. What is the rate?

Solution:

If on $395140 a tax of $4725 is to be raised, on $1 as much tax must be raised as $395140 is contained times in $4725, which is .0119+, or about $.0119. This would be called $0.012, or 12 mills on the dollar.

Work:

```
                .0119+
     395140 ) 4725.0000
              395140
              -------
               773600
               395140
               -------
               3784600
               3556260
               -------
                228340
```

To the pupil:

If the rate is made 12 mills here, and the tax is all collected, it will bring a little more than $4725. Why?

PROBLEM:

In Akron, the valuation was $34,146,250. The tax to be raised was:—city, $560,340; county, $40240; state, $36120. There were 7809 polls taxed at $2. What should the rate be to allow a little overlay for incidental expense?

Work and Explanation:

Tax to be raised:

city	$560340
county..........	40240
state............	36120
Total....	$636700
Less 7809 polls @ $2..........	15618
Amount to be raised on city property..	$621082

```
                .01810 +
34146250 ) 621082.00000
           341462 50
           279619 500
           273170 000
              6449 5000
              3414 6250
                34 8750
```

This gives us a rate of a little more than 18.1 mills. To meet the overlay, we may call it 18.2 mills.

To find the overlay

For incidentals that will be raised by making the rate 18.2 mills, if all the tax is collected:—

First, find the total that will be raised

34,146,250 × $0.0182 = $621461.75

Then subtract total tax called for........... 621082.

This leaves for overlay......... $379.75

INDIRECT TAXES.

In the last section we dealt with direct taxes. This section deals with taxes placed upon goods and collected before they are sold to the consumer.

They are of two kinds, **customs** or **duties,** and **excises** or **internal revenue.**

Excises or **internal revenue** are taxes levied on certain domestic goods, as, manufactured tobacco, liquors, and the like.

WHY THESE TAXES ARE NEEDED.

The national government needs money to pay:—

1. Interest on the public debt.
2. To support an army and navy; to build vessels, and keep up arsenals and forts.
3. To pay pensions.

4. To improve the rivers and harbors.

5. To pay the salaries of its officers; as, the president, cabinet officers, judges, ministers to foreign countries, congressmen, etc.

Indirect taxes levied by the government on imported goods or merchandise are called **duties** or **customs.**

Duties are of two kinds, **specific** and **ad valorem.**

A **specific** duty is one levied at a specified sum per yard, gallon, ton, etc.

An **ad valorem** duty is one levied at a certain percentage of the value of the goods, at the port of export.

A **custom house** is a government office where duties are collected and where vessels are entered and cleared. Nearly every seaport of consequence has a custom house. So also have important towns near the Canadian and Mexican boundaries.

Tare is an allowance made for the weight of bags, barrels, or cases, in which merchandise is shipped.

Leakage is an allowance made for loss of liquids from casks, barrels, etc., in shipping.

Breakage is an allowance made for the loss of liquids from bottles in shipping.

Net weight is the weight after allowance has been made for tare, leakage, or breakage. Duties are reckoned on net weights.

NOTE.—Teach indirect taxes at the time you take up that part of commercial geography which deals with imports and exports. Problems are not for their practical value, but simply to give a clear knowledge of what is done.

PROBLEM:

Find the duty on 420 sacks of coffee, each weighing 175 lb. gross, and invoiced at 9¢ per pound. The tare is 500 lb. and the duty 20% ad valorem.

WORK AND EXPLANATION:

Weight of 420 sacks is 420 × 175 lb. = 73500 lb.
Tare is 500
Net weight................73000 lb.
Its value is 73000 × 9¢ = $6570.
The duty is 20% of $6570 = $1314.

PROBLEM:

Find the duty on 4 doz. bottles of cologne, allowing 4% for leakage and 3% for tare. The invoice value is 90¢ a bottle and the duty is 25% ad valorem and 20¢ specific. Find the total cost per bottle.

WORK AND EXPLANATION:

Leakage and tare are 4% + 3% = 7%.
4 doz. bottles = 48 bottles.
The invoice value of 48 bottles is 48 × 90¢ = $43.20
Tare and leakage are 7% of $43.20 = $ 3.024
Value on which duty is paid...... $40.176

Ad valorem duty is 25% of $40.176 = $10.044
Specific duty is 48 × 20¢ = 9.60
Total duty $19.644

The total cost is
Invoice value.........$43.20
Ad valorem duty...... 10.04
Specific duty......... 9.60
$62.84

The total cost per bottle is $\frac{1}{48}$ of $62.84, or $1.31−.

NOTE.—A little work in duties may have some value for pupils who live in or near ports of entry, or for those whose parents are interested in the import or export trade. As a rule, it is not profitable to spend much time on it.

INSURANCE.

TALK:

In recent years insurance, in some form, has grown to be of interest to almost everyone. In view of this fact, it seems wise that the older pupils be given some elementary knowledge of it, and of the problems it presents.

Insurance is a guarantee to indemnify for loss or damage for a certain time, or to pay a definite amount on the happening of a named contingency.

Property insurance is a guarantee to indemnify against loss or damage to property.

It includes **fire, marine, live-stock, transit,** etc.

Life insurance is a guarantee to indemnify against loss by death, sickness, or injury.

It includes ordinary **life insurance, accident, sick benefit,** etc.

"Old line" insurance is where the insurance is taken in the large life corporations.

"Fraternal" insurance is where the policy is issued by some fraternal organization.

The **policy** is the written agreement between the insurance company and the person taking out the insurance.

NOTE.—Use actual insurance policies here, if they can be procured. If not, the illustration in this HELP may be used.

The **insured,** or **assured,** is the person on whom the policy is issued. (This is in life insurance.)

The **insurer, assurer,** or **underwriter,** is the company or person who issues the policy.

The **beneficiary** is the person named in the policy to whom the insurance money is to be paid when due.

The **risk** is the sum in which the insurer is liable in case of loss. It is the *face* of the policy.

The **premium** is the sum paid for the insurance. It is reckoned on the risk.

The risk taken differs. On **personal property** or **goods in transit,** the risk usually covers the full value. On other property, companies usually insure at from two-thirds to three-fourths of the full value. This, in a measure, protects them from deliberate destruction of property by the insured.

The **rate** is the per cent of the risk used to compute the premium.

NOTE.— Fire insurance is sometimes renewed once each year, though it may be for other periods. Life insurance may be for life or a period of years, the premium to be paid annually or oftener.

Endowment policies are common in life insurance. They may be for any period of years, as 10, 15, 20, 30, etc., with a premium payable each year.

They may be for any of these periods and have only one premium payment large enough to cover the whole. The premiums may be paid annually for only a part of the time the policy runs, as, a 10 payment 20 year policy. Premium payments on this would be made annually for 10 years. The policy becomes due in 20 years.

Life policies are payable only at death. This form of policy may call for a single payment premium, premium annually for a term of years, or premium annually until death.

Dividends.—Most large insurance companies put their rates on life insurance high enough to meet all contingencies that may arise. The death rate may for some reason be increased, or the company not be able to get as high a rate of interest for its loans. In consequence, a **surplus** often arises. This surplus is disposed of by the payment of dividends to the policy holders.

Industrial Insurance is a name given to policies written for any amount from a few dollars up.

INSURANCE TABLES.

Life insurance companies issue books which show at a glance the premium to be paid on any kind of policy taken out at any age. They are carefully compiled and in the main are the same for similar policies. This table is a good illustration.

The premiums named are for $1000, with annual payments.

Life Policies Payable at Death.						Endowments Payable as Indicated, or at Death, if Prior.					
Age.	Payments for Life.	20 Payments.	15 Payments.	10 Payments.	1 Payment.	Payable in 10 Years.	Payable in 15 Years.	Payable in 20 Years.	Payable in 25 Years.	Payable in 30 Years.	Age.
21	17.70	24.82	29.39	38.84	301.54	100.11	63.27	45.43	35.21	28.79	21
22	18.15	25.32	29.97	39.60	307.21	100.23	63.40	45.58	35.37	28.97	22
23	18.62	25.84	30.58	40.39	313.08	100.35	63.54	45.74	35.54	29.17	23
24	19.11	26.38	31.21	41.21	319.14	100.49	63.69	45.90	35.72	29.39	24
25	19.63	26.95	31.87	42.05	325.41	100.63	63.84	46.07	35.91	29.63	25
26	20.17	27.54	32.55	42.93	331.89	100.78	64.01	46.25	36.12	29.88	26
27	20.74	28.15	33.25	43.84	338.58	100.93	64.18	46.44	36.35	30.14	27
28	21.34	28.78	33.98	44.78	345.50	101.10	64.37	46.65	36.59	30.42	28
29	21.97	29.44	34.74	45.75	352.64	101.27	64.56	46.87	36.85	30.73	29
30	22.63	30.12	35.53	46.76	360.02	101.45	64.76	47.10	37.13	31.07	30
31	23.32	30.83	36.34	47.81	367.64	101.64	64.98	47.35	37.43	31.44	31
32	24.05	31 58	37.19	48.89	375.51	101.84	65.20	47.62	37.76	31.83	32
33	24.82	32.36	38.07	50.01	383.63	102.06	65.44	47.92	38.12	32.25	33
34	25.63	33.17	38.99	51.17	392.02	102.28	65.71	48.24	38.50	32.71	34
35	26.49	34.01	39.94	52.38	400.68	102.51	65.99	48.58	38.92	33.21	35
36	27.39	34.90	40.93	53.64	409.63	102.76	66.29	48.95	39.37	33.76	36
37	28.35	35.83	41.97	54.94	418.87	103.03	66.62	49.36	39.87	34.36	37
38	29.36	36.81	43.06	56.29	428.42	103.33	66.99	49.82	40.42	35.01	38
39	30.43	37.84	44.20	57.70	438.29	103.65	67.40	50.32	41.02	35.73	39
40	31.57	38.92	45.39	59.17	448.49	104.01	67.85	50.87	41.68	36.52	40
41	32.78	40.07	46.65	60.71	459.05	104.41	68.34	51.48	42.41	37.38	41
42	34.07	41.29	47.97	62.33	469.97	104.86	68.89	52.15	43.22	38.32	42
43	35.45	42.58	49.37	64.02	481.26	105.36	69.51	52.88	44.10	39.35	43
44	36.91	43.94	50.84	65.79	492.88	105 92	70.20	53.71	45.07	40.48	44

PROBLEMS.

PROBLEM:

A hotel valued at $15000 was insured for $\frac{2}{3}$ of its value. The rate was 2%. What was the premium?

WORK AND EXPLANATION:

Insured value was $\frac{2}{3}$ of \$15000 = \$10000. Premium was 2% of \$10000 = \$200.

PROBLEM:

An agent offered to insure my goods worth \$40000, at $\frac{3}{4}$ of their value. The premium was \$450. What was the rate?

WORK AND EXPLANATION:

The insured value was $\frac{3}{4}$ of \$40000 = \$30000.
The rate equals \$450 ÷ \$30000 = .015, or $1\frac{1}{2}$%.

PROBLEM:

A ship load of goods worth \$75000 was insured for $\frac{2}{3}$ of its value at $2\frac{1}{2}$%. The company reinsured $\frac{7}{8}$ of its risk at 2%. What was the premium retained by the first company?

NOTE.—When a company has more risk on property than it cares to carry, it often lets another company take part of the risk. This is reinsurance. In such cases, where there is loss, the owner looks to the first company for payment. The first company looks to the second to pay its share.

WORK AND EXPLANATION:

The first risk is $\frac{2}{3}$ of \$75000 = \$50000.
The second risk is $\frac{7}{8}$ of \$50000 = \$43750.
The first premium is $2\frac{1}{2}$% of \$50000 = \$1250.
The second premium is 2% of \$43750 = \$875.
\$1250 − \$875 = \$375, premium retained by first company.

PROBLEM:

A manufacturing plant was insured in one company for \$38000 and in another for \$40000. A fire injured the property to the extent of \$19500. What part of it should each company pay?

NOTE.—Two or more companies often issue policies on the same property. They share losses in proportion to the face of their respective policies.

WORK AND EXPLANATION:

They divide the loss in proportion to $38000 and $40000, or (dividing both by 2000 does not change ratio) in proportion to 19 and 20. The sum of the parts is 39.

1 part = $\frac{1}{39}$ of $19500, or $500.

19 parts = 19 × $500, or $9500, the part of loss paid by first company.

20 parts = 20 × $500, or $10000, the part paid by second company.

PROBLEM:

The premium for the insurance on my house at $1\frac{1}{4}$% is $25. What is the face of the policy?

WORK AND EXPLANATION:

$1\frac{1}{4}$% or $\frac{5}{4}$% of face of policy = $25.
1% of face = $\frac{4}{5}$ of $25, or $20.
100% of face = 100 × $20, or $2000.

PROBLEM:

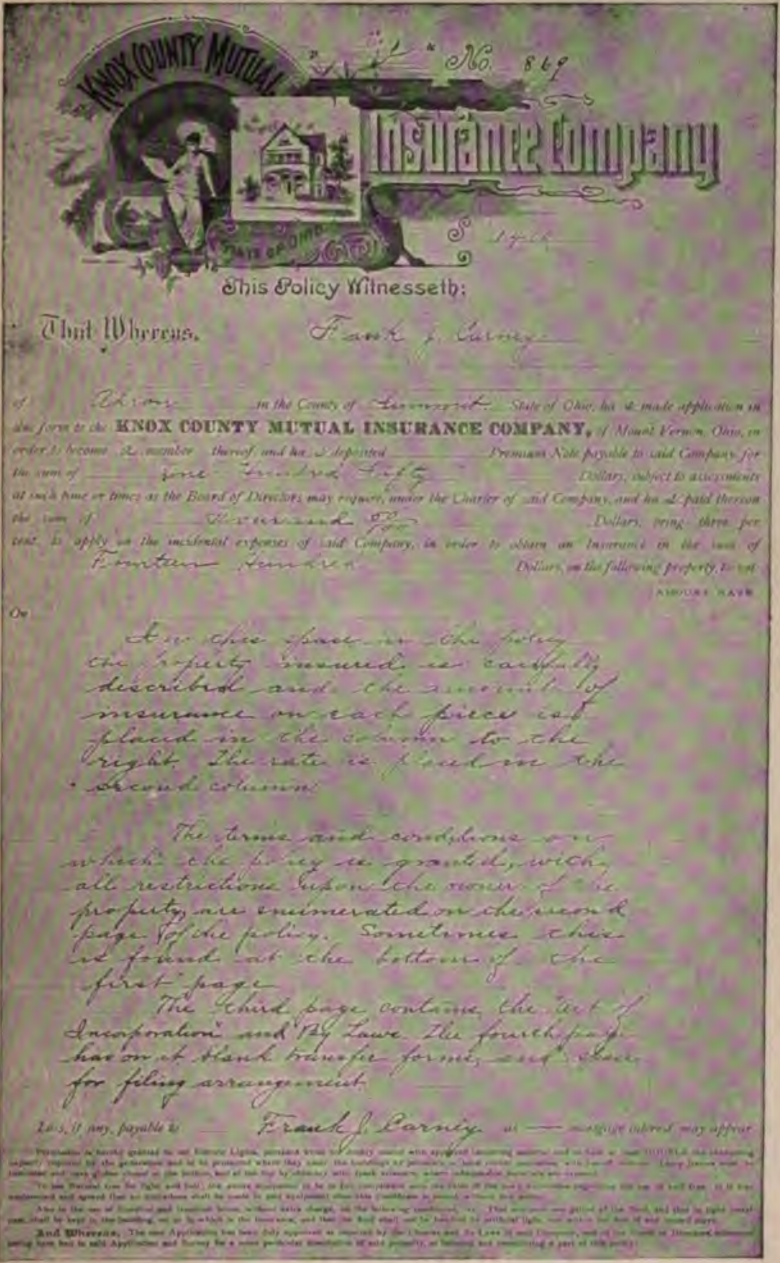
At the age of 20, I took out a $5000 life insurance policy, payable in 20 years, with semi-annual premium payments of $23.40 per $1000. How much money will I have paid the company in 20 years?

NOTE.—The premium for life insurance is based upon elaborate and carefully drawn mortality tables, which show the average years of life to be expected by people in good health at any age. They are made up from actual observance.

Life insurance companies issue tables showing the premium per $1000 for policies taken out at any age.

WORK AND EXPLANATION:

Premium on $1000 for 1 year is 2 × $23.40 = $46.80.
Premium on $5000 for 1 year is 5 × $46.80 = $234.00.



KNOX COUNTY MUTUAL

Insurance Company

No. 869

$ 1400

STATE OF OHIO

This Policy Witnesseth:

That Whereas, Frank J. Carney

of Akron in the County of Summit State of Ohio, ha s made application in due form to the **KNOX COUNTY MUTUAL INSURANCE COMPANY,** of Mount Vernon, Ohio, in order to become a member thereof, and ha s deposited Premium Note payable to said Company for the sum of one hundred fifty Dollars, subject to assessments at such time or times as the Board of Directors may require, under the Charter of said Company, and ha s paid thereon the sum of Four and 50/100 Dollars, being three per cent. to apply on the incidental expenses of said Company, in order to obtain an Insurance in the sum of Fourteen hundred Dollars, on the following property, to wit:

AMOUNT RATE

On

In this space in the policy the property insured is carefully described and the amount of insurance on each piece is placed in the column to the right. The rate is placed in the second column.

The terms and conditions on which the policy is granted, with all restrictions upon the owner of the property, are enumerated on the second page of the policy. Sometimes this is found at the bottom of the first page.

The third page contains the "Act of Incorporation" and "By Laws". The fourth page has on it blank transfer forms, and space for filing arrangement.

Loss, if any, payable to Frank J. Carney as — mortgage interest may appear

POLICY IN A MUTUAL FIRE INSURANCE COMPANY.

Premium on \$5000 for 20 years is 20 × \$234 = \$4680.

By most policies issued now, I would receive a small rebate each year after the first or second, called "participating in the dividends."

PROBLEM:

Mr. Garvin took out a policy for \$2000 at the age of 32. His semi-annual premium was \$24.90 per \$1000. In 3 years he died. How much more did his beneficiary receive than he had paid the company?

WORK AND EXPLANATION:

Annual premium on \$1000 is 2 × \$24.90 = \$49.80.
Annual premium on \$2000 is 2 × \$49.80 = \$99.60.
Premium on \$2000 for 3 years is 3 × \$99.60 = \$298.80.
\$2000 − \$298.80 = \$1701.20.

ANNUITIES.

Annuity policies are usually secured by a single cash premium, and insure the holder a specified sum each year for life. This may be made in quarterly or semi-annual payments.

Perpetual annuities are those that continue forever.

Contingent annuities depend on the happening of some contingency, as the death of the individual.

Annuities in arrears are those on which payments are unpaid after they become due.

Annuities in reversion are those that begin at some future time.

Certain annuities begin and terminate at specified times.

The **present value** of an annuity is the sum which at compound interest will amount to its final value.

The **final value** of an annuity is the sum to which all its payments at compound interest will amount at the termination of the annuity.

ANNUITY TABLE.

Showing the present value of an annual anuity of $1 from 1 to 40 years at 3½% and 4%, compound interest.

YR.	3½ PER CENT	4 PER CENT	YR.	3½ PER CENT	4 PER CENT
1	0.96618	0.96154	21	14.69797	14.02916
2	1.89969	1.88610	22	15.16713	14.45112
3	2.80164	2.77509	23	15.62041	14.85684
4	3.63708	3.62990	24	16.05837	15.24696
5	4.51505	4.45182	25	16.48152	15.62208
6	5.32855	5.24214	26	16.89035	15.98277
7	6.11454	6.00206	27	17.28537	16.32959
8	6.87396	6.73275	28	17.66702	16.66306
9	7.60769	7.43533	29	18.03577	16.98372
10	8.31661	8.11090	30	18.39205	17.29203
11	9.00155	8.76048	31	18.73628	17.58849
12	9.66333	9.38507	32	19.06887	17.87355
13	10.30274	9.98565	33	19.39021	18.14765
14	10.92052	10.56312	34	19.70068	18.41120
15	11.51741	11.11839	35	20.00066	18.66461
16	12.09412	11.65230	36	20.29049	18.90828
17	12.65132	12.16570	37	20.57053	19.14258
18	13.18968	12.65930	38	20.84109	19.36786
19	13.70984	13.13394	39	21.10250	19.58449
20	14.21240	13.59033	40	21.35507	19.79277

PROBLEMS.

1. Find the present value of an annuity for ten years of $700 at 3½%.

SOLUTION :

By the table the present value of $1 for 10 yr. at 3½% is $8.31661.

The present value of $700 is 700 × $8.31661, or $5821.63.

2. What is the present value of an annuity of $300 for 4 yr. at 4% ?

SOLUTION :

By the table the present value of $1 for 4 yr. at 4% is $3.6299.

The present value of $300 is 300 × $3.6299, or $1088.97.

3. The present value of an annuity for 20 yr. at 4% is $2718.07. What is the annuity?

SOLUTION:

The present value of $1 for 20 yr. at 4% is $13.59033.

Then $2718.07 is the present value of as many dollars as 13.59033 is contained times in $2718.07, or $200.

NOTE.—For computations on life annuities, carefully prepared tables are used. These tables show the average expectancy of life for anyone from 1 yr. up. They are based upon the average death rate through a long period of years. One of the most famous of these tables is known as the Carlisle Table, because based on the average for a period of years at Carlisle, England.

STOCKS AND BONDS.

TALK:

Work in stocks and bonds is difficult for most students chiefly because the terms used are not made clear. Then, too, most pupils have no knowledge of the way a corporation is formed, nor why it is so organized.

If the teacher will become a little familiar with the subject, a simple, clear statement of the points to be considered will help the pupils greatly. Use actual shares of stock for illustration, if they can be procured. Do the same with bonds and their interest coupons. If real ones cannot be procured, do the next best thing by using the illustrations of these papers that are given here.

This subject, like some others given in these later pages, should not consume too much of the student's time. The practical value, to most pupils, is very limited.

Explain how, in order to carry on many business enterprises, two or more men put their money together and each shares the profits or losses in proportion to the *money he has put into the business. Tell how corporations are formed with a certain number of thousands of dollars of capital stock. Every $50 or $100 of capital stock is represented by a certificate called one or as many* **shares of stock** *as is indicated on it. These shares are bought by individuals, who are then said to hold stock in the company to the amount represented in the shares they hold. The money received from the sale of these shares is used to do the business of the company.*

A **corporation** is an organization of people authorized by law to do business as one person.

Stock is the capital of the corporation. It is represented by shares in certificate form, each certifying that the person named therein holds the specified number of shares of stock.

A **share** is one of the equal parts into which the capital stock is divided. In most companies the value of a share is $100.

A **certificate of stock** is a written paper signed by the proper officers of the corporation, naming the number of shares to which the person named therein is entitled, and the original value of the same.

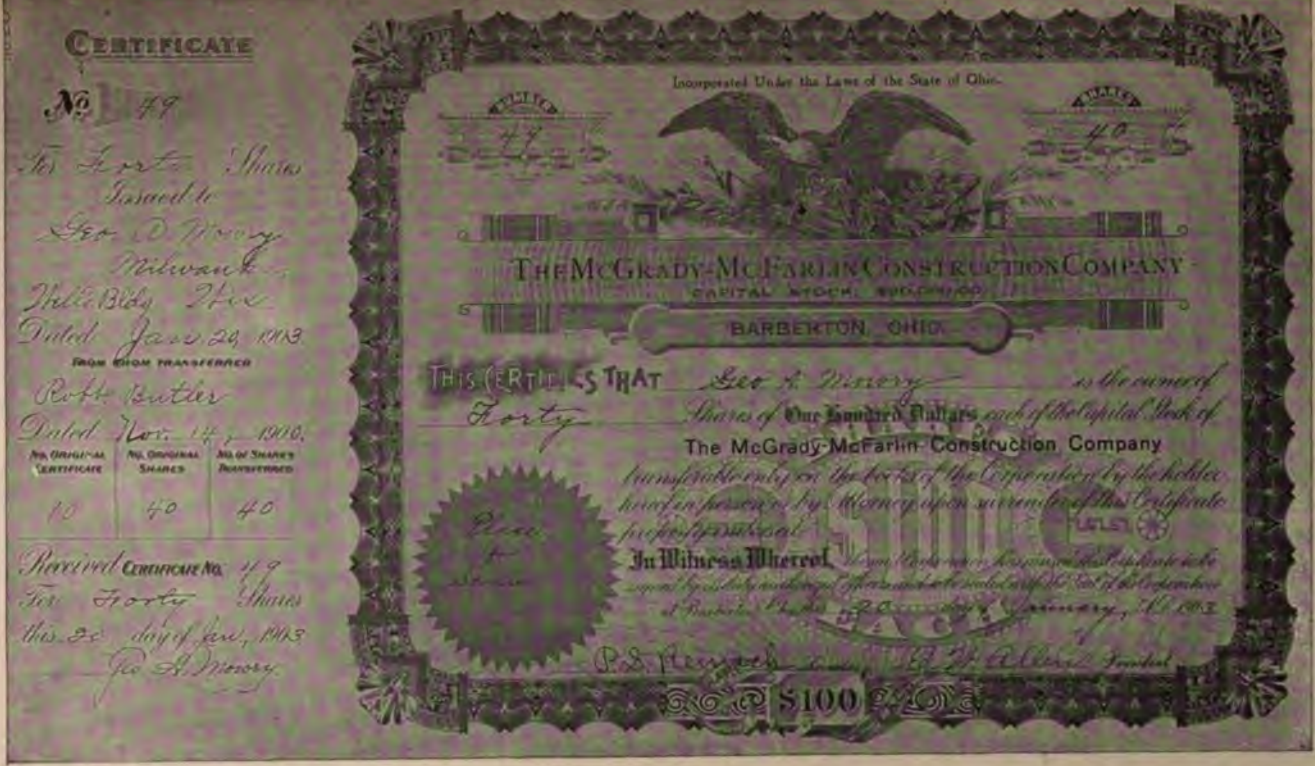
The **par value** of a share of stock is the value named in the certificate of stock.

When a corporation is prosperous, its shares of stock often sell for more than the value named in the certificate of stock. They are then said to be **above par,** or **at a premium.** In times of business depression, often these shares of stock sell below their face value. They are then said to be **below par,** or **at a discount.**

The **market value** of a share of stock is the value for which it sells in the open market.

A **stock broker** is one who makes a business of buying and selling stocks and bonds. He charges a commission for this which is called **brokerage.**

A **surplus** is a part of the earnings of a corpora-



CERTIFICATE

No. 49

For Forty Shares

Issued to

Geo. A. Mowry

Milwaukee,

Wells Bldg. Wis.

Dated Jan. 20, 1903

FROM WHOM TRANSFERRED

Robt. Butler

Dated Nov. 14, 1900.

NO. ORIGINAL CERTIFICATE	NO. ORIGINAL SHARES	NO. OF SHARES TRANSFERRED
10	40	40

Received CERTIFICATE NO. 49

For Forty Shares

this 20 day of Jan., 1903

Geo. A. Mowry

Incorporated Under the Laws of the State of Ohio.

49

40

THE McGRADY-McFARLIN CONSTRUCTION COMPANY

CAPITAL STOCK [illegible]

BARBERTON, OHIO

THIS CERTIFIES THAT Geo. A. Mowry is the owner of Forty Shares of One Hundred Dollars each of the Capital Stock of The McGrady-McFarlin Construction Company transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

In Witness Whereof, [illegible] at Barberton, Ohio, [illegible] 20 [illegible] January, A.D. 1903

President

$100

CERTIFICATE OF STOCK FOR FORTY SHARES — SHOWING STUB OF RECEIPT BOOK.

ion that are set aside to meet possible losses, for im- rovements, etc.

A **dividend** is that part of the net earnings of the orporation that is distributed among the stock- olders.

The **gross earnings** of a corporation are its total eceipts from all sources.

The **net earnings** are the profits remaining when ll expenses, losses, interest and debts due, are paid.

An **assessment** is a sum levied proportionate to tock held by stockholders, to help out the business vhen it is not prospering, or when more money is eeded to carry it on. It is levied as so many lollars on each share at its par value.

The **directors** are those shareholders elected by ll to manage the affairs of the corporation.

A **bond** is, in form, a carefully drawn, interest- earing, promissory note. Ordinarily, it runs for a eriod of years with interest often payable semi-an- ually. It is more formal than the ordinary promis- ory note. Bonds are usually issued by national, state, ounty, or local governments, or by corporations, vhen they wish to raise large sums of money for mmediate use.

Interest coupons are notes for the interest due on he bond from time to time. Each note is for the nterest of one interval of time. They are attached o the margin of the bond.

A ten-year bond for $500, with interest payable nnually, has ten coupons in its margin; if interest s payable semi-annually it has 20 coupons in the nargin, etc.

If a bond for $500 is made at 4%, interest pay- ble annually, each coupon would call for $20 in- erest.

The coupons are usually detached as they become lue and the interest collected.

The terms **above par, par value, below par,** etc, apply also to bonds.

Coupon bonds are usually made payable to bearer. A **registered bond** is made payable to order, and is recorded with the owner's name on the books of the corporation. It is transferable only by indorsement.

Difference between Stocks and Bonds.

TALK:

Many books present the subject without distinguishing stocks from bonds. Remember the following facts about each and you will have little trouble:—

Bonds—A bond runs for a specified time. It bears a specified interest, and is an absolute promise to pay the face of it at maturity. It matures at a definite time, and at that time the holder is paid its **face value** and no more, by the organization that issued it, unless such organization is insolvent, or has repudiated its debts.

Stocks—A certificate of stock is no promise to pay. It simply shows that the holder owns as much stock in the corporation as is shown by the face of the certificate. It bears no interest and has no date of maturity.

The interest returns of the bond holder are certain and definite. The returns of the stockholder, dividends, are uncertain and depend on the profits of the business.

Consequently, no table can be arranged to show at what rate stocks can be bought to yield a definite return; but with bonds, tables may be made which show at a glance what the return will be from a purchase made at any rate.

Note.—Be careful of what your arithmetic says on this subject.

PREMIUM AND DISCOUNT ON BONDS.

If a bond bears a rate of interest higher than what the investor expected to realize, it sells at a premium, and the longer it has to run, the higher the premium.

If the rate is lower than the investor expected to make, the bond sells at a discount, and the longer it has to run, the greater the discount.

The longer a bond has to run the more a purchaser can afford to pay for it, as will appear from the problems given later.

Business men pay a higher price for bonds with interest payable semi-annually, than for those with interest payable annually. Most bonds bear interest payable semi-annually; and all bond tables are based on semi-annual interest. Some bonds have interest payments quarterly.

PROBLEMS IN STOCKS.

PROBLEM:

Five men, A, B, C, D, and E combined to build and operate a cotton mill. $800000 capital was needed. They formed a stock company and divided the capital stock into 8000 shares of $100 each. They took shares as follows: A, 2000 shares; B, 2000 shares; C, 3000 shares; D, 500 shares, and E, 500 shares. No dividend was declared the first year, but at the end of the second year there were $64000 in profits, and a dividend was declared. What per cent was the dividend and what did each receive as his share?

WORK AND EXPLANATION:

Query? $64000 is what % of $800000?
$64000 is $\frac{64000}{800000}$ of 100% = 8%.

NOTE.—At this stage of his study, the pupil does not need such minute analysis as earlier. His mind should be ready now to deal with larger concepts.

The face value of A's 2000 shares is 2000 × $100 = $200000.

He receives 8% of $200000 = $16000.

The face value of B's 2000 shares is 2000 × $100 = $200000.

He receives 8% of $200000 = $16000.

The face value of C's 3000 shares is 3000 × $100 = $300000.

He receives 8% of $300000 = $24000.

The face value of D's 500 shares and E's 500 shares, respectively, is 500 × $100 = $50000.

Each receives 8% of $50000 = $4000.

Proof:

They all receive the sum of $16000, $16000, $24,000, and $8000, or $64000, the whole of the profits.

Note.— The form below is equally good. For quick work it is to be preferred.

At 8% 1 share of stock yields $8.
A's 2000 shares yield 2000 × $8 = $16000.
B's 2000 shares, etc.

PROBLEM:

For the company named in the preceding problem, there was little profit at the end of the third year and C bought the stock of B and D at a premium of 3%. What did he pay for it?

Work and Explanation:

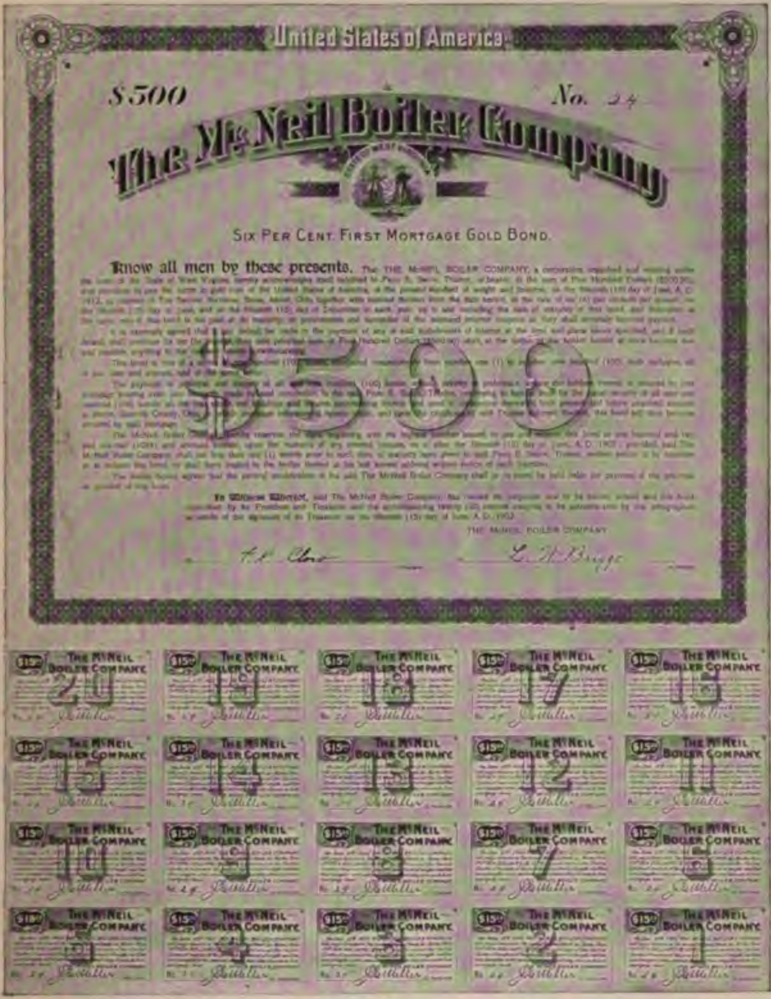
B's stock at par is worth 2000 × $100 = $200000.
D's stock at par is worth 500 × $100 = 50000.

B's and D's stock at par is worth $250000.

C pays for it 3% premium, or 103 per cent, of its par value. 103% of $250000 = $257500.

PROBLEM:

Miss Seeber owns twenty shares in a company whose capital stock is $100000, shares $100 each.



United States of America

$500 No. 24

The McNeil Boiler Company

SIX PER CENT. FIRST MORTGAGE GOLD BOND.

Know all men by these presents. That THE McNEIL BOILER COMPANY,

THE McNEIL BOILER COMPANY

A COUPON BOND.

(See Illustration of Coupons from This Bond.)

Last year there was a profit of $10000 to be divided. What dividend was declared? What was Miss Seeber's share?

Work and Explanation:

Query? $10000 is what % of $100000?

$10000 is $\frac{1}{10}$, or 10% of $100000.
Hence, the dividend declared was 10%.
1 share yields $10.
20 shares yield 20 × $10 = $200.

PROBLEM:

What will 15 shares of New York, New Haven, and Hartford R. R. stock cost me at 168?

Work and Explanation:

At 168, one share costs $168.
15 shares cost 15 × $168, or $2520.

Note.—Explain to the pupil that the quotation for any stock represents its market value. It is usually above or below par. Here it was 68 above par. This means that each 100-dollar share is selling in the market for $168.

PROBLEM:

Michigan Central R. R. stock is quoted at 98. Its average dividends for some years have been 5%. How much must I invest to get an income of $450 from it?

Work and Explanation:

1 share at 5% yields $5.
To yield $450 it takes $\frac{450}{5}$, or 90 shares.
90 shares at 98 cost 90 × $98 = $8820.
∴ I must invest $8820.

PROBLEM:

J. Pickard bought ten shares of Ill. Cent. R. R. stock at 96. What does he get when a dividend of 6% is declared? What per cent is that on his investment?

Work and Explanation:

1. 1 share at 6% yields \$6.
   10 shares yield 10 × \$6 = \$60.

2. Each share at 96 costs \$96.
   Each share yields \$6.

Query? \$6 is what % of \$96?
$6 is $\frac{6}{96}$ of 100%, or $6\frac{1}{4}$.
∴ the investment yields $6\frac{1}{4}$%.

PROBLEM:

I am offered stock that pays 8%. What should I pay for it to bring me 6% on my investment?

Work and Explanation:

1 share yields \$8.
Query? \$8 is 6% of what number?
1% = \$$1\frac{1}{3}$.
100% = 100 × \$$1\frac{1}{3}$, or \$$133\frac{1}{3}$.
∴ I can pay \$$133\frac{1}{3}$ per share, or buy when it is quoted at $133\frac{1}{3}$.

Note.—Should this corporation ever close up, I would only get par value, or \$100 a share, for this stock. Consequently, I must consider that element in buying.

PROBLEM:

After a bank has declared a scrip dividend of 5%, I hold 42 shares. The par value is \$100. How many shares had I at first?

Work and Explanation:

The par value of 42 shares is \$4200.

The dividend was 5% of par value of my original shares.

Then 105% of par value of my original shares = \$4200.

100% of par value of original shares is $\frac{100}{105}$ of \$4200 = \$4000.

\$4000 = par value of 40 shares.

∴ I had 40 shares at first.

NOTE.— Explain that a *scrip* dividend is the name given to a dividend paid in new shares. When a company wishes to increase its capital stock it sometimes uses the profits in that way. Instead of declaring a cash dividend, it issues new shares of stock to each stockholder to the amount of what his share of a cash dividend would be.

STOCK QUOTATIONS.

The stock quotations in the daily newspapers on Dec. 10, 1902 were: —

New York.	
American Sugar	120½
American Wool, preferred	76
C. & N. W. R. R	220¾
N. Y. Gen'l Electric	179
Maryland Coal, preferred	100
Chicago Union Traction	14
Canadian Pacific R. R.	128⅛
Baltimore & Ohio R. R.	98¾
Amalgamated Copper	58⅜
Chicago.	
Swift & Co.	124½
National Biscuit Co.	45⅝
National Biscuit Co., preferred	102
Diamond Match	138

NOTE.— These quotations represent the value of each 100-dollar share of stock. For instance, a 100-dollar share of Diamond Match stock was worth in the market $138 on Dec. 10, 1902.

The pupil will notice in this table that some of the stocks are **preferred.** This necessitates an explanation of the terms, common stock and preferred stock.

Common stock is the ordinary stock of a corporation, which has no preference, in the payment of dividends, over any other.

Preferred stock is stock which is given a preference over the common stock. Ordinarily, a dividend is paid on the preferred stock before any is paid on the common shares.

PROBLEMS IN BONDS.

PROBLEM:

A city issued bonds to the sum of $500000, to build a public library. The bonds were for 20 years, interest at 6%, payable annually. (a) How many interest coupons should be attached to each bond? (b) If the bonds sold at 2% premium, what did they bring? (c) What did I pay for six 1000-dollar bonds? (d) What is my income from them? (e) What per cent is that on my investment?

WORK AND EXPLANATION:

(a) Since the bonds run for 20 years and the interest is payable annually, there should be one coupon for each year, or 20 in all.

(b) At 2% premium, they sold for 102% of their face value. 102% of $500000 = $510000.

(c) The par value of six 1000-dollar bonds is $6000.
I pay 102% of par value, or $6120.

(d) 6% of $6000 = $360.

(e) On an investment of $6120, I get an income of $360. $360 is $\frac{360}{6120}$ of 100% = $5\frac{15}{17}$%.

NOTE.—Remember that in 20 years, when the bonds become due, I only get the par value, or $6000. They cost me now $6120.

PROBLEM:

What can I afford to pay for bonds that pay 6% interest and have 10 years to run to get 4% on my investment?

Work and Explanation:

A 6% bond yields $6 a year.

$6 is 4% of $150.

At maturity, I get only par value, $100.

Consequently, if I buy at $150, I will lose $50 in 10 years.

In order that I may not lose, I can only pay $150, less the sum which, put at compound interest for 10 years at 4%, amounts to $50.

At 4% for 10 yr., at compound interest, $33.79 amounts to $50. (Use Comp. Int. Table.)

$150 − $33.79 = $116.21, the price I can afford to pay.

BOND QUOTATIONS.

Newspaper quotations for bonds on Dec. 10, 1902, were as follows:—

New York.

United States refunding 2s, registered	107¾
United States refunding 2s, coupon	108¼
United States 3s, registered	107¾
United States 3s, coupon	107¾
United States new 4s, registered	135½
United States new 4s, coupon	135½
United States old 4s, registered	108¾
United States old 4s, coupon	109½
United States 5s, registered	103¾
United States 5s, coupon	103¾
Adams Express 4s	104½
C. B. & Q.s joint	95⅝
Northern Pacific 3s	72½

Chicago.

Metropolitan gold 4s	100¼
Metropolitan extension 4s	94⅞
Western Stone 5s	96

PROBLEM:

At the above quotations, I buy 10 one thousand dollar Adams Express 4s, 5 five hundred dollar Metropolitan gold 4s, and 7 one hundred dollar Northern Pacific 3s. What does the whole cost me?

WORK AND EXPLANATION:

10 one thousand dollar Adams Express cost
10000 × $1.04½ = $10450
5 five hundred dollar Metropolitan 4s cost
2500 × $1.00¼ = $ 2506.25
7 one hundred dollar Northern Pacific cost
700 × $.72½ = $ 507.50
The whole cost is.............. $13463.75

NOTE.—The multiplicand in each case above is the market value of one dollar of the bond, as shown by the quotation. The terms 2s, 4s, etc., designate the rates of interest paid on the bonds; 2s mean 2% interest, 4s mean 4% interest, etc.

TALK:

Have the pupil take a daily paper, turn to the financial page, note the market price of stocks and bonds, and make and solve problems therefrom. Problems similar to those in the preceding pages embrace about all that the average pupil should be asked to consider. The intricate problems found in many of the arithmetics are impractical and are not to be considered in grammar school work.

In connection with this work, a few simple problems on the commission paid to stock brokers may be given. The commission is usually ¼% or ⅛% on the face value of the stock or bond.

Here is a sample problem:—

What will be the cost of 40 one hundred dollar Western Stone 5s at 96, brokerage ⅛%?

Cost of bonds is 40 × $96, or $3840
Brokerage is ⅛% of $4000, or 5
Total cost is............. $3845

THE STOCK EXCHANGE.

TALK:

The pupils will be interested in knowing something of the stock exchange in a large city. For extended information, read any of the large cyclopedias. Then give the pupils a talk on the subject. For those who find it difficult to procure the desired information, the following brief summary is given:—

The **stock exchange** of a city is an organization of brokers, for the purpose of dealing in stocks, bonds, etc. The membership is limited and each applicant pays a fee for admission. When the membership is completed, new members can only come in by purchasing a place. In this way, places have cost as high as $24000.

At each day's session, which lasts from 10 A. M. to 3 P. M., the securities to be dealt in are read off by the presiding officer or a clerk. When a stock is called, those wishing to sell make their offer, while those desiring to buy make an offer. Whenever a sale is made, the secretary records the sale.

A Few Stock Exchange Terms:

Watered stock are shares distributed among shareholders for which there is no capital stock. It does not increase the value of the stock. When there is a consolidation of corporations, watered stock is often issued.

Buying long is buying stock for future delivery with the expectation of a rise in value before delivery.

Selling short is selling stock to be delivered at some future date. This is stock which the seller does not own at the time, but which he expects to buy at a lower rate before date of delivery.

A **bull** is a broker who is "long" on stock and holds it for an advance in price. He tries to advance the price, hence he is called a bull, from his desire to "toss up" the price.

A **bear** is a broker who is "short" on stock that he has sold for future delivery. He wishes to buy as cheaply as he can, so he tries to "tear down" the price. From this desire to "tear down," he is called a bear.

A **margin** is a deposit which the client leaves with his broker to protect the latter in case of fluctuations in the stock bought for his customer.

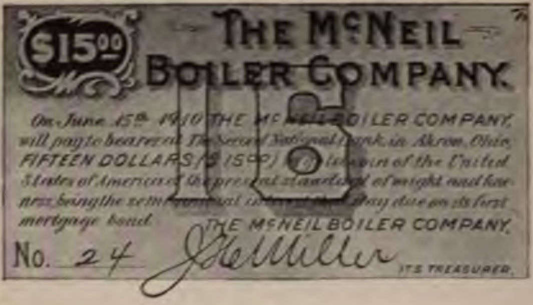
A **corner** is the control of the price of a security or article of value. This is done by one person, or a combination of people, who buy all of the article or security offered for sale and hold it for a rise. When the time of those who bought "short" comes for delivery, they must pay what the men who have the corner ask. Here is where the latter make their profits.

Stock sold **cash** is to be delivered on the day sold.

Stock sold **regular** is to be delivered next day. Stock bought regular is to be paid for next day. Transactions are all "regular way," unless otherwise specified.

Buyer three means that the buyer of stock has the next three days on any one of which he may demand delivery.

Seller three means that the seller has his option of delivering the stock on any of the three following days.


$15.00

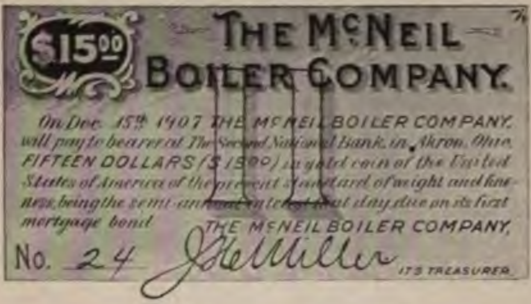
THE McNEIL BOILER COMPANY.

16

On June 15th 1910 THE McNEIL BOILER COMPANY, will pay to bearer at The Second National Bank in Akron, Ohio, FIFTEEN DOLLARS ($15.00) in gold coin of the United States of America of the present standard of weight and fineness, being the semi-annual interest that day due on its first mortgage bond.

THE McNEIL BOILER COMPANY,

No. 24 J.H.Miller ITS TREASURER.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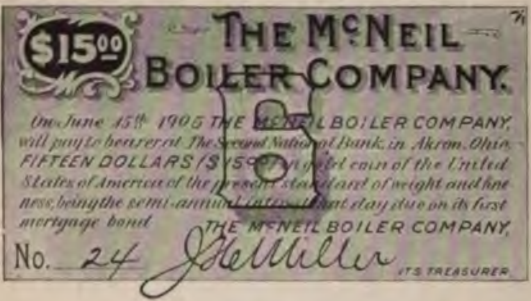
THE McNEIL BOILER COMPANY.

11

On Dec. 15th 1907 THE McNEIL BOILER COMPANY, will pay to bearer at The Second National Bank in Akron, Ohio, FIFTEEN DOLLARS ($15.00) in gold coin of the United States of America of the present standard of weight and fineness, being the semi-annual interest that day due on its first mortgage bond.

THE McNEIL BOILER COMPANY,

No. 24 J.H.Miller ITS TREASURER.


$15.00

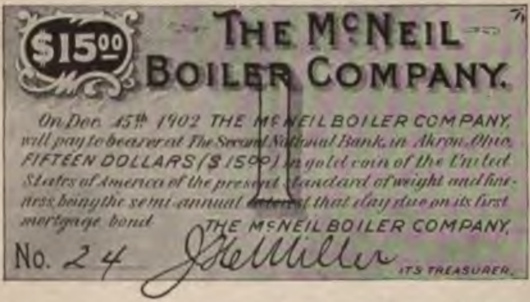
THE McNEIL BOILER COMPANY.

6

On June 15th 1905 THE McNEIL BOILER COMPANY, will pay to bearer at The Second National Bank in Akron, Ohio, FIFTEEN DOLLARS ($15.00) in gold coin of the United States of America of the present standard of weight and fineness, being the semi-annual interest that day due on its first mortgage bond.

THE McNEIL BOILER COMPANY,

No. 24 J.H.Miller ITS TREASURER.


$15.00

THE McNEIL BOILER COMPANY.

1

On Dec. 15th 1902 THE McNEIL BOILER COMPANY, will pay to bearer at The Second National Bank in Akron, Ohio, FIFTEEN DOLLARS ($15.00) in gold coin of the United States of America of the present standard of weight and fineness, being the semi-annual interest that day due on its first mortgage bond.

THE McNEIL BOILER COMPANY,

No. 24 J.H.Miller ITS TREASURER.

THE illustration here shows the actual size of the first vertical row of interest coupons on the bond illustrated in another part of this book.

Read the terms of that bond. What is its number? Does that number appear on all of the coupons? Why? How many of these bonds were issued on June 15, 1902? When do they become due? When may they be paid if the maker desires it? How much of the face value must the maker pay if he pays these bonds before June 15, 1902? What rate of interest do these bonds draw?

The interest on this bond is payable every six months, so there are 20 coupons, numbered in the order in which they come due. Each calls for the payment of $15 in gold coin of the present standard of value. Why at the present standard of value?

Notice also that the payment of this bond is secured by a first mortgage on all the buildings and other property of the maker. That is because this is the bond of a private corporation. When a national, state, county, or city government issues bonds it gives no mortgage. Why?

PART EIGHT.

EQUATIONS. PROPORTION.
RATIO. PARTNERSHIP.
AVERAGES.
AVERAGE OF PAYMENTS.

"When you set out to do anything, never let anything disturb you from doing that one thing. This power of putting the thought on one particular thing, and keeping it there for hours at a time takes practice; and it takes a long time to get into the habit."

—Thomas A. Edison.

THOROUGH TRAINING.

THIS is the age of the trained man and the trained woman. That is the thing I want to write on your hearts. There was a time in this country when opportunities were so great, and there was so much to be done, that any man or any woman who had a good heart and a good character, and a strong arm, might achieve a certain degree of success. I am not saying that that time has entirely passed. I hope it will be long before it is entirely passed. But this I am saying to you, that if I were a young man or a young woman, going out into the world to-day, I should not dare to go, unless I had given myself every possible educational opportunity; unless I had made myself absolutely master of the thing I wanted to do.

I tell you to-day that the tragedy of modern life is the tragedy of the half-educated man or woman. It is the tragedy of the man or woman who wants to do something and cannot do anything well.

HAMILTON W. MABIE,
In address to business college students.

EQUATIONS.

TALK:

As a means of expressing the parts of a solution to a problem, the equation is used in all arithmetic work. From the simplest statements of the child in early number work to the most complex work of the advanced student, the equation is in constant use.

The primary pupil makes all his use of the equation without the formality of naming it, and the results are just as good. The older pupil may, if the teacher sees fit, pursue the elementary discussion of the equation here, as a basis for information, or for his later work in advanced mathematics.

An **equation** is a statement of equality between two numerical expressions, as, $2 + 2 = 4$.

The parts on either side of the sign of equality are called the **members.**

When a member is separated into parts by either of the signs + or −, the parts are called **terms.**

ILLUSTRATIONS:

12 inches = 1 foot. Here, each member is a term. $4 + 4 = 8$. Here, 4, 4, and 8 are all terms. An area 4 ft. wide and 6 ft. long = 24 sq. ft. Here, each member is a term, multiplication being understood in the first.

TRUTH OF EQUATIONS.

Some equations are always true and may be used in the solution of any problem, as,

1. $4 + 4 + 4 = 3 \times 4$.
2. 3 feet = 1 yard.

3. 4 times my age = 3 times my age + once my age.

4. 1728 ÷ 12 = 144.

Other equations are true only when found in certain problems; hence, they can only be used in the solution of those problems, as,

1. Length of a board walk = 140 ft.
2. The cost of a piano = $700.
3. $\frac{1}{4}$ of my money − $\frac{1}{2}$ of yours = $\frac{3}{4}$ of Elmer's.

CHANGES IN EQUATIONS.

Sometimes the given form of an equation is not desired. It may be changed to the desired form by one or more of the following ways:—

1. Turned Around.—This means putting the right member where the left was found, and placing the left where the right was, as,

$6 + 3 = 3 \times 3$, which may be written $3 \times 3 = 6 + 3$, and is still a true statement of the equality.

This change is made on the principle that

Equals are equals in whatever order they are placed.

2. Simplified.—This is done in the same way that other numerical expressions are simplified, as, from the preceding equation,

$6 + 3 = 3 \times 3$. Performing the indicated operations, we have $9 = 9$.

The requisite here is that only similar terms be found in each member, as,

$\frac{1}{2}$ day + $\frac{1}{3}$ day = 12 hr. + 8 hr., which simplified is, $\frac{5}{6}$ day = 20 hr.

3. Multiplied.—An equation may be multiplied by multiplying each member by the same number, as,

$4 = 2 \times 2$. Multiply both members by 2 and we have $8 = 2(2 \times 2)$, or $8 = 8$. In the form $8 = 8$, we have both multiplied and simplified the original equation.

This may be done on the principles that

1. Equals multiplied by equals give equals.

2. Multiplying all the terms of an equation by the same number, multiplies the equation by that number.

To show the simple form which this application of the equation may take, note this: —

What is the cost of 3 pencils @ 2¢ each?

Cost of 1 pencil = 2¢.
Cost of 3 pencils = 6¢.

We get the second equation from multiplying the first by 3.

4. *Divided.*— An equation may be divided by finding the same fractional part of each of its members, as,

$6 = 3 + 3$. Divide by 3. Then $2 = 1 + 1$.

This is done on the principles that

1. Equals divided by equals give equal quotients.

2. Dividing the terms of a member by a number, divides the member by that number.

Applied to a simple problem, we have: —

If 4 pencil tablets cost 12¢, what is the cost of each?

Cost of 4 tablets = 12¢.
Cost of 1 tablet = 3¢.

For the second equation which contains the desired result, we simply divide the first by 4.

NOTE.—If the teacher gives pupils as many simple problems to be performed by the use of equations, as will acquaint them with the use of the equation, it is time well spent.

RATIO.

TALK:

Many of the new arithmetics very properly introduce, and carry along, the idea of ratio in an informal way, at an early stage of the pupil's work in arithmetic. Any proper system of teaching primary numbers must include the ratio idea.

The work in the first part of this "Help" on Measures for Little People embraces the ratio plan. Whenever the child makes a comparison of 6 inches with 1 foot, 2 quarts with 1 gallon, etc., he uses the ratio idea. Consequently, he learns to think in the ratio way from the first, without the formality of calling it so.

The use of ratio is very common. When you hear people say "99 out of 100," "4 out of 5," "I pay 5% interest (5 out of 100)," "$\frac{4}{5}$ of my time is spent at home (meaning 4 parts of 5)," etc., we do not ordinarily say they use the ratio idea, yet it is nothing less.

There need be no time set apart for the formal teaching of ratio. When the teacher deems it advisable to define it, and dwell upon it in particular, she may do so. For convenience, at such a time, the matter which follows is included here.

Ratio is a comparison of the magnitudes of numbers of the same kind.

We may compare numbers of the same kind in two ways: By their difference; and by the number of times one is found in the other. The latter is a comparison of ratio.

The number taken for the standard of comparison is called the **consequent**.

The number compared with the consequent is called the **antecedent.**

SIGN OF A RATIO.

Ratio may be expressed by the sign :, as in 3:4, which is read, 3 is to 4, or the ratio $\frac{3}{4}$, or the ratio 3 to 4.

It may be expressed as a division, as,

$$\tfrac{3}{2}, \text{ or } 3 \div 2, \text{ or } 2)3.$$

It is usually expressed in one of these ways: 3:4, or $\frac{3}{4}$.

In each of the above, 3 is the antecedent. The consequent in both is 4.

THE VALUE OF A RATIO.

The value of a ratio is found by dividing the antecedent by the consequent.

The value of the ratio 4 : 2 is $4 \div 2 = 2$. The value is 2.

The value of the ratio $8 to $2 is 4. This means that in $8 there are 4 times $2, not $4 × $2. From this, it is seen that *the value of a ratio is always abstract.*

The numbers used in the ratio, as shown above, may both be abstract, or may both be concrete. The value is never concrete.

OTHER NAMES USED.

The numbers compared in a ratio are called the **terms.**

The numbers of a ratio considered together are called a **couplet.**

When the ratio is written in this way, $4 : $2, it is called an **indicated ratio.**

The value is called a **numerical ratio.**

ILLUSTRATION:

$$\underbrace{\$10 : \$5 =}_{\text{Indicated ratio.}} \qquad \underbrace{2.}_{\text{Numerical ratio.}}$$

Remember:

1. **The numerical ratio is always found by division.**
2. **The numbers of a ratio must be of the same kind.**

There can be no comparison of numbers such as 6 years with 4 cents.

WRITING A RATIO.

Whenever an indicated ratio has concrete numbers for its terms, it may be written as an indicated ratio whose terms are abstract, as,

(concrete) 24 horses : 6 horses.
(abstract) 24 : 6.

In either case, the value is 4. Hence, the ratio is the same whether the terms are concrete or abstract, so long as they are numerically equal.

For all practical work in solving problems where ratio is used, it is more convenient to consider the terms as abstract.

FINDING THE MISSING PART.

Where one part of a ratio is not written, we may find it in the way shown here:—

PROBLEM:

What is the ratio of 15 days to 3 days? Here we are to find the value.

STATEMENT AND WORK:

15 days : 3 days = (?)
15 : 3 = 5.

PROBLEM:

6 is the ratio of what number to 10?

STATEMENT AND WORK:

(?) : 10 = 6.

Inversely, to work above: 6 × 10 = 60.

PROBLEM:

10 is the ratio of 40 years to what?

Statement and Work:

$$40 \text{ years} : ? \text{ years} = 10.$$
$$40 : ? = 10.$$
$$40 \div 10 = 4.$$

Since the required result is to be years, we name the 4, *years*, thereby changing back to the concrete.

X FOR THE UNKNOWN QUANTITY.

TALK:

It is not necessary that the pupil know anything of the word algebra to make intelligent and practical use of x for the unknown quantity. Instead of using the ? for the unknown quantity, teach him early to use x.

In the preceding problems in ratio we have used ? to represent the number to be found. The ? properly belongs at the end of an interrogation, and not to represent the unknown quantity. For the latter purpose, the letter x has come into common use.

EXAMPLE:

20 is the ratio of what number to 9?

Statement:

$$x : 9 = 20.$$
$$20 \times 9 = 180.$$

Remember that the use of x always means that we are to find the number it represents.

PROBLEMS.

PROBLEM:

What is the ratio of a loan of $10 for 4 mo. to a loan of $5 for 6 mo.?

Explanation:

$10 for 4 mo. = 4 × $10, or $40 for 1 mo.
$ 5 for 6 mo. = 5 × $6, or $30 for 1 mo.

Then $40 : $30 = x.

Find x.

$$\frac{40}{30} = 1\tfrac{1}{3}.$$

$$\therefore x = 1\tfrac{1}{3}.$$

We may indicate the above work in a shorter way:—

$$4 \times 10 : 5 \times 6 = x.$$
$$40 : 30 = 1\tfrac{1}{3}.$$

PROBLEM:

A surface has an area of 150 sq. ft. and is 30 ft. long. What is the ratio of its width to that of a surface of 200 sq. ft. which is 40 ft. long?

Explanation:

Width of 1st surface = $\frac{150}{30}$ ft.
Width of 2d surface = $\frac{200}{40}$ ft.

$$\text{Then } \frac{150}{30} : \frac{200}{40} = x.$$

$$\frac{150 \times 40}{30 \quad 200} = 1.$$

Inspection shows the result in the above to be 1.

AXIOMS.

For much work in ratio as ratio, in proportion, or with equations, the common axioms are valuable. Their truth any pupil in the upper grammar grades may prove for himself.

They are given here:—

1. If equals are added to equals, the sums are equal.

$$2 + 4 = 3 + 3.$$

$$\text{Then } 2 + 4 + 5 = 3 + 3 + 5.$$

2. Numbers equal to the same number or equal numbers are equal to each other.

$$4 = 2 + 2$$
and
$$3 + 1 = 2 + 2$$
$$\text{Then, } 4 = 3 + 1.$$

3. If equals are subtracted from equals, the remainders are equal.

$$6 = 4 + 2$$
$$\text{Then, } 6 - 3 = 4 + 2 - 3.$$

4. If equals are multiplied by equals, the products are equal.

$$3 \times 4 = 6 \times 2$$
$$\text{Then, } 3 \times 4 \times 4 = 6 \times 2 \times 4.$$

5 If equals are divided by equals, the quotients are equal.

$$40 = 20 \times 2$$
$$\text{Then, } 40 \div 10 = 20 \times 2 \div 10.$$

PROPORTION.

TALK:

It cannot be said that proportion has much practical use judging from the manner in which arithmetic is taught in most schools at present. Its chief uses seem to be in work demanding a knowledge of the more advanced mathematics.

Like some other subjects in this "Help," it is included because some teachers consider it important. In schools where the foundation for geometry and physics is laid, a study of this subject will be valuable.

A proportion is an expression of equality between ratios.

The ratio 4 : 5 is the same as 8 : 10. We express this in a proportion in this way: —

$$4 : 5 :: 8 : 10.$$

The double colon, : :, is used here to denote the equality. We may write it,

$$4 : 5 = 8 : 10,$$

or

$$\frac{4}{5} = \frac{8}{10}.$$

TERMS USED.

In every proportion there are four terms. These are named from left to right.

The first and fourth terms are called the extremes.

The second and third terms are called the means.

$$\underbrace{4 : \underbrace{24 :: 5}_{\text{means}} : 30}_{\text{extremes}}$$

THE RATIOS.

While the two terms of a ratio must be of the same kind, the ratios of a proportion need not.

We may have a proportion like this: —

4 days : 8 days : : 3 men : 6 men,

or

\$4 : \$9 : : 8 : 18.

The ratios may be —

both concrete,
both abstract,
one abstract, the other concrete.

In solving problems in proportion, the terms may all be considered as abstract and the result named from inspection for what is wanted.

The product of the means equals the product of the extremes.

When the terms of any proportion are written as abstract, the above is true.

TO ILLUSTRATE:

1. 4 : 6 : : 8 : 12.
   Product of means = 6 × 8, or 48.
   Product of extremes = 4 × 12, or 48.
2. \$8 : \$16 : : 5 : 10.
   Consider the terms as abstract.
   Then product of means = 16 × 5, or 80,
   product of extremes = 8 × 10, or 80.
3. 4 mo. : 9 mo. : : \$18 : \$40.50.
   $4 : 9 :: 18 : 40\frac{1}{2}$.
   $9 \times 18 = 162$.
   $4 \times 40\frac{1}{2} = 162$.

Either extreme is equal to the product of the means divided by the other extreme.

TO ILLUSTRATE:

1. 10 : 25 : : 6 : 15.
   $\frac{25 \times 6}{10} = 15.$ $\frac{25 \times 6}{15} = 10.$
2. 4 men : 20 men : : \$1 : \$5.
   $\frac{20 \times 1}{5} = 4.$ $\frac{20 \times 1}{4} = 5.$

PROBLEMS.

PROBLEM:

Find the missing term in $4 : 18 :: 10 : x$.

$$\frac{18 \times 10}{4} = 45.$$

$$\therefore x = 45.$$

PROBLEM:

Four men working 6 days complete a piece of work. How long will it take six men?

6 men : 4 men : : 6 days : x days.

$$\frac{4 \times 6}{6} = 4$$

$$\therefore x = 4 \text{ days.}$$

PROBLEM:

If 7 tons of coal cost \$28, what will 9 tons cost?

$$7 \text{ tons} : 9 \text{ tons} :: \$28 : \$x.$$

$$\frac{28 \times 9}{7} = 36$$

$$x = \$36.$$

NOTE.—The two preceding problems illustrate the impractical side of proportion in ordinary simple problems. Any pupil will solve them by the usual way while he is stating the proportion. For instance, for the last one, how simple to say,

"If 7 tons cost \$28, one ton costs \$4 and 9 tons cost \$36." or "If 7 tons cost \$28, 9 tons cost \$36."

The proportion is in the last solution but how infinitely more practical it is to make no reference to it in a formal manner at all. If you are untangling intricate compound arithmetical curiosities, proportion may serve you, but for the grammar school pupil it does not appear to be time well spent.

Either mean is equal to the product of extremes divided by the other mean.

TO ILLUSTRATE:

$$4 : 10 :: 6 : 15.$$

$$\frac{4 \times 15}{10} = 6. \qquad \frac{4 \times 15}{6} = 10.$$

PROBLEMS.

PROBLEM:

Find the missing term in

$$9 : x :: 12 : 4.$$

$$\frac{9 \times 4}{12} = 3.$$

$$\therefore x = 3.$$

PROBLEM:

A piece of work can be done by 7 men in 4 days. How many days should it take 6 men?

$$7 \text{ men} : 6 \text{ men} :: x \text{ days} : 4 \text{ days}.$$

$$\frac{7 \times 4}{6} = 4\tfrac{2}{3}$$

$$\therefore x = 4\tfrac{2}{3} \text{ days}.$$

NOTE.—The way to find the fourth term of a proportion when three are given, has long been popularly known as solving problems by "the rule of three," or because of the frequency with which it was used a generation ago, it was often called "the golden rule of three." To-day the average pupil leaving the grammar school does not know what is meant by the name.

PROBLEM:

If 4 barrels of flour cost $15, what will 42 barrels cost?

STATEMENT AND WORK:

$$4 \text{ bbls.} : 42 \text{ bbls.} :: \$15 : \$x.$$
$$\tfrac{42 \times 15}{4} = 157\tfrac{1}{2}$$
$$\therefore\ x = \$157\tfrac{1}{2}.$$

PROBLEM:

A pole 9 ft, high casts a shadow of $12\frac{1}{2}$ ft. At the same time, a near-by tree casts a shadow of 28 ft. How tall is the tree?

STATEMENT AND WORK:

$$12\tfrac{1}{2} \text{ ft. shadow} : 28 \text{ ft. shadow} :: 9 \text{ ft.} : x \text{ ft.}$$
$$\tfrac{28 \times 9}{12.5} = 20.16$$
$$\therefore x = 20.16 \text{ ft.}$$

PROBLEM:

A railroad contractor has 120 men at work and has provisions on hand to supply them for 3 months. He increases the force to 200 men. How long will his provisions last them?

STATEMENT AND WORK:

$$200 \text{ men} : 120 \text{ men} :: 3 \text{ months} : x \text{ months}.$$
$$\tfrac{120 \times 3}{200} = 1\tfrac{4}{5}.$$
$$\therefore x = 1\tfrac{4}{5} \text{ mo., or } 1 \text{ mo. } 24 \text{ da.}$$

NOTE.—In problems like the preceding, the pupil must use judgment in stating his proportion. He knows that the food will not last 200 men as long as it will 120 men. He must state his proportion accordingly.

PROBLEM:

A train goes 92 miles in 3 hours. At the same rate, in what length of time will it go 122 miles?

Statement and Work:

$$92 \text{ miles} : 122 \text{ miles} :: 3 \text{ hr.} : x \text{ hr.}$$

$$\frac{122 \times 3}{92} = 3\tfrac{45}{46}$$

$$\therefore x = 3\tfrac{45}{46} \text{ hr., or } 3 \text{ hr. } 58\tfrac{16}{23} \text{ min.}$$

PROBLEM:

10 men in 26 days earn $500. What will 5 men earn in 4 days at the same rate?

Statement and Work:

$$\left.\begin{array}{r} 10 \text{ men} \\ 26 \text{ days} \end{array}\right\} : \left\{\begin{array}{l} 5 \text{ men} \\ 4 \text{ days} \end{array}\right\} :: \$500 : \$x.$$

$$\frac{5 \times 4 \times 500}{10 \times 26} = 38\tfrac{6}{13}.$$

$$\therefore x = \$38\tfrac{6}{13}.$$

PROBLEM:

A farmer has a bin 8 ft. wide, 10 ft. long, and 6 ft. deep, that holds 384 bu. of wheat. How deep must he make another that is 15 ft. long and 10 ft. wide, so that it will hold 720 bushels?

Statement and Work:

$$\left.\begin{array}{r} 8 : 10 \\ 10 : 15 \\ 720 : 384 \end{array}\right\} :: 6 \text{ ft.} : x \text{ ft.}$$

$$\frac{10 \times 15 \times 384 \times 6}{8 \times 10 \times 720} = 6.$$

$$\therefore x = 6 \text{ ft.}$$

Note:—Use cancellation in all work in proportion. It is one of the places where cancellation can be used to great advantage.

PARTITIVE PROPORTION.

TALK:

Many problems in partitive proportion are given in the arithmetics. As a rule, the problems are not of a practical or useful nature. Since many pupils will be called upon to work with them, nevertheless, a few are given in this book.

Problems.

PROBLEM:

B's money is equal to 4 times what A has. They both have $2000. How much has each?

SOLUTION:

$$\begin{array}{r l}
\text{A's money} & = 1 \text{ A's money.} \\
\text{B's money} & = 4 \text{ A's money.} \\
\hline
\text{A's and B's money} & = 5 \text{ A's money.} \\
5 \text{ A's money} & = \$2000. \\
\text{A's money} & = \$400.
\end{array}$$

4 A's money, or B's money = 4 × $400, or $1600.

PROBLEM:

Divide $840 between two men in such a way that they take it in the ratio 5 to 7.

SOLUTION:

Divide it into 5 + 7, or 12 equal parts.

1 part = $\frac{1}{12}$ of $840, or $70.

5 parts = 5 × $70 = $350.

7 parts = 7 × $70 = $490.

∴ one receives $350, the other $490.

This problem may be made very simple by using x to equal the value of one of the parts. Then,

$$\begin{aligned}
5x + 7x &= \$840 \quad (1) \\
12x &= \$840 \quad (2) \text{ (Divide by 12.)} \\
x &= \$\ 70 \quad (3) \\
5x &= \$350 \quad (4) \text{ (Multiply (3) by 5.)} \\
7x &= \$490 \quad (5) \text{ (Multiply (3) by 7.)}
\end{aligned}$$

Such problems may also be solved by proportion, as, in the preceding problem,

$$12 : 5 : : \$840 : \$x.$$

$$\frac{5 \times \overset{70}{840}}{12} = 350.$$

$$x = \$350.$$

$$12 : 7 : : \$840 : \$x.$$

$$\frac{7 \times \overset{70}{840}}{12} = 490.$$

$$x = \$490.$$

PROBLEM:

A man willed $3900 to his three sons so that the youngest had $100 less than the second son, and the second had $100 less than the oldest. What was the share of each one?

SOLUTION:

To make this simple, we will let the share of the youngest equal x. Then,

Share of youngest $= x.$
Share of second $= x + \$100.$
Share of oldest $= x + \$100 + \$100.$
(Adding) What all get $= 3x + \$300.$
Then, $3x + \$300 = \$3900.$
$3x = \$3600.$
$x = \$1200.$

Substituting the value of x,
the youngest gets $1200,
the second gets $1200 + $100, or $1300,
the oldest gets $1200 + $100 + $100 = $1400.

PROBLEM:

Divide $260 into parts proportional to $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$.

EXPLANATION:

Since the fractions have no common name, reduce to a common denominator. Then we have $\frac{4}{12}$, $\frac{6}{12}$, and $\frac{3}{12}$, or 13 parts in all.

1 part = $\frac{1}{13}$ of \$260, or \$20.
4 parts = 4 × \$20, or \$80.
6 parts = 6 × \$20, or \$120.
3 parts = 3 × \$20, or \$60.

PARTNERSHIP.

When two or more persons join their capital and services for business purposes, the gains or losses to be divided as agreed, they are said to be in **partnership.**

A **special** or **silent** partner is one who is not active in the business, and whose liability is limited by the amount he has invested.

General partners are those who conduct the business.

A **resource** or **asset** is anything belonging to the firm and having a commercial value.

A **liability** is a debt owed by the firm.

The **net worth** of a firm is the excess of its assets over its liabilities.

When a firm is unable to pay its debts in full, it is said to be **insolvent** or **bankrupt.**

There are two kinds of partnership, simple and compound.

A partnership is said to be **simple** when each partner invests his money for the same time.

A partnership is said to be **compound** when partners invest their money for different periods of time.

These terms are used merely to designate the difference as used in arithmetic. In the business world, the two are merely spoken of as **partnerships.**

When partnership profits or losses are to be settled, the time the capital has been invested is an element to be considered, if the partnership is compound.

PROBLEMS.

PROBLEM:

Three men engage in the cattle trade for a year. A puts in $2600, B puts in $4000, and C puts in $3500. At the end of a year their profits are $5050. What part of it should each take?

SOLUTION No. 1:

It should be divided in proportion to the sums invested, or 2600, 4000, and 3500. To simplify, divide each by 100 and the parts are proportional to 26, 40, and 35.

The number of parts = 26 + 40 + 35, or 101.

Then A receives,

$$101 : 26 :: \$5050 : \$x.$$
$$\frac{26 \times 5050}{101} = 1300.$$
$$x = \$1300.$$

B receives,

$$101 : 40 :: \$5050 : \$x.$$
$$\frac{40 \times 5050}{101} = 2000.$$
$$x = \$2000.$$

C receives,

$$101 : 35 :: \$5050 : \$x.$$
$$\frac{35 \times 5050}{101} = 1750.$$
$$x = \$1750.$$

SOLUTION No. 2:

The fractional method is simpler when the proportional parts are found, as,

A receives $\frac{26}{101}$ of $5050, or $1300.
B receives $\frac{40}{101}$ of $5050, or $2000.
C receives $\frac{35}{101}$ of $5050, or $1750.

PROBLEM:

A bankrupt owes four creditors as follows: A, \$150; B, \$200; C, \$100; and D, \$150. His property is worth \$486. How much of it should each creditor receive?

SOLUTION:

Dividing the sums due by 50, they are proportional to 3, 4, 2, and 3.

\$486 must be divided into $3 + 4 + 2 + 3$, or 12 equal parts.

Then, A receives $\frac{3}{12}$ of \$486, or \$121.50.
B receives $\frac{4}{12}$ of \$486, or \$162.00.
C receives $\frac{2}{12}$ of \$486, or \$81.
D receives same as A, or \$121.50.

The last part of the solution may be simplified in this way:—

1 of the equal parts = $\frac{1}{12}$ of \$486, or \$40.50.
A receives 3 × \$40.50, or \$121.50.
B receives 4 × \$40.50, or \$162.
C receives 2 × \$40.50, or \$81.
D receives \$121.50.

PROBLEM:

A and B pay \$36 rent for a pasture. A grazes on it 4 horses, and B grazes 10 sheep. They estimate that 1 horse eats as much as 2 sheep. What part of the rent should each pay?

SOLUTION:

1 horse eats as much as 2 sheep.
4 horses eat as much as 8 sheep.

Hence, the rent should be proportioned as 8 to 10, or 18 equal parts.

A should pay $\frac{8}{18}$ of \$36, or \$16.
B should pay $\frac{10}{18}$ of \$36, or \$20.

PROBLEM:

A and B form a partnership. A invests $2500 and B $3000. Ten months later A adds $500 to his investment. At the end of a year the net profits are $2010. What should each receive?

SOLUTION:

Use of $2500 for 1 yr. = use of 12 × $2500, or $30000 for 1 mo. Use of $500 for 2 mo. = use of 2 × $500, or $1000 for 1 mo. A's investment = $30000 + $1000, or $31000 for 1 mo. B's investment = the use of 12 × $3000, or $36000 for 1 mo. Dividing the investment of each for 1 mo. by 1000, we find the parts proportioned to 31 and 36.

A receives $\frac{31}{67}$ of $2010, or $930.
B receives $\frac{36}{67}$ of $2010, or $1080.

AVERAGES.

The **average** of a set of numbers is a number which can be put in place of each of them without changing the sum.

The sum of 3, 6, and 12 is 21. If we put 7 in place of each, the sum is 21. Hence, 7 is said to be the average of the three numbers 3, 6, and 12.

To find the average, the numbers or quantities must be of the same kind.

To find the average,

1. **Find the sum of the quantities.**
2. **Divide the sum by the number of quantities added.**

PROBLEMS.

PROBLEM:

Find the average cost of soap at 5¢, 6¢, and 10¢ a bar.

Work and Explanation:

5¢ + 6¢ + 10¢ = 21¢.
21¢ ÷ 3 = 7¢.

The number of values considered is 3. The average value is 7¢.

PROBLEM:

In one week a merchant received cash as follows: Monday, $214.25; Tuesday, $156.93; Wednesday, $236.50; Thursday, $187.90; Friday, $168.10; and Saturday, $326. What were his average daily receipts?

Explanation:

For the week he received the sum of the daily receipts, or $1289.68. The average receipts were $\frac{1}{6}$ of $1289.68, or $214.95.

PROBLEM:

I mix 20 lb. of coffee @ 20¢, 15 lb. @ 30¢, and 20 lb. @ 40¢. What do I gain by selling the mixture @ 40¢?

Explanation:

20 lb. @ 20¢ = $4.00.
15 lb. @ 30¢ = 4.50.
20 lb. @ 40¢ = 8.00.
55 lb. cost $16.50.
55 lb. @ 40¢ = $22.00.
I gain $22.00 − $16.50, or $5.50.

PROBLEM:

What is the average cost of the mixture in the preceding problem?

Solution:

Since 55 lb. of the mixture cost $16.50, one pound of the mixture costs $\frac{1}{55}$ of $16.50, or 30¢.

Note.—The preceding problems include about all the features of average that the pupil will ordinarily meet. They will enable him to do intelligent work on any problem that is apt to be presented to him.

AVERAGE OF PAYMENTS.

TALK:

While work in average of payments does not have a practical value for many pupils, any of them may have occasion to use it in later life. There is nothing difficult about it, and a few lessons in it may be helpful.

Introduce the subject in this way:—

1. The use of $3 for 2 mo. is equal to the use of $1 for how many months?

The use of $3 for 2 mo. equals the use of $1 for 3 times 2 mo., or 6 mo.

2. The use of $4 for 1 mo. will balance the use of $2 for how many months?

The use of $4 for 1 mo. will balance the use of $2 for 2 mo., because $2 is ½ of $4. Hence, it must be used twice as long as $4.

3. $100 for 2 years will balance $50 for how long?

$50 is ½ of $100; hence, it must be used twice as long as the $100, or 4 yr.

The **average of payments** is the way of finding the time at which two or more sums of money, due on different dates, may be paid so that neither debtor nor creditor lose by the transaction.

This is sometimes called **equation of payments.**

The **average time,** or **equated date,** is the day on which the whole sum owed may be paid without loss to either party.

The **term of credit** is the time allowed on a debt item before it is due.

PROBLEMS.

PROBLEM:

A merchant owes a wholesale house \$300, due in 3 mo.; \$150 due in 4 mo.; and \$200 due in 6 mo. What is his average term of credit?

EXPLANATION:

\$300 for 3 mo. = 3 × \$300, or \$ 900 for 1 mo.
\$150 for 4 mo. = 4 × \$150, or \$ 600 for 1 mo.
\$200 for 6 mo. = 6 × \$200, or \$1200 for 1 mo.
\$650 for the periods above = \$2700 for 1 mo.

The time of cr. on \$2700 will be as many mo. as 2700 ÷ 650, or $4\frac{2}{13}$. $4\frac{2}{13}$ mo. = 4 mo. $4\frac{8}{13}$ da.

NOTE.— Where the fraction of a day is more than $\frac{1}{2}$, call it 1 day.

PROBLEM:

W. A. Clark bought of Browning, King & Co. these bills of goods: Jan. 10, \$400 due in 60 da.; Feb. 10, \$600 due in 90 da., and March 1, \$300 due in 30 da. What is the average time of payment?

EXPLANATION:

1st bill is due Jan. 10 + 60 da. = Mar. 11.
2d bill is due Feb. 10 + 90 da. = May 12.
3d bill is due Mar. 1 + 30 da. = Apr. 1.

Assume that all are settled on May 12.

Then,

The first bill is over due May 12 for 62 days.
The second bill is over due May 12 for 0 days.
The third bill is over due May 12 for 42 days.
\$400 for 62 da. = 400 × 62 da. for \$1, or 24800 da.
\$600 for 0 da. = 600 × 0 da. for \$1, or 0 da.
\$300 for 42 da. = 300 × 42 da. for \$1, or 12600 da.
Average term of cr. on \$1300 = \$1 for 12600 da.

Then I owe the interest on $1 for 12600 da.

The int. on $1300 is due for $\frac{1}{1300}$ of 12600 da., or 10 da.

.·. the average date is 10 da. before May 12, or May 2.

PROBLEM:

Wilhelmina Case bought a piano for $700 on 6 months credit. She paid $300 cash. In what time should the balance be paid?

SOLUTION:

$700 for 6 mo. = 6 × $700, or $4200 for 1 mo.

The balance, or $400, should be paid in as many months as credit on $400 will equal $4200 for 1 mo.

This is $\frac{1}{400}$ of 4200, or 10½.

.·. the balance should be paid in 10½ mo.

REMARK:

The preceding problems include all the phases of the subject ordinarily needed. The pupil who has gone over these carefully should be able to do intelligent work in averaging any set of payments.

PART NINE.

POWERS AND ROOTS.

> "*The world generally gives its admiration not to the man who does what nobody else ever attempts to do, but to the man who does best what multitudes do well.*"
>
> —*Macaulay.*

MOTHER AND TEACHER.

When mothers understand what true education is, and realize what a power for good every good teacher is in the life of a child, and set themselves to secure the one and to preserve the other, public opinion and school officials will be with them. Then we shall have schools that are homes where children live, each free to unfold himself, through his activities, and each teacher has leisure to study every pupil and to choose the methods which appeal to his temperament, and fit his intellectual gifts.

But the first step in bringing the home and the school together must consist in the closer relations of mothers and teachers. It is strange, that, though allies in the most important work of the world, they only occasionally understand each other. . . .

The various associations of mothers and teachers should be of value in uniting them in meetings, under any name, which bring them close to one another in the genuine exchange of views and interchange of views about children.

The Success Library, Vol. II.

SQUARES AND SQUARE ROOTS.

The **square** of a number is the product found from multiplying a number by itself.

Thus, 9 is the square of 3, because 3 multiplied by itself gives the product 9.

The **exponent 2** (^{2}) is used to show that the square of the number preceding it is to be found.

Thus, 8^2 means that the square of 8 is to be found.

$8^2 = 64$ reads, "Eight squared equals sixty-four,"
or, "The square of eight equals sixty-four,"
or, "Eight to the second power equals sixty-four,"
or, "The second power of eight is sixty-four."

$9^2 = 81$. $10^2 = 100$. $12^2 = 144$. $100^2 = 10000$.

Let the pupil find all the squares from 1 to 100, and learn the squares of numbers up to 20.

The **square root** of a number is one of the two equal factors of that number.

Thus, 5 is the square root of 25, because it is one of the two equal factors (5 and 5) of that number.

The **radical sign** ($\sqrt{}$) is used to show that the square root of the number after it is to be found.

Thus, $\sqrt{49}$ shows that the square root of 49 is to be found.

$\sqrt{49} = 7$ is read, "The square root of 49 is 7."

$\sqrt{100} = 10$ $\sqrt{144} = 12$ $\sqrt{225} = 15$.

Let the pupil give the square roots of 4; 9; 16; 49; 64; 81; 100.

Notice:

$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$
$10^2 = 100$ $11^2 = 121$ $12^2 = 144$ $18^2 = 324$
$100^2 = 10000$ $125^2 = 15625$ $150^2 = 22500$ $175^2 = 30625$
$1000^2 = 1000000$ $10000^2 = 100000000$

The pupil will notice from the above that,

The square of any number contains twice as many figures as its root, or twice as many less one.

That is the reason we point off the number, of which we are to get the square root, into periods of two figures each. For each period of two figures each, there will be but one figure in the root.

Thus, 875639 is pointed off 87·56·39. We begin with the units' place to point off and should there be a decimal involved, begin from the decimal point.

Thus, 93634.0728 would be pointed off 9·36·34.07·28.

PERFECT AND IMPERFECT SQUARES.

A **perfect square** is a number that can be resolved into two equal factors.

Thus, 9 is a perfect square, because it can be resolved into the two equal factors 3 and 3. 7, 19, 23, 2323, cannot be resolved into two equal factors.

It seems strange that we cannot find two numbers which, if multiplied together, will produce those numbers, yet such is the case. They are called **imperfect squares** because their square roots can be found only *approximately*.

Below are some perfect squares with their factors. Notice the number of like factors in each.

$$36 = 2 \times 2 \times 3 \times 3.$$

$$81 = 3 \times 3 \times 3 \times 3.$$

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3.$$

Below are some numbers, with their factors, that are not perfect squares. Notice the number of like factors in each.

$$72 = 2 \times 2 \times 2 \times 3 \times 3.$$
$$162 = 3 \times 3 \times 3 \times 3 \times 2.$$

The pupil will notice that in every case *the perfect square has an even number of like factors*, while every number that is not a perfect square has not. Now he will readily see that the factors of every perfect square may be grouped into two groups of like factors, and the product of each group is the same. Also, that *the product of each group is the square root of the number.*

TO ILLUSTRATE:

Find the square root of 3025.

$3025 = 5 \times 5 \times 11 \times 11.$

The two groups (5×11) (5×11).

The square root $= 5 \times 11$, or 55.

Remember:

To find the square root of a perfect square,

1st. Resolve the number into its prime factors.

2d. Group half of these factors.

3d. The product is the square root.

For the Pupil:

Find the square roots of the following perfect squares: —

8100 1225 625 225 196.

Finding the Square Root of Any Number.

The square root of 144 is 12. Let us see how we found it.

$12 = 1$ ten $+ 2$ units.

12^2 is the same as $(10 + 2)^2$.

Let us square (10 + 2), that is, multiply 10 + 2 by 10 + 2.

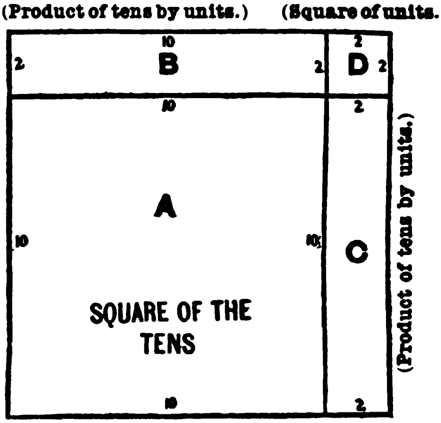
$$\begin{array}{l} 10 + 2 \\ \underline{10 + 2 \qquad\quad} \\ 10^2 + (10 \times 2) \\ \underline{\qquad + (10 \times 2) + 2^2} \\ 10^2 + 2\,(10 \times 2) + 2^2 \end{array}$$

Then, $12^2 = 10^2 + 2\,(10 \times 2) + 2^2$

From this we learn that,

The square of any number made up of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

This can be shown by the following diagram:—



A (the square of tens) = 100
B + C (twice the product of tens by units) = 40
D (square of units) = 4

A + B + C + D (square of tens and units) = 144

That is, the square = tens² + 2 (tens × units) + units².

EXAMPLE.

Find the square root of 1024.

Full Explanation:

Pointing off, we have 10·24.

Since there are two periods in the square, we will have two figures in the root.

The greatest square of even tens in 1024 will make a square of 900 units, and its square root is 30, or 3 tens.

$$
\begin{array}{r|l}
 & 10{\cdot}24\;)\,\underline{30+2=32} \\
\text{tens}^2 \text{ or } 30^2 = & \underline{900} \\
2\times\text{tens}=2\times30=60\ (\text{Trial divisor}) & 124 \\
2\times\text{tens}+\text{units}=2\times30+2=62 & \underline{124}
\end{array}
$$

We see that the square root of 1024 will be somewhere between 30 and 40. Now, to find the units' figure of the root.

The tens' square disposes of 900 units.

There are 1024 − 900, or 124 units left.

$$2 \times \text{tens, or } 2 \times 30 = 60.$$

Use this product as the *trial divisor.*

60 is contained in 124 about 2 times. Place the 2 in the root.

$$2 \times \text{tens} + \text{units} = 2 \times 30 + 2 = 62.$$

Multiply out. Subtract.

Square root = 32.

SHORTER EXPLANATION.

$\sqrt{45369} = ?$

SOLUTION:

$$
\begin{array}{r|l}
 & 4{\cdot}53{\cdot}69\;)\,\underline{213} \\
 & \underline{4} \\
41 & 53 \\
 & \underline{41} \\
423 & 1269 \\
 & \underline{1269}
\end{array}
$$

(1.) Point off the number into periods of two figures each, as before.

(2.) The square root of the first period is 2. 2 × 2 = 4. Write the 2 in the root and subtract the 4 from 4. Bring down the next period, 53.

(3.) 2 × 2 = 4. (Remember the 4 is to be used as a trial divisor, being 2 × the *tens.*)

4 is contained in 5 about 1 time. Place 1 in the root, also on the right of the 4 in the divisor. Multiply 41 by 1. Subtract and bring down the next period.

(4.) 2 × 21 = 42. 42 is the *trial divisor.* 126 ÷ 42 = *about* 3 times. Place the 3 in the root also at the right of the 42 in the divisor. Multiply out.

Square root = 213.

A DECIMAL INVOLVED.

EXAMPLE:

Find the square root of 593.8969.

```
        5·93.89·69 ) 24.37
        4
  44  | 193
      | 176
  483 | 1789
      | 1449
  4867 | 34069
       | 34069
```

EXPLANATION:

The process is exactly the same where a decimal is involved, except in pointing off periods. Begin at the decimal point and point off to the right and left.

EXAMPLE:

$\sqrt{42849} = ?$

```
     4·28·49 ) 207
     4
 407 | 2849
     | 2849
```

EXPLANATION:

Where the period brought down will not contain the divisor even 1 time, place a zero in the root, one in the divisor, and bring down another period. Proceed then as ever.

XAMPLE:

Find the square root of .02.

ORK:

```
  .02·00·00(.141+
   1
24 | 100
   |  96
  281 | 400
      | 281
```

EXPLANATION:

We can annex as many zeros to the right of a decimal as we wish, without changing its value. In this example enough zeros are annexed o make 3 periods.

In the root there should be one decimal place for ach period in the decimal square.

XAMPLE:

$$\sqrt{\frac{676}{729}} = ?$$

ORK:

```
   6·76)26        7·29)27
   4              4
46 | 276       47 | 329
   | 276          | 329
```

ence, $\sqrt{\frac{676}{729}} = \frac{26}{27}$

EXPLANATION:

To find the square root of a fraction, find the roots of the numerator and denominator separately. If the denominator is not a perfect square, multiply both terms of the fraction by a number that will make it a perfect square. Another way is to change he fraction to a decimal and find the square root of he decimal.

USES OF SQUARE ROOT.

Some uses of square root may be seen from the ollowing problems: —

1. If a square piece of land is $\frac{1}{16}$ of a square mile, ow much fence will be required to fence it?

2. 1728 soldiers are placed in the form of a square. ow many soldiers on each side?

3. What are the two equal factors of 3025?

4. Find the area of a triangular piece of ground 6 rd. by 12 rd. by 13 rd. (The work for such problems is found under mensuration.)

CUBES AND CUBE ROOTS.

The cube of a number is the product of that number used three times as a factor. The cube of a number is also called the third power of the number.

Thus, the cube of 3 is 27, because 27 is the product of 3 × 3 × 3.

The cube of a number is indicated by writing the exponent 3 above and to the right of the number, as 4^3 means the cube of 4, or 4 to the third power.

The **cube root** of a number is one of the three equal factors of that number.

Thus, 5 is the cube root of 125, because $5 \times 5 \times 5 = 125$.

The **radical sign** with a figure 3 over it ($\sqrt[3]{\quad}$) means that the cube root of the number following it is to be taken.

$\sqrt[3]{125}$ reads, "The cube root of 125."

Notice:

$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$
$10^3 = 1000$ $11^3 = 1331$ $12^3 = 1728$ $18^3 = 5832$
$100^3 = 1000000$ $150^3 = 3375000$

In finding the cube root of numbers, we point off the numbers into periods of three figures each, and for each period in the cube there will be one figure in the cube root.

Thus, 289383 will be pointed off into two periods—289·383—and we readily see there will be only 2 figures in the root.

The cube root of 1728 is 12. *Let us see how we found it.*

$$\begin{array}{l}
12 = 1 \text{ ten} + 2 \text{ units} \\
12^3 = (10+2)^3 \\
(10+2)^3 \text{ means } 10+2 \times 10+2 \times 10+2
\end{array}$$

$$\begin{array}{l}
10+2 \\
\underline{10+2} \\
10^2 + (10\times 2) \\
\underline{\qquad + (10\times 2) + 2^2} \\
10^2 + 2\,(10\times 2) + 2^2 \\
\underline{10+2} \\
10^3 + 2\,(10^2\times 2) + (10\times 2^2) \\
\underline{\qquad\qquad (10^2\times 2) + 2\,(10\times 2^2) + 2^3} \\
10^3 + 3\,(10^2\times 2) + 3\,(10\times 2^2) + 2^3
\end{array}$$

That is, *the cube of any number made up of tens and units equals*—

The cube of the tens + three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units,

or

$$\text{tens}^3 + 3\,(\text{tens}^2 \times \text{units}) + 3\,(\text{tens} \times \text{units}^2) + \text{units}^3.$$

EXAMPLE:

Find the cube root of 19683.

WORK:

$$\begin{array}{rr|l}
 & & \multicolumn{1}{l}{19{\cdot}683\ \underline{)\ 20+7=27}} \\
\text{tens}^3 \text{ or } 20^3 = & & \multicolumn{1}{l}{8000} \\ \cline{3-3}
 & & 11683 \\
\text{Trial divisor } 3\times\text{tens}^2 = 3\times 20^2 = & 1200 & \\
3\times(\text{tens}\times\text{units}) = 3\times(20\times 7) = & 420 & \\
\text{units}^2 \text{ or } 7^2 = & 49 & 11683 \\ \hline
 & 1669 &
\end{array}$$

Explanation:

(1.) Separate the number into periods. We find we have two periods, hence we will have 2 figures in the root.

(2.) 8000 is the largest cube contained in 19683. The cube root of 8000 is 20 (2 tens). Place the 2 in the root. 19683 − 8000 = 11683.

(3.) 8000 is the tens^3. 11683 = 3 (tens^2 × units) + 3 (tens × units^2) + units^3. The units' figure (to be found) is one factor of 11683. Taking the units out, we have left 3 × ten^2 + 3 (tens × units) + units^2, which is the other factor.

(4.) 3 × ten^2 (1200) is the largest part of the largest factor of 11683, so we will use it as a *trial divisor.* 11683 ÷ 1200 = about 7 times.

(5.) Now, we can complete the divisor.

1200 + 420 + 49 = 1669, the complete divisor. Multiplying 1669 by the units' factor, we have 1669 × 7, or 11683.

The cube root = 20 + 7 = 27.

In actual practice, we omit the zeros in the cube of the tens in the first part, using only the number without the ciphers.

EXAMPLE:

Find the cube root of 1.225043.

Work:

$$\begin{array}{r|l} & 1.225\cdot043\;\underline{)\,1.07\quad} \\ & 1 \\ \hline \text{trial divisor } 3 \times 10^2 = 300 & 225 \\ \hline \textit{trial divisor } 3 \times 100^2 = 30000 & 225043 \\ 3 \times (100 \times 7) \;=\; 2100 & \\ 7^2 \;=\; 49 & \\ \hline 32149 & \underline{225043} \end{array}$$

Explanation:

Begin at the decimal point and point off to thi right and left. In the decimal there are two periods, so there will be two figures in the root decimal. Proceed as before. You will find the trial divisor 300 is not contained in 225, so bring down another period and annex two zeros at the right of the trial divisor.

EXAMPLE:

Find the cube root of .0163956, carrying the root to 3 decimal places.

Work:

```
         .016·395·600).254 +
            8
      ------------
   1200 | 8395
    300 |
     25 |
   ---- |
   1525 | 7625
        |---------
        |  770600
 187500 |
   3000 |
     16 |
 ------ |
 190516 |  762064
        |---------
        |    8536
```

Explanation:

After the process is understood, this short method of writing the work may be used by the pupil.

EXAMPLE:

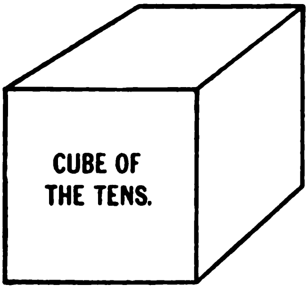
Find the cube root of $\frac{1728}{19683}$.

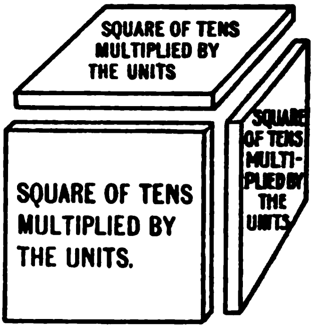
Work

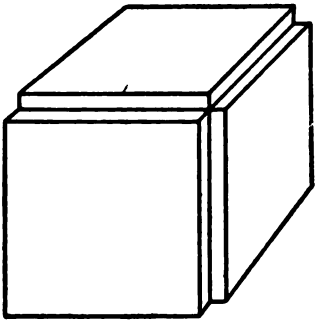
$$\sqrt[3]{\frac{1728}{19683}}=\frac{\sqrt[3]{1728}}{\sqrt[3]{19683}}=\frac{12}{27}.$$

For the Pupil:

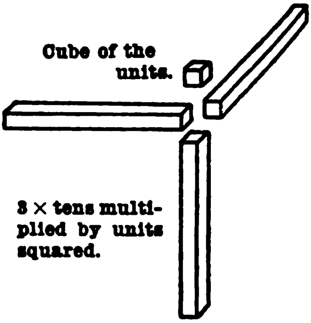
1. What is meant by a power of a number? by a root?
2. Give the second and third powers of numbers to 12 by writing results only.
3. State how you find the cube root of a fraction.

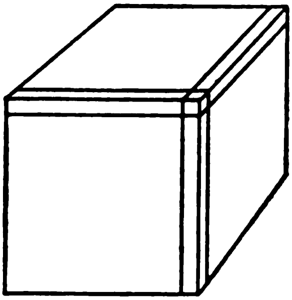






Cube of the tens + 3 × the square of the tens multiplied by the units.





Cube of the tens + 3 × tens squared multiplied by the units + units squared multiplied by 3 × tens + cube of the units.

CUBES GEOMETRICALLY.

For graphic illustration the geometrical representation of the cube of units and tens in the drawings is helpful.

TO ILLUSTRATE:

Find the cube of 12.

EXPLANATION:

The cube of $(10 + 2) = 10^3 + 3\,(10^2 \times 2) + 3\,(10 \times 2^2) + 2^3$.

The cube at the top and to the left may represent one whose edge is 10 in.

The illustration to the right of the first may represent the three square solids each 10 in. in length and width, but only 2 in. thick.

The next illustration shows the preceding two combined.

The fourth illustration shows, first, the three rectangular solids each 10 in. long, but only 2 in. wide, and 2 in. thick; second, the small solid which represents the cube of 2 and is only 2 in. on an edge.

The last illustration shows the completed large cube which includes all the others.

$$\begin{array}{rr} \text{Then,}\quad 10^3 = & 1000 \\ 3 \times (10^2 \times \ 2) = & 600 \\ 3 \times (\ 2^2 \times 10) = & 120 \\ 2^3 = & 8 \\ \hline & 1728 \end{array}$$

CUBES OF HIGHER DEGREES.

If any root is to be found higher than the cube root, factor the root figure indicated and use the processes already given.

EXAMPLE:

$$\sqrt[8]{6561} = ?$$

The factors of the root figure 8 are 2, 2, 2.

Thus, we find the square root of 6561, then the square root of that result, and then the square root of the third result, which will be the 8th root of 6561.

$$\begin{array}{r|l} & 65{\cdot}61\ (\underline{81} \\ & \underline{64} \\ 161 & 161 \\ & \underline{161} \end{array}$$

$$\sqrt{6561} = 81$$

$$\sqrt{81} = 9$$

$$\sqrt{9} = 3$$

Hence, $\sqrt[8]{6561} = 3.$

EXAMPLE:

Find the 6th root of 64.

The sixth root is the same as the square root of the cube root. (Find either the square root first or the cube root first.)

$$\sqrt[3]{64} = 4$$

$$\sqrt{4} = 2$$

Hence, $\sqrt[6]{64} = 2$

EXAMPLE:

$$\sqrt[16]{43046721} = ?$$

The 16th root of a number is the same as the square root of the square root of the square root of the square root of that number.

WORK:

$$\sqrt{43046721} = 6561$$

$$\sqrt{6561} = 81$$

$$\sqrt{81} = 9$$

$$\sqrt{9} = 3$$

$$\therefore \sqrt[16]{43046721} = 3.$$

PART TEN.

THE METRIC SYSTEM.

"*Inquire about everything that you do not know; since, for the small trouble of asking, you will be guided in the road of knowledge.*"

—*From the Persian.*

MENTAL ARITHMETIC.

Let every teacher be thoroughly skilled in this subject. It is not enough that the teacher is able in some way to obtain the answers to the questions proposed. He should be able to give the reason for every step in the process he takes to obtain them, and to do it in a clear and concise manner. It is this which constitutes the discipline of this branch for the mind.

— David Page.

Speaking of the time spent in the study of a book on mental arithmetic, he says: "In three weeks I had mastered it; and I had gained in that time more knowledge of the principles of arithmetic than I had ever acquired in all my life before. I no longer 'saw through a glass darkly.'"

— David Page.

METRIC SYSTEM.

TALK:

In many schools little is done with the metric system. In others it is given a prominent place. The discussion as to the value of teaching it is indulged in by business and commercial men as well as educators, but thus far no satisfactory conclusion has been reached. It is each teacher's privilege to teach as much of it as she sees fit.

If it is taught, do not occupy the pupil's time with changing the metric measures to their equivalents in our system. Such work discourages the child with the difficulty of the subject and is of little practical or disciplinary value. Do not try to teach the subject unless you have at least a meter stick. If possible, have a set of metric weights and measures. If each pupil can have a stick on which the decimeters, centimeters, and millimeters are marked, it will help wonderfully.

The metric system is also known as the French system, because the French nation was the first to adopt it. In that country it is used exclusively. Many other nations have adopted it and are bringing it into use. The United States has also officially recognized it, but as yet it is not coming into general use here. Its principal use in this country is for scientific measurements.

The metric system is based upon the decimal scale. Consequently, all of its measures are based on the scale of ten.

The **meter** is the standard of measurement for the system. It is about 39.37 inches long. The standard meter is a bar of platinum which is carefully preserved in the archives at Paris. Exact copies of the meter and other measures have been procured by the governments that have officially adopted the system. Comparisons are made with the originals under certain conditions of temperature and atmospheric pressure.

The unit for dry and liquid measures is the **liter.** The unit of weight is the **gram.**

HIGHER UNITS.

Units of measure or weight that are larger than the base units are called by the Greek prefixes: **Deca** (10), **Hecto** (100), **Kilo** (1000), and **Myria** (10000). For instance, the Decameter is 10 meters; the Decaliter is 10 liters, etc. The Hectometer is 100 meters; the Hectogram is 100 grams, etc.

LOWER UNITS.

Units of measure or weight that are smaller than the base units are named by prefixing the Latin numerals: **deci** ($\frac{1}{10}$), **centi** ($\frac{1}{100}$), **milli** ($\frac{1}{1000}$). For instance, the decimeter is $\frac{1}{10}$ of a meter; the decigram is $\frac{1}{10}$ of a gram; etc. The centimeter is $\frac{1}{100}$ of a meter; the centiliter is $\frac{1}{100}$ of a liter, etc.

REMARK.—Notice that the measures higher than the base unit begin with capitals. The base unit and those below it begin with small letters. The same is true in writing the abbreviations. Remember this and it will prevent confusion.

Many authors of Arithmetics use small letters to begin all the names. This, it seems, often leads to trouble on the part of pupils who are not familiar with the subject. It is to be regretted that there is not greater uniformity.

Another point of difference is in the use of c or k in the measures Decameter and Hectometer. The use of c is preferred and the words should be taught in that way regardless of the book in use.

MEASURES OF LENGTH.

TABLE.

1 **millimeter** (mm) = .001 meter
1 **centimeter** (cm) = .01 meter
1 decimeter (dm) = .1 meter
1 **meter** (m) *standard unit*
1 Decameter (Dm) = 10 meters
1 Hectometer (Hm) = 100 meters
1 **Kilometer** (Km) = 1000 meters
1 Myriameter (Mm) = 10,000 meters

or

10 **millimeters** = 1 **centimeter** (cm)
10 **centimeters** = 1 decimeter (dm)
10 decimeters = 1 meter (m)
10 meters = 1 Decameter (Dm)
10 Decameters = 1 Hectometer (Hm)
10 Hectometers = 1 **Kilometer** (Km)
10 **Kilometers** = 1 Myriameter (Mm)

The measures in *heavy face* type are those commonly used.

REDUCTION.

Since this is a decimal system, units of any denomination may be reduced to the next higher denomination by removing the decimal point one place to the left, as, 425 meters = 42.5 Decameters = 4.25 Hectometers, etc.

Units of any denomination may be reduced to the next lower denomination by removing the decimal point one place to the right, as, 4.25 Hectometers = 42.5 Decameters = 425 meters = 4250 decimeters, etc.

In general:

1. To reduce to a higher denomination,

Remove the decimal point as many places to the left as there are denominations between, inclusive of the denomination to be changed.

2. To reduce to a lower denomination,

Remove the decimal point as many places to the right as there are denominations between, inclusive of the denomination to be changed.

EXAMPLES.

1. How many meters in 4260 mm?

EXPLANATION:

According to the preceding, we count mm, cm, and dm, or three places. Pointing off three places to the left, we have 4.260 m, or 4.26 m.

2. How many cm in 1 Km?

EXPLANATION:

From the preceding rule, we count Km, Hm, Dm, m, dm, or five places. Pointing off five places to the right, we have 100000 cm.

Before *adding* or *subtracting* write the quantities in the same unit.

To the Teacher:

Let the pupil make and solve many problems similar to the preceding two. If he becomes familiar with the reduction in linear measure, his trouble with the whole metric system is removed, since all of the reductions are made in a similar way.

PROBLEMS.

1. Add: 446 mm, 293 cm, 7 m and 40 Km.

WORK AND EXPLANATION:

Writing them in the same unit, we have:—

$$\begin{array}{r} .446\ \text{m} \\ 2.93\phantom{6\ \text{m}} \\ 7.\phantom{376\ \text{m}} \\ 40000.\phantom{376\ \text{m}} \\ \hline 40010.376\ \text{m} \end{array}$$

NOTE.—For convenience, we write them in the base unit. The result may now be reduced to any unit by merely changing the place of the decimal point.

2. I walked 743.4 m before breakfast and 1.23 Km in the afternoon. How much longer was my afternoon walk?

Work and Explanation:

Writing them in the same denomination, we have:

$$\begin{array}{r} 1230.\ \text{m} \\ -\ 743.4\phantom{\ \text{m}} \\ \hline 486.6\ \text{m} \end{array}$$

3. What is the cost of 396 cm of cloth at $1.80 a meter?

Explanation:

396 cm equal 3.96 m, then 3.96 m cost 3.96 × $1.80 = $7.13.

4. The value of 12 m of silk is $49.20. What is the value of 4 dm of it?

1st Way:

12 m = 120 dm.
4 dm is $\frac{4}{120}$, or $\frac{1}{30}$ of 120 dm.
120 dm is worth $49.20.
4 dm is worth $\frac{1}{30}$ of $49.20, or $1.64.

2d Way:

1 m is worth $\frac{1}{12}$ of $49.20, or $4.10.
4 dm is .4 m.
.4 m is worth .4 × $4.10, or $1.64.

5. At $14 per front meter, what is the value of a lot having 40 meters frontage?

Solution:

The lot is worth 40 × $14, or $560.

SURFACE MEASURES.

The principal unit for surface measurement is the square meter.

A square meter may be said to be made up of 100 smaller squares each 1 dm on a side, just as a square foot may be said to be made up of 144 smaller squares, each 1 inch on a side.

Table.

1 square millimeter (qmm) = 0.000001 of a square meter.
1 **square centimeter** (qcm) = 0.0001 of a square meter.
1 square decimeter (qdm) = 0.01 of a square meter.
1 **square meter** (qm), Base unit.
1 square Decameter (qDm) = 100 square meters.
1 square Hectometer (qHm) = 10000 square meters.
1 **square Kilometer** (qKm) = 1000000 square meters.

In the measurement of land, the following units are used: —

1 **square meter** is called a **centar** (ca).
1 square Decameter is called an **ar** (a).
1 square Hectometer is called a hectar (ha).

Observe in the general table that each unit is 100 times as large as the next smaller unit. In linear measure, it was only 10 times as large.

Then, to change from one unit to the next, the decimal point must be moved two places.

PROBLEMS.

1. I have a field 100 meters wide and 140 meters long. How many square meters in it? How many ars in it?

1st Way:

The field contains 100 × 140 sq. meters, or 14000 sq. meters. 100 sq. meters make 1 ar.

Then, in 14000 sq. meters there are as many ars as 100 is contained times in 14000, or 140.

2d Way:

A field 100 meters wide and 1 meter long contains 100 sq. meters, or just 1 ar.

And a field 140 times as long contains 140 ars.

2. If land is worth $400 an ar, what is the value of a piece 120 meters wide and 500 meters long?

SOLUTION:

There are 120 × 500 sq m, or 60000 sq m in all.
60000 sq m = 600 ars.
600 ars are worth 600 × $400, or $240000.

---

3. I sold 120 ars of forest for $24000. What was the selling price per centar?

SOLUTION:

1 ar sold for $\frac{1}{120}$ of $24000, or $200.
1 centar sold for $\frac{1}{100}$ of $200, or $2.

MEASURES OF VOLUME.

The **cubic meter** is the principal unit of volume. It is a cube one meter long, one meter wide, and one meter high.

TABLE.

1 cubic millimeter (cmm) = 0.000000001 of a cubic meter.
1 cubic centimeter (ccm) = 0.000001 of a cubic meter.
1 cubic decimeter (cdm) = 0.001 of a cubic meter.
1 cubic meter (cbm), Base unit.

Observe that each unit is 1000 times as large as the next smaller unit. Hence,

A *cubic meter* contains 1000 cubic decimeters.
A cubic decimeter contains 1000 cubic *centimeters.*
A cubic centimeter contains 1000 cubic millimeters.

A cubic meter, when used in measuring wood, is called a **ster.**

10 decisters (dst) = *1 ster* (st). 10 sters = 1 Decaster (Dcst).

PROBLEMS.

1. How many cbm in 423 cdm?

SOLUTION:

Since there are 1000 cdm in 1 cbm, in 423 cdm, there are as many cbm as 1000 is contained times in 423, or .423.

∴ in 423 cdm there are .423 cbm.

---

2. John Collins has a bin of wheat 4 m long, 3 m wide, and 2 m deep. What is it worth at $14 per cbm?

SOLUTION:

The contents of the bin are 2 × 4 × 3 cbm, or 24 cbm.

At $14 per cbm, it is worth 24 × $14, or $336.

---

3. How many sters in a pile of wood that is 3.4 m long, 2.1 m wide, and 2.3 m high?

SOLUTION:

The pile contains 3.4 × 2.1 × 2.3 cbm, or 16.422 cbm. Since 1 cbm equals 1 ster, there are 16.422 sters in the pile.

---

4. What will be the cost of building a wall 4.3 dm thick, 2.3 m high, and 1 Km long at 50¢ a cbm?

SOLUTION:

Reducing the dimensions to meters, the wall contains .43 × 2.3 × 1000 cbm, or 989 cbm.

989 cbm @ 50¢ = $494.50, the cost of the wall.

MEASURES OF CAPACITY.

The **liter** is the principal unit of capacity. It is a cube whose dimensions are 1 decimeter; consequently, it is exactly the same in size as a cubic decimeter.

TABLE.

1 milliliter (ml)	= 0.001 of a liter.
1 **centiliter** (cl)	= 0.01 of a liter.
1 deciliter (dl)	= 0.1 of a liter.
1 **liter,**	Base unit.
1 Decaliter (Dkl)	= 10 liters.
1 **Hectoliter** (Hl)	= 100 liters.
1 Kiloliter (Kl)	= 1000 liters.

Observe that each unit is 10 times as large as the ext smaller unit.

PROBLEMS.

1. If a liter of molasses is worth 10¢, what is the alue of 8 Hectoliters?

OLUTION :

In 8 Hectoliters there are 8 × 100 liters, or 800 iters. 800 liters are worth 800 × 10¢, or $80.

---

2. A commission merchant has a bin 4 m long and .7 m wide. How deep should it be to hold 950 Hl f grain?

OLUTION :

Since a liter is of the same size as a cubic decimeter, or convenience, change the dimensions to decimeters. Then,

If the bin were 1 dm deep, it would hold 40 × 37 dm, or 1480 cdm = 1480 liters.

950 Hl = 95000 liters.

To hold 95000 liters, it must be as many dm deep s 1480 is contained times in 95000, or 64.19.

64.19 dm = 6.419 m.

∴ the bin must be 6.419 meters deep.

3. Wheat is worth 18¢ a Decaliter. What is the value of as much as the bin in the preceding problem will hold?

SOLUTION:

95000 liters = 9500 Decaliters.
9500 Decaliters @ 18¢ = $1710.

---

4. How many liters of water in a cylindrical measure whose base is 132 qcm, and whose depth is 42 cm?

SOLUTION:

The cubic contents of the vessel are 42 × 132 ccm, or 5544 ccm.

1 ccm = .001 cdm, or .001 liters.

5544 ccm = as many liters as 1000 is contained times in 5544, or 5.544.

∴ the vessel contains 5.544 liters.

MEASURES OF WEIGHT.

The gram is the principal unit of weight of this system. The gram is the weight of a cubic centimeter of pure water whose temperature is just a little above the freezing point. That is, its point of greatest density,—the only point when that weight of the water can be put into that space.

TABLE.

1 **milligram** (mg) = 0.001 of a gram.
1 centigram (cg) = 0.01 of a gram.
1 decigram (dg) = 0.1 of a gram.
1 **gram**, Base unit.
1 Decagram (Dkg) = 10 grams.
1 Hectogram (Hg) = 100 grams.
1 **Kilogram** (Kg) = 1000 grams.
1 **metric ton** (t) = 1000 kilograms.

Observe that each unit is 10 times as large as the ext smaller one, except the metric ton, which is 1000 imes as large as the Kilogram.

Remember :

1. **A cubic centimeter of water weighs a gram.**
2. **A liter of water weighs a Kilogram.**
3. **A cubic meter of water weighs a ton, or 1000 Kg.**

PROBLEMS.

1. A French farmer sold in Havre 4200 Kg of hay t $15 per ton. What did he get for it ?

OLUTION:

In 4200 Kg there are 4.2 tons.
4.2 t bring 4.2 × $15, or $63.

2. A grocer sells sugar in one day as follows: 8 Kg, 3.9 Kg, 4.14 Kg, 32 g, 14.2 g, and 29 g. How many grams did he sell in all ?

Work and Explanation :

Write each sale as grams.

78000 g.
3900
4140
32
14.2
29
86115.2 g.

Proceed as in simple addition.

3. I bought 4.29 Hg of distilled water in Paris and paid ½¢ per gram for it. What did it cost ne ?

Solution :

4.29 Hg = 429 g
429 g @ ½¢ = $2.15.

4. A chemist finds that the water of a lake con-

tains 2 g of potash to each Hg. How much potash in a ton of the water?

SOLUTION:

1 t = 1000 Kg = 10000 Hg.

10000 Hg contain 10000 × 2 g of potash, or 20000 g potash.

20000 g = 20 Kg.

HANDY METRIC TABLES.

The following tables give the equivalents of both the metric and common systems, and will be found convenient for reference whenever it is found necessary to change from one system to the other:—

LENGTH.

	APPROXIMATE EQUIVALENT.	ACCURATE EQUIVALENT.
1 inch	2½ cubic centimeters	2.539
1 centimeter	0.4 inch	0.393
1 yard	1 meter	0.914
1 meter (39.37 inches)	1 yard	1.093
1 foot	30 centimeters	30.479
1 kilometer (1,000 meters)	⅝ mile	0.621
1 mile	1½ kilometers	1.609

WEIGHT.

1 gram	15½ grains	15.432
1 grain	0.064 gram	0.064
1 kilogram (1,000 grams)	2.2 pounds avoirdupois	2.204
1 pound avoirdupois	½ kilogram	0.453
1 ounce avoirdupois (437½ grains)	28⅓ grams	28.349
1 ounce troy, or apothecary (480 grains)	31 grams	31.103

BULK.

1 cubic centimeter	1.06 cubic inch	0.060
1 cubic inch	16⅓ cubic centimeters	16.386
1 liter (1,000 cubic centimeters)	1 United States standard quart	0.946
1 United States quart	1 liter	1.057
1 fluid ounce	29½ cubic centimeters	29.570

SURFACE.

1 hectare (10,000 square meters)	2½ acres	2.471
1 acre	0.4 hectare	0.40

It may not be generally known that we have in the nickel five-cent piece of our coinage a key to the tables of linear measures and weights.

The diameter of this coin is two centimeters, and its weight is five grams. Five of them placed in a row will, of course, give the length of the decimeter; and two of them will weigh a decagram. As the Kiloliter is a cubic meter, the key to the measure of length is also the key to the measure of capacity.

Any person, therefore, who is fortunate enough to own a five-cent nickel, may carry in his pocket the entire metric system of weights and measures.

BRIEF HISTORY OF THE METRIC SYSTEM.

This system of weights and measures had its origin in France about 112 years ago. In 1799, the French government invited other governments to send representatives to an international convention which met at Paris. Representatives were present from many European countries.

As a result of the work of the convention, the metric system was based on the length of the fourth part of a terrestrial meridian. The ten-millionth part of this arc was chosen as the unit of length and called a **meter.** The unit of capacity was made a cube whose edge is the tenth part of a meter, and it was called a **liter.** A liter of distilled water at its greatest density was called a **Kilogram.** The one-thousandth part of this was taken as the unit of weight, and called a **gram.**

PROBLEMS IN COMPARISON.

TALK:

These problems are included for those who care to make the transitions from one system to the other. When the pupil has completed the other work problems like these may be given if the teacher thinks it advisable.

1. How many yards in 427 meters?

SOLUTION:

Since one meter is 1.093 yd., in 427 meters there are 427 × 1.093 yd., or 466.711 yd.

2. How many miles in 47 Kilometers?

SOLUTION:

In 1 Kilometer there is .621 of a mile and in 47 Kilometers there are 47 × .621 miles, or 29.187 miles.

---

3. How many Kilometers in 25 miles?

SOLUTION:

25 miles equal 25 × 1.6 Kilometers, or 40 Kilometers.

---

4. What is the cost of 14 liters of milk at 3¢ a quart?

SOLUTION:

Since one quart equals 1.057 liters, 14 liters equal as many quarts as 1.057 is contained times in 14 liters, or 13.24.

13.24 qt. @ 3¢ = 40-¢.

---

5. Find the value of 80 Hectares of land at $40 an acre.

SOLUTION:

Since one Hectare equals 2.471 acres, 80 Hectares equal 80 × 2.471 acres, or 197.68 acres.

197.68 acres @ $40 = $7907.20.

---

6. How many Kilometers in 29 miles?

SOLUTION:

Since one mile equals 1.6 Kilometers, 29 miles equal 29 × 1.6 Kilometers, or 36.4 Kilometers.

---

7. A field is 44 Dm. long and 32 Dm. wide. How many miles around it?

SOLUTION:

The distance around the field is 2 × 44 Dm. + 2 × 32 Dm., or 152 Dm.

152 Dm. = 1.52 Km.

1.52 × .621 miles = .94392 miles.

PART ELEVEN.

MENSURATION.

BEGINNER'S ALGEBRA.

"It is not a question, how much a man knows, but what use he can make of what he knows; not a question of what he has acquired, and how much he has been trained, but of what he is and what he can do."

—J. G. Holland.

WRITTEN ARITHMETIC.

THIS everybody demands of the teacher; and he is scarcely in danger of being without fair pretensions in this branch. He should, however, know it by its *principles*, rather than by its rules and facts. He should so understand it, that if every arithmetic in the world should be burned, he could still make another, constructing its rules and explaining their principles. He should understand Arithmetic so well, that he could teach it thoroughly though all text-books should be excluded from his schoolroom. This is not demanding too much.

Arithmetic is a *certain science*, and used every day of one's life,—the teacher should be an entire master of it. And when he reflects that without Arithmetic the wonderful exchanges made through the net-work of modern business must be reduced to the simple barter of barbarous times; that without Arithmetic the manufacture and manipulation of delicate or highly effective machinery must cease; that the almost miraculous processes of the chemical and physical laboratory must be suspended; and that without the proportion of numbers, architecture, sculpture, painting, and even poetry and music must all lose their charms, then he can comprehend to what an extent Arithmetic lies at the foundation of modern civilization and contributes to the physical, intellectual, and even spiritual welfare of mankind; then he can see *why* the teacher should be an entire master of it.

—DAVID PAGE.

MENSURATION.

TALK:

The custom of leaving all work in mensuration until pupils are about to leave the grammar grades has happily disappeared in most schools. To-day the pupil early becomes familiar with the simpler phases of the work in lines, surfaces, and solids. Pupils enjoy it when the presentation is concrete, and they quickly grasp the meaning of terms used.

With the new methods of teaching the subject, the old plan of working by rule for an answer has to some extent gone out of use here, as well as in the other parts of arithmetic. Then, too, since the child is to know and deal with lines, surfaces, and solids throughout his life, it is well that he becomes familiar with them as early as possible.

Do not attempt to teach mensuration without objects. Cut pieces of paper, cardboard, or wood to illustrate what you are teaching. Model solids from clay, if necessary. The piece of paper and pin used for a compass in "Measures for Little People" may be made to do good service here.

Mensuration is the measuring of lines, surfaces, and solids.

LINES AND ANGLES.

A **line** has only one dimension, length. The end of a line is a **point.** A point has position only.

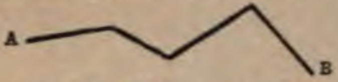
A **straight line** is one that does not change its direction at any point. It is the shortest distance between two points.

A————————B The line AB is a straight line.

A **curved line** is one that changes its direction at every point.

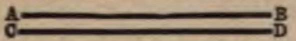
MN is a curved line.

A **broken line** is one that changes its direction at some points but not at all.



AB is a broken line.

Parallel lines are lines which will never meet no matter how far extended.

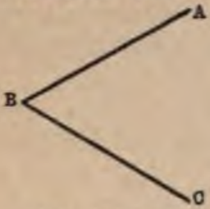


AB and CD are parallel lines.

A **horizontal line** is one parallel to the horizon.

A **vertical line** is one which makes an angle of 90° with the horizon.

An **angle** is the difference in direction between two lines which meet.

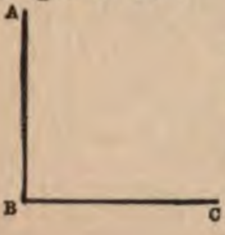


The angle ABC here is the difference in direction between the lines AB and BC, which meet at B. We read it as angle ABC, or angle CBA, placing the letter at the vertex in the middle.

The **vertex** of an angle is the point where the lines meet. B is at the vertex of the angle illustrated.

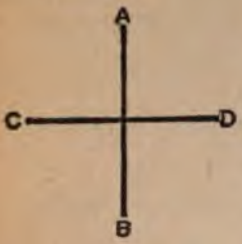
The **sides** of an angle are the lines that meet to form it.

Perpendicular lines are those which make an angle of 90°.



The line AB is perpendicular to the line CB, and the line CB is perpendicular to the line AB.

NOTE.—Teach the pupil that in whatever direction the lines extend, so long as the angle formed is 90°, the lines are perpendicular to each other. Many pupils get the mistaken idea that the lines must extend just as they do in the illustration.

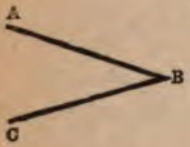


Here the line AB is perpendicular to the line CD, and CD is perpendicular to AB.

A **right angle** is one of 90°, or one whose sides are perpendicular to each other.

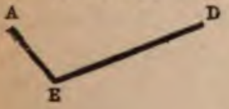
The angles shown in the two preceding drawings are all right angles.

An **acute angle** is one less than 90°.



Angle ABC is an acute angle.

An **obtuse angle** is one greater than a right angle.



Angle AED is an obtuse angle.

PLANE SURFACES.

A **surface** has two dimensions, length and breadth.

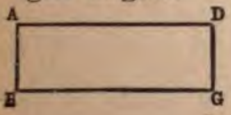
A **plane surface** is one that does not change its direction. A straight line laid upon it in any direction will touch at all points.

Note.—Let the pupil apply the edge of a ruler to a table. Try a warped board. Ask the pupil to notice a carpenter test a planed board.

A **quadrilateral** is a plane figure having four sides.

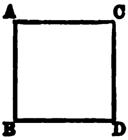
A **parallelogram** is a quadrilateral whose opposite sides are parallel.

A **rectangle** is a parallelogram whose angles are right angles.



ADEG is a rectangle.

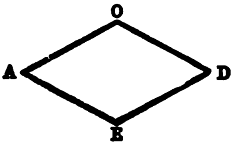
A **square** is a rectangle whose sides are equal.



ABCD is a square.

A **rhomboid** is a parallelogram whose angles are not right angles.

A **rhombus** is a rhomboid whose sides are equal.



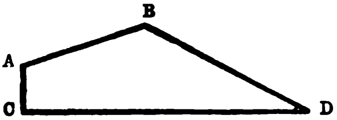
AODE is a rhombus.

A **trapezoid** is a quadrilateral which has only two of its sides parallel.

Its area is equal to that of a rectangle having the same altitude and a length equal to one-half the sum of the parallel sides.

NOTE.—Have the pupil illustrate this with slips of paper cut to show it.

A **trapezium** is a quadrilateral which has no two sides parallel.



ABCD is a trapezium.

TO FIND THE AREA OF A RECTANGLE.

The area of a surface 1 ft. long and 1 ft. wide = 1 sq. ft.

The area of a surface 2 ft. long and 1 ft. wide = 2 sq. ft.

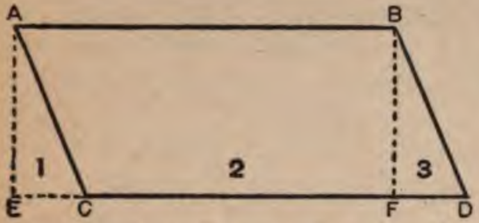
The area of a surface 2 ft. long and 2 ft. wide = 4 sq. ft.

Illustrate this by drawings and actual measurements. Follow it with illustrations for larger areas.

To summarize the result, we have,

The area of a rectangle equals the product of the base multiplied by the altitude.

TO FIND THE AREA OF A PARALLELOGRAM.



Let ABCD be any parallelogram with the base CD. Draw perpendiculars AE and BF to the base, or base extended. Then, triangles 1 and 3 are equal. (Let the pupil tell why or show it.) So, if we take off triangle 3 and leave triangle 1, we have a rectangle ABEF, which is equal to parallelogram ABCD, since it has the same altitude and base.

Then, to find the area of a parallelogram,

Take the product of the base multiplied by the altitude.

TO FIND THE AREA OF A TRAPEZOID.

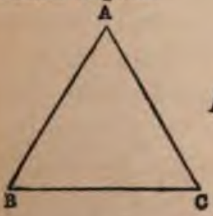
In the same way as in the preceding, illustrate how the area of a trapezoid is found.

The area of a trapezoid is the product of the altitude multiplied by one half the sum of the bases.

TRIANGLES.

A **triangle** is a plane figure terminated by three straight lines.

An **equilateral triangle** is one that has its three sides equal.



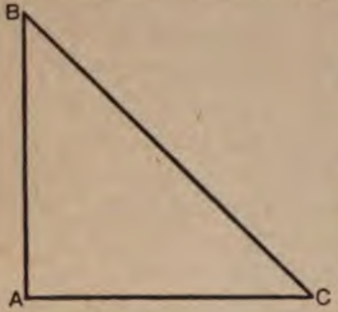
ABC is an equilateral triangle.

An **isosceles triangle** is one that has two of its sides equal.

Consequently two of its angles are equal.

A **scalene triangle** is one that has no two sides equal.

Scalene and **isosceles** are terms not in common use.



A **right triangle** is one that has one angle that is a right angle.

ABC is a right triangle.

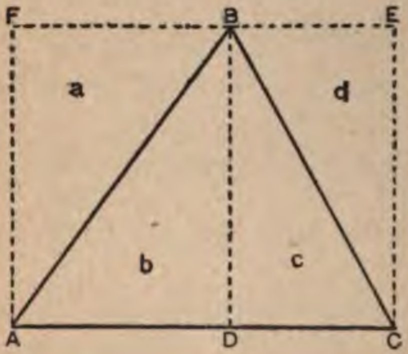
In a right triangle the side opposite the right angle is called the **hypotenuse.** The side BC is the hypotenuse of the above right triangle.

An **equiangular triangle** is one that has three equal angles.

An **acute angle triangle** is one whose angles are less than right angles.

An **obtuse angle triangle** is one having one angle greater than a right angle.

TO FIND THE AREA OF A TRIANGLE.



Take the triangle ABC and draw the dotted lines. Now notice that triangles a and b are equal, since each is one half of the same rectangle, and that triangles c and d are equal, since each is one half of the same rectangle.

The two rectangles make the large rectangle having AC for a base.

The area of the triangles b and c, or of the large one ABC, is equal to one half the area of the large rectangle, FACE. *From this we get the rule,*

The area of a triangle is one half the product of the base multiplied by the altitude.

The altitude BD is the same as the sides EC and FA of the large rectangle.

NOTE.—In the preceding, AC is considered the base of the triangle ABC. Any side of a triangle may be considered the base. The *altitude* of a triangle is the perpendicular distance from the apex of the angle opposite the base to the base, or the base extended.

When the altitude is not given:

In many cases, the altitude of a triangle is not known. Where this is true, the pupil who has not studied geometry should follow the rule given here to find the area. He will understand why it is true when he has studied geometry.

Remember:

(a) **Find one half the sum of the sides.**

(b) **From this, subtract each side separately.**

(c) **Find the product of the one half sum and the three remainders.**

(d) **Take the square root of this product.**

The result is the number of surface units in the triangle.

PROBLEMS.

I. Where the altitude is given.

1. What is the area of a triangular field whose base is 40 rods and altitude 34 rods?

WORK:

$\frac{1}{2} \times 40 \times 34 = 680.$

Hence, the field has 680 sq. rd. in it.

2. A board in the form of a triangle has a base of 14 in. and an altitude of 20 in. What is its area?

WORK:

$\frac{1}{2} \times 14 \times 20 = 140.$

Hence, it has 140 sq. in., or nearly 1 sq. ft. of surface.

II. Where the sides only are known.

1. A triangular lot is 6 rods on one side, 4 rods on another, and 8 rods on the third. What is its area?

WORK:

(a) $\frac{1}{2}$ of $(6 + 8 + 4) = 9$

(b) $9 - 6 = 3$
$9 - 8 = 1$
$9 - 4 = 5$

(c) $9 \times 3 \times 1 \times 5 = 135$

(d) $\sqrt{135} = 11.61.$

Hence, the area is 11.61 sq. rods.

NOTE.—The steps, a, b, c, d, correspond to the steps named in the rule.

---

2. What is the area of a piece of triangular cardboard whose sides are 12, 8, and 6 inches, respectively?

WORK:

(a) $\frac{1}{2}$ of $(12 + 8 + 6) = 13$

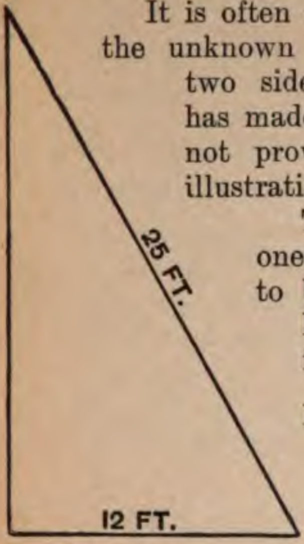
(b) $13 - 12 = 1$
$13 - 8 = 5$
$13 - 6 = 7$

(c) $13 \times 1 \times 5 \times 7 = 455$

(d) $\sqrt{455} = 21.3 +.$

Hence, the area is 21.3 sq. in.

To find the third side of a right triangle.



It is often convenient to know how to find the unknown side of a right triangle when two sides are given. Until the pupil has made a study of geometry, he cannot prove the truth of it except by illustration.

There are two rules needed here; one where one of the short sides is to be found; the other, where the long side, or hypotenuse is to be found.

Remember:

1. To find one of the short sides,

(a) Find the square of each known side.

(b) Find the square root of the difference of these squares.

2. To find the long side or hypotenuse,

(a) Find the square of each known side.

(b) Find the square root of the sum of these squares.

PROBLEMS.

1. Find the third side of the right triangle in the last illustration.

WORK:

(a) $25^2 = 625$. $12^2 = 144$.

(b) $625 - 144 = 481$. $\sqrt{481} = 22$, almost.

Hence, the third side is almost 22 ft. long.

2. Find the hypotenuse of a triangle whose other two sides are 4 in. and 6 in., respectively.

WORK:

(a) $4^2 = 16$. $6^2 = 36$.

(b) $16 + 36 = 52$. $\sqrt{52} = 7.2$.

Hence, the hypotenuse is 7.2 inches.

3. I have one fourth of a quarter section of land. I wish to put a fence diagonally across it. How long will the fence be?

NOTE.—Illustrate this by diagram.

WORK AND EXPLANATION:

Each side of the land is 80 rds. long. Hence, I know two sides of the right triangle are each 80 rods. Then, to find the long side,

(a) $2 \times 80^2 = 12800$.

(b) $\sqrt{12800} = 113.13+$.

Hence, the length of the fence will be a little more than 113.13 rods.

To illustrate that the square on the hypotenuse equals the sum of the squares on the other two sides,

Take a large sheet of paper and on it draw a right triangle whose two shorter sides are 3 inches and 4 inches, respectively. Now from the rule, you find that the hypotenuse is just 5 inches. Next, draw the squares on the sides of the triangle. Divide each square into smaller squares each one inch on a side.

Now, count the small squares in each of the three large squares. You find that,

The large square on the shortest side of the triangle has 9 small squares.

The large square on the next short side of the triangle has 16 small squares.

Both the large squares considered have 25 small squares.

The large square on the hypotenuse has 25 small squares.

NOTE.—What has just been illustrated is known as the Pythagorean proposition, because Pythagoras was the first to demonstrate it. It is often stated,

The square on the hypotenuse equals the sum of the squares on the other two sides.

REGULAR POLYGONS.

A **polygon** is a plane figure having many sides and many angles.

A **regular polygon** is one having equal sides and equal angles.

The **perimeter** of a polygon is the sum of the sides.

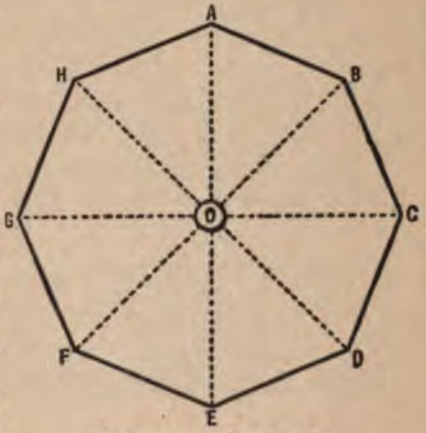
Regular polygons may be named as follows:—

No. of Sides.	Name.
3	equilateral triangle.
4	square.
5	regular pentagon.
6	regular hexagon.
7	regular heptagon.
8	regular octagon.
9	regular nonagon.
10	regular decagon, etc.

Note.—Let the pupil draw figures to illustrate each of the above. For this work his compass again becomes valuable.

A regular polygon can be divided into as many isosceles triangles as it has sides, as shown here.

If a line be drawn from the center, O, to the middle of the base of any triangle, as to the middle of AB, it represents the **altitude of the triangle,** or the **apothem of the polygon.**



Regular Octagon.

Then, to find the area of the polygon, we need only find the area of one triangle and multiply by the number of triangles.

This is equivalent to taking the product of the sum of the bases, or the perimeter of the polygon, by the apothem and dividing by two. *Hence, to find the area of any regular polygon,*

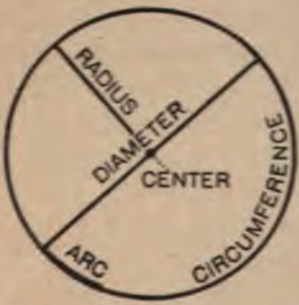
Divide the product of the perimeter multiplied by the apothem by two.

CIRCLES.

A **circle** is a plane surface bounded by a curved line, every point of which is equally distant from a point within, called the center.

The **circumference** of a circle is its bounding line.

NOTE.—Be careful to avoid giving the pupil the common impression that the circumference is the circle.



The **diameter** of a circle is a line from circumference to circumference, through the center.

The **radius** of a circle is any straight line from the center to the circumference. It is half the diameter.

The **arc** of a circle is any part of its circumference.

A **chord** is any line which terminates in the circumference without passing through the center.

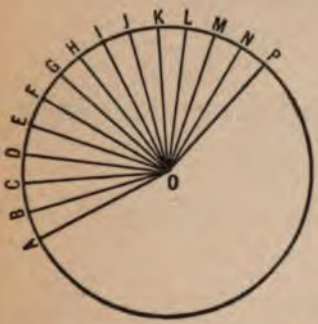
CIRCUMFERENCE OF A CIRCLE.

The circumference of any circle may be found if the diameter is known, by multiplying the diameter by 3.1416.

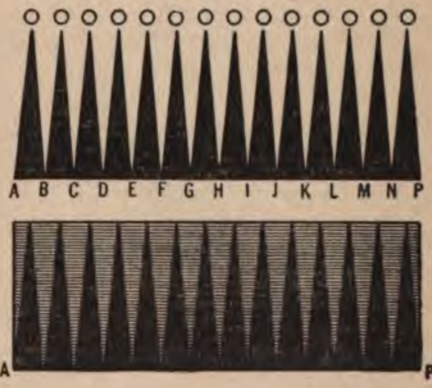
Conversely, if you know the circumference of any circle, you can find the diameter, by dividing the circumference by 3.1416.

NOTE.—For ordinary purposes $3\frac{1}{7}$ may be used in place of 3.1416. The form $\frac{22}{7}$ is convenient.

AREA OF A CIRCLE.



Draw a regular polygon of 6 sides. Draw one of 24 sides. Notice that the greater the number of sides the more the polygon approaches a circle in shape. Now, imagine the circle in the picture to be cut on the lines OA, OB, OC, etc., leaving just the margin at the circumference uncut. If we take that portion of the circle AOP and lay the circumference, AP on a flat surface, it will look like the first picture here. The smaller the sections the nearer AP will be to a straight line when laid on the flat surface. If half the circle is taken for that and the other half is cut in a similar way, the two parts may be fitted together to form a rectangle like that in the second picture here. The width of this rectangle is the same as the radius of the circle, and the length is equal to half the circumference.



Then, to find the area of a circle,

Multiply the radius by one half the circumference.

Remember:

1. *To find the circumference of a circle,*
   Multiply the diameter by $3\frac{1}{7}$.
2. *To find the diameter of a circle,*
   Divide the circumference by $3\frac{1}{7}$.
3. *To find the area of a circle,*
   Multiply $\frac{1}{2}$ the circumference by the radius.

PROBLEMS.

1. Find the circumference of a wheel whose diameter is 2 feet.

SOLUTION:

The circumference is $3\frac{1}{7} \times 2$ ft., or $6\frac{2}{7}$ ft.

---

2. The circumference of a tree is 4 ft. 6 in. What is its diameter?

SOLUTION:

The diameter is as many feet as $3\frac{1}{7}$ is contained times in 4 ft. 6 in., or 1ft. $5\frac{2}{11}$ in.

---

3. What is the area of a circle whose diameter is 4 feet?

SOLUTION:

The radius is 2 ft.
The circumference is $3\frac{1}{7} \times 4$ ft., or $12\frac{4}{7}$ ft.
The area in sq. ft. is $\frac{1}{2}$ of $2 \times 12\frac{4}{7}$, or $12\frac{4}{7}$.

---

4. A cow is tied to a stake with a rope 30 ft. long. How many square yards can she graze on?

SOLUTION:

She can graze on a circle whose radius is 10 yards.
The circumference of the circle is $3\frac{1}{7} \times 20$ yd., or $62\frac{6}{7}$ yd.
The area in sq. yd. is $\frac{1}{2}$ of $10 \times 62\frac{6}{7}$, or $314\frac{2}{7}$.

5. A landscape gardener wishes to arrange a circular plot so that it will contain 1 acre. What will be the diameter of the plot?

Note.—**For such problems, let the pupil use the formula that the area equals 3⅐ multiplied by the radius squared.**

Solution:

An acre contains 160 sq. rd. or 43560 sq. ft.

The square of the radius of the circle equals $\frac{43560}{3\frac{1}{7}}$ ft., or 13860 ft.

The radius equals $\sqrt{13860}$ ft., or 117.72 + ft.

The diameter is 2 × 117.72 + ft., or about 235¾ ft.

SIMILAR SURFACES.

Similar surfaces are those having the same form regardless of size.

The area of a rectangle 2 ft. on a side is 4 sq. ft. The area of a rectangle 4 ft. on a side is 16 sq. ft. The 4 ft. rectangle has 4 times the area of the 2 ft. rectangle.

Then rectangles are not proportional to their dimensions, but to the *squares* of their dimensions.

Try other rectangles by making diagrams and marking off the area of each in squares of 1 foot on a side. Notice that the area is always proportional to the squares of the respective dimensions.

Try rectangles having the same width but different lengths. Notice that the areas are proportional to the squares of the lengths.

From all this we deduce the rule that,

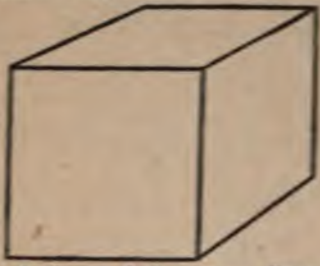
Similar surfaces are to each other as the squares of their like dimensions.

With the same rectangles, compare the similar sides with the areas. *You find that the*

Similar sides are proportional to the square roots of the areas.

SOLIDS.

Solids have three dimensions, length, breadth, and depth.



A CUBE 1 IN. ON A SIDE.

A **cube** is a solid having six equal squares for faces.

The cube is the unit of volume.

Volume or **solid contents** is the number of cubic units in a solid.

From the pupil's early work in arithmetic, he is familiar with the basic principle of the measurement of solids, which is that,

The units of volume in a rectangular solid are equal to the continued product of its three dimensions.

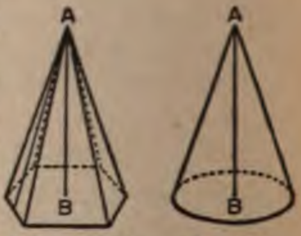
PYRAMIDS AND CONES.

A **pyramid** is a solid whose base is either triangular, square, octagonal, etc., and whose sides are triangles meeting in a vertex.

It is named from the form of its base, as a square pyramid, hexagonal pyramid, etc.

A **cone** is a solid whose base is a circle and whose convex surface terminates in a point.

The **slant height of a pyramid** or *cone* is the shortest distance from the vertex to the edge of the base.



A hexagonal pyramid. A cone.

The **altitude** of a cone or pyramid is the perpendicular distance from the vertex to the base. With the base, it forms a right triangle of which the slant height is the hypotenuse. The line AB shows the altitude in the illustrations.

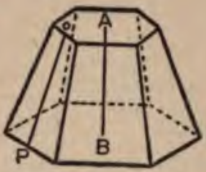
To find the contents of a pyramid or cone,

Multiply the area of the base by $\frac{1}{3}$ of the altitude.

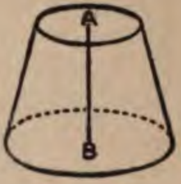
To find the slant surface,

Multiply the circumference by $\frac{1}{2}$ the slant height.

A **frustum** of a pyramid or cone is one which has the top cut off parallel to the base.



Frustum of hexagonal pyramid.



Frustum of cone.

To find the volume of a frustum of pyramid or cone,

(a) **Add the areas of the bases.**

(b) **Find the product of the bases and extract the square root.**

(c) **Add the sum and square root found and multiply this by $\frac{1}{3}$ of the altitude.**

NOTE.—The surface where the top has been cut off is the second base.

THE SPHERE.

Every pupil is familiar with the sphere. His ball, marbles, etc., are all spheres. So is the globe in the schoolroom.

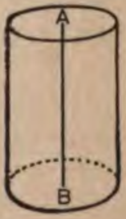
A **sphere** is a solid bounded by a curved surface every point of which is equally distant from the center.

To find the surface of a sphere,

Multiply the circumference by the diameter.

To find the volume of a sphere,

Multiply the surface by $\frac{1}{6}$ of the diameter.



A **cylinder** is a prism whose bases are equal parallel circles.

To find the lateral area of a cylinder,

Multiply the circumference of one base by the altitude.

To find the volume of a cylinder,

Multiply the area of one base by the altitude.

The parallelograms forming the convex surface of a cylinder are infinite in number and infinitely narrow. Consequently, the surface exposed is convex.

NOTE.—The pupil may have demonstrated to him the truth of these formulas for the surface and volume of solids in much the same way that we illustrated the area of a circle; but, with pupils who have not studied geometry, it is not time profitably spent.

It is not likely that the pupils will ever be called upon to use these formulas much, so no problems are added. The teacher who teaches this part of mensuration can easily find problems in plenty. Each result may easily be proved.

SIMILAR SOLIDS.

Similar solids are those having the same form without regard to size.

Cubes are similar solids, regardless of relative size. So are spheres.

Compare a cube 1 in. on a side with one 2 in. on a side. Compare one 2 in. on a side with one 3 in. on a side; with one 4 in. on a side; 5 in. on a side.

You notice that the 2 in. cube has 8 cu. in. of volume; the 3 in. cube has 27 cu. in. of volume; etc. From this the conclusion is that,

Similar solids are to each other as the cubes of their like dimensions and,

Corresponding dimensions of similar solids are to each other as the cube roots of their volumes.

ALGEBRA FOR BEGINNERS.

TALK:

Algebra is too often looked upon as a "bugbear" by pupils just beginning the work. If the first few lessons on the subject can be made to appear as a review of arithmetical processes with the substitution of letters for figures, the rest of the work will be easy.

In fact, the pupil will naturally be pleased at each new phase of the work, for he learns to solve problems that are more difficult when limited to figures. He will like the freedom given to his reasoning power, how if x equals this or that, $2x$ will equal, etc. Algebra has a fascination about it that arithmetic has not, yet everything depends upon the start.

Proceed very slowly at first. Get the few general truths about the signs, the fundamental operations, the parenthesis, etc., well fixed in the pupil's mind, and then let the development be more rapid.

Any hesitancy on the part of the pupil to understand operations should be overcome by frequent substitutions of figures for letters. In so doing, the process is explained.

THE FIRST LESSONS.

$5 + 5$. Here we have used what process? (*Pupil*) Addition.

Suppose we wish to perform the same process with a and b. How would you write it? (*Pupil*) $a + b$.

In algebra, we use letters as well as figures in all the processes. We cannot always perform the operations as we might with figures, so many times we have to simply *indicate* the process.

Thus, a divided by b is indicated as $\frac{a}{b}$, or $a \div b$.

How would you show that m and n are to be added? x and y? b and c?

$5 - 3$. Here we have used what process? How would you show that b is to be taken from a? c from x?

Suppose we wish to multiply 4 by 8, how would we show the process? (*Pupil*) 4×8. 8 4's.

If we show that a is multiplied by b we would write ab.

Show that c is multiplied by d. x by y.

Who can give us two forms for showing that a is to be divided by b? (*Pupil*) $a \div b$, $\frac{a}{b}$.

In algebra, we use both forms.

To the Teacher:

Give problems like the following to make the pupil more proficient in using letters instead of figures:

1. Mary has a oranges and Susan has b oranges. How would you express the number of oranges both have?

2. If apples cost b cents a peck, what will 3 pecks cost? How would you express the cost of the 3 pecks? What will c pecks cost?

3. If 9 books cost c dollars, what will 1 book cost?

4. If a is the cost of b dinners, what is one meal worth? etc.

The first letters of the alphabet a, b, c, d, e, f, etc. are used to denote **known numbers** in algebra, while the last letters, x, y, z, are used for **unknown numbers,** or numbers which are found by solving problems.

TO ILLUSTRATE:

1. John has 468 sheep and Frank has 1391. How many have both?

SOLUTION:

We know how many John has, so we may designate the number by a. Frank's may be expressed by *b.* We do not know, without working the problem, how many both have, but we may call the number x.

Then, $x = a + b$

$x = 468 + 1391$

$x = 1859.$

2. Mr. Moore has \$40 and Mr. Gray has 3 times as much. How much money have both?

SOLUTION :

Let a = money Mr. Moore has.

Then, $3a$ = money Mr. Gray has.

x = money both have.

$x = a + 3a$

or, $x = \$40 + 3a$

or, $x = \$40 + \$120$

or, $x = \$160.$

NAMES OF PARTS OF QUANTITIES.

There are four parts of numbers that we have to deal with in algebra, (1) the sign, (2) the literal part, (3) the exponent, and (4) the coefficient.

The Sign.

There are but two signs, the plus (+) and the minus (−). The minus sign is never omitted, so when no sign is expressed, the plus sign is understood.

The Literal Part.

The literal part is the part indicated by one or more letters used as factors.

In $3ab$, ab is the literal part.

The Exponent.

The exponent has been used in arithmetical problems, denoting powers of numbers. It is a small figure written to the right and above any factor to show the power to be taken.

In $9ax^2$, 2 is the exponent, showing that x is to be raised to the second power.

The Coefficient.

The coefficient is the figure on the left of the literal part and indicates how many times the literal part is to be taken.

In $4ab$, 4 is the coefficient, and it shows that ab is to be taken 4 times.

If no coefficient is expressed, 1 is understood.

TERMS.

The **terms** of an expression in algebra are the parts separated by the plus or minus signs.

In $4a + 3b - 9c$, the terms are $4a$, $3b$, and $9c$.

The literal part of each term gives the name to the term.

In $9a$, the a gives the name to the term.

Suppose we add $4a$, $5a$, and $6a$.

$$4a + 5a + 6a = 15a.$$

Here the terms are similar.

Suppose we add $5a$, $3b$, and $6c$.

$$5a + 3b + 6c = 5a + 3b + 6c.$$

Here the terms are dissimilar.

The literal parts show whether or not the terms are similar. If they have the same name, they can be added; if they have different names, the process can only be expressed.

Remember:

1. **Only similar numbers can be added or subtracted to make one term.**
2. **In the addition or subtraction of dissimilar numbers, the operation can only be expressed.**

For the Pupil:

1. Can 4 sheep and 8 oranges be added?
2. Can 4 sheep and 8 sheep be added?
3. Can $4a$ and $3b$ be added to form one term? What is the only thing that can be done?

4. What part of a term gives it its name?

5. $5b + 3b = ?$ $6a + 3b = ?$
$4ab + 2ab = ?$ $3ab - bc = ?$

6. Define term, similar term, dissimilar term, coefficient, exponent, sign, literal part.

EXPRESSIONS.

An **algebraic expression** is one or more terms taken together as one quantity.

Thus, a, $a + c + d$, $\sqrt{x - y}$, are expressions.

A number or quantity containing but one term is called a **monomial.**

$5a$, 2, $3c$, $-15x$ are all monomials.

A number containing two or more terms is called a **polynomial.**

$6a + 4b - 3c$, $16ab + 9cd$, $3c + 4d + 3f + 3c^2 + 9x - 4d$, are polynomials.

A polynomial containing two terms is called a **binomial**; three terms, a **trinomial.**

$3ab + 5cd$ is a binomial.

$3xy + 4x - 2yz$ is a trinomial.

POSITIVE AND NEGATIVE NUMBERS.

The signs plus (+) and minus (−) are used in algebra to express opposite meanings or conditions.

I climb 10 feet. If I express the distance I climb as + 12 feet, − 12 feet means a descent of 12 feet.

I gain $4. − $4 express a loss of $4.

I walk east 3 miles. If + 3 miles means east from a point, − 3 miles means west of the point.

If + 10° means above zero, − 10° means below zero.

Remember:

The signs + and − express opposite conditions or meanings.

If a number is preceded by a plus sign or no sign at all, we call it a **positive number.**

3, 5, $+3a$, $+8$, and 9 are all positive numbers.

If a number is preceded by a minus sign, we call it a **negative number.**

-3, $-5a$, $-4c$, and -10 are all negative numbers.

For the Pupil:

What kinds of numbers are these:—

16, -24, -2, $3ab$, $-4a$, and $16b$?

ADDITION

TO ILLUSTRATE:

$36a + 90a + 35a = ?$

$$\begin{array}{r} 36a \\ 90a \\ 35a \\ \hline 161a \end{array}$$

Remember:

To add positive numbers,

Add the coefficients and write the sum before the common literal part.

TO ILLUSTRATE:

Add $-36a$, $-90a$, $-35a$.

$$\begin{array}{r} -36a \\ -90a \\ -35a \\ \hline -161a \end{array}$$

Remember:

To add negative numbers,

Add the coefficients and write the sum preceded by the minus sign, before the literal part.

TO ILLUSTRATE:

Add $-36a$, $5a$, $16a$, $-2a$.

$$\begin{array}{rcr} -36a & \quad & 5a \\ -\ \ 2a & & 16a \\ \cline{1-1}\cline{3-3} -38a & & 21a \\ 21a & & \\ \cline{1-1} -17a & & \end{array}$$

Remember:

To add negative and positive numbers,

1. Add the negative and positive numbers separately, subtract the smaller from the larger, and give the result the sign of the minuend.

2. If the terms are dissimilar, merely indicate the operation performed.

For the Pupil:

Add or express the operation in the following:—

1. $-7, +13, -81, +106, 7000.$
2. $2ab, 4xy, -2bc, -3ab.$
3. $\$92, -\$28, -\$56, -\$102.$
4. $2xy, -12xy, -26xy, 100xy.$
5. $16b, -81b, -22b, -28b.$

Add orally:—

(1)	(2)	(3)	(4)	(5)	(6)
-7	14	-92	-17	63	-25
-2	-3	-3	-21	4	9
			-2		

(7)	(8)	(9)	(10)	(11)	(12)
$12b$	$-16x$	$32x$	$-15b$	-64	-13
$-4b$	$-4x$	$-3x$	$3b$	45	12

THE PARENTHESIS.

If we wish to consider two or more terms as one term, we enclose them in a parenthesis ( ).

$12 + (5 + 3)$ means that 5 and 3 together affect what is outside the parenthesis.

Remember:

In solving problems in which the parenthesis is concerned,

Consider the parenthesis first.

To Remove the Parenthesis:

$$\left\{\begin{array}{l} 10 + (3 + 2) = 15 \\ 10 + 3 + 2 = 15 \end{array}\right. \qquad \left\{\begin{array}{l} 10a + (3a + 2a) = 15a \\ 10a + 3a + 2a = 15a \end{array}\right.$$

Notice what sign precedes the parenthesis and the change, if any, that occurs when the parenthesis is removed.

$$\begin{cases} 10 - (3 + 2) = 5 \\ 10 - 3 - 2 = 5 \end{cases} \qquad \begin{cases} 10a - (3a + 2a) = 5a \\ 10a - 3a - 2a = 5a \end{cases}$$

Notice above what sign precedes the parenthesis Notice the sign within the parenthesis and the change due to the removal of the parenthesis.

$$\begin{cases} 10 - (3 - 2) = 9 \\ 10 - 3 + 2 = 9 \end{cases} \qquad \begin{cases} 10a - (3a - 2a) = 9a \\ 10a - 3a + 2a = 9a \end{cases}$$

Notice above what sign precedes the parenthesis in each case. Observe the sign within the parenthesis and the change that takes place due to the removal of the parenthesis.

Remember:

1. If a parenthesis is preceded by a plus sign,

The parenthesis may be removed without changing a sign in the expression within the parenthesis.

2. If a parenthesis is preceded by a minus sign,

The parenthesis may be removed, but every sign in the expression within the parenthesis must be changed.

For the Pupil:

Remove the parenthesis in each of the following, and make the changes necessary:—

1. $19 + (2 + 3) = ?$
2. $26 - (4 + 9) = ?$
3. $36a + (9a - 7a) = ?$
4. $-(36 + 2) + 3 = ?$
5. $36y - (6y + 5y) = ?$
6. $43 + (12 - 4) = ?$
7. $18d + (7d - 4d) = ?$

SUBTRACTION.

TO ILLUSTRATE:

Subtract 30 from 80.

$$\begin{array}{r} 80 \\ \underline{30} \\ 50 \end{array}$$

Both minuend and subtrahend are preceded by + here and the work is simple subtraction, but it is better in algebra to subtract by thinking of the sign before the subtrahend 30 as being changed.

Then add as we learned to add positive and negative numbers.

$$\begin{array}{r} 80 \\ -30 \\ \hline 50 \end{array}$$

TO ILLUSTRATE:

Subtract -40 from -80.

The best way is to change the sign before the subtrahend 40 and add.

$$\begin{array}{r} -80 \\ 40 \\ \hline -40 \end{array}$$

TO ILLUSTRATE:

Subtract $-40a$ from $6a$.

The best way is to change the sign before the subtrahend $-40a$ and add.

$$\begin{array}{r} 6a \\ 40a \\ \hline 46a \end{array}$$

Remember:

To subtract algebraic numbers,

1. Change the sign of subtrahend and proceed as in addition.

2. If the terms are dissimilar, merely indicate the operation performed.

TO ILLUSTRATE:

1. Subtract $3a$ from $4a$.

$$\begin{array}{r} 4a \\ -3a \\ \hline a \end{array}$$

2. From $8a - 7b + 5c - 19d$ take $5a - 4b - 7c + 4d$.

$$\begin{array}{r} 8a - 7b + 5c - 19d \\ -\ 5a + 4b + 7c - 4d \\ \hline 3a - 3b + 12c - 23d \end{array}$$

For the Pupil:

Remove the parentheses in

(1) $18 - (6 + 4)$

(2) $16x - (12 + 3x)$

(3) $12b + (6 - 2)$

Copy and subtract:—

(1) From $3x - 5y^2 - z$ take $6x + 5y^2 + 3z$.

(2) $17b + 5c - 3c + 3b = ?$

(3) From $-25 + (x - y)$ take $3 + (x - y)$.

MULTIPLICATION.

We know that $3a$ means $3 \times a$, or $a + a + a$.

Then, $3\ (a + b\ )$ means $3a + 3b$.

And, $2\ (a + c + d + e) = 2a + 2c + 2d + 2e$.

Notice that when we multiply a number in algebra, we multiply each term in the multiplicand by the multiplier.

MULTIPLYING WHEN THE MULTIPLIER IS POSITIVE.

TO ILLUSTRATE:

1. $4\ (8 + 4 - 3) = ?$

$$\begin{array}{l} 8 + 4 - 3 \\ 4 \\ \hline 32 + 16 - 12 \end{array}$$

In writing the multiplication, the multiplier may be placed below the term farthest to the left as given above, or it may be written under the term at the extreme right, as,

$$\begin{array}{r} 8 + 4 - 3 \\ 4 \\ \hline 32 + 16 - 12 \end{array}$$

2. Multiply $2ab - 3bc + 4c$ by $3d$.

$$\begin{array}{l} 2ab - 3bc + 4c \\ \underline{3d \qquad\qquad\qquad} \\ 6abd - 9\,bcd + 12cd \end{array}$$

MULTIPLYING WHEN THE MULTIPLIER IS NEGATIVE.

Let a represent a gain. Then $-a$ is a loss.

$$\begin{aligned} \text{Then, } 2 \times +a &= +2a \\ 2 \times -a &= -2a \\ \text{But } -2 \times -a &= +2a \end{aligned}$$

The Multiplication Sign Law:—

Multiplying numbers with like signs gives plus; with unlike signs, minus.

TO ILLUSTRATE:

Multiply $7m - 3n$ by $2a$.

$$\begin{array}{l} 7m - 3n \\ \underline{2a \qquad\quad} \\ 14am - 6an \end{array}$$

IMPORTANT PRINCIPLES.

1. Multiply $a + b$ by $a + b$.

Let a and b be any numbers. Then here we are going to get the square of the sum of two numbers.

$$\begin{array}{l} a + b \\ \underline{a + b} \\ a^2 + ab \\ \underline{\quad + ab + b^2} \\ a^2 + 2ab + b^2 \end{array}$$

NOTE.—$a \times a = aa$ or a^2. We use the exponent to save repetition.

From the work, the following principle is derived:—

The square of the sum of two numbers is equal to the square of the first, plus two times the first by the second, plus the square of the second.

---

2. Multiply $a - b$ by $a - b$.

Let a and b be any two numbers. Then here we are going to get the square of the difference of two numbers.

$$\begin{array}{l} \quad a - b \\ \quad a - b \\ \hline a^2 - ab \\ \qquad - ab + b^2 \\ \hline a^2 - 2ab + b^2 \end{array}$$

From the work, the following principle is derived:—

The square of the difference of two numbers is equal to the square of the first, minus two times the first by the second, plus the square of the second.

3. Multiply $a - b$ by $a + b$.

Let a and b be any two numbers. Then here we are going to find the product of the sum and difference of two numbers.

$$\begin{array}{l} a - b \\ a + b \\ \hline a^2 - ab \\ \quad + ab - b^2 \\ \hline a^2 - b^2 \end{array}$$

From the work, the following principle is derived:—

The product of the sum and difference of two numbers is equal to the square of the first minus the square of the second.

For the Pupil:

1. State the 3 principles of multiplication.
2. Give the law for signs in multiplication.
3. Multiply $2a - 3b$ by $4a + 3b$.
4. Multiply $9a - 3c - 3d$ by $15a + 1$.

DIVISION.

In arithmetic we learned that the product of the divisor and quotient gives the dividend; in fact, examples in division were proved that way.

Thus,

(1) $+ 2a \times + 3a = + 6a^2$,

then, $+ 6a^2 \div + 2a = + 3a$.

(2) $-2a \times +3a = -6a^2$,
then, $-6a^2 \div -2a = +3a$.

(3) $-2a \times -3a = +6a^2$,
then, $+6a^2 \div -2a = -3a$.

(4) $+2a \times -3a = -6a^2$,
then, $-6a^2 \div +2a = -3a$.

From the above, the following Law of Signs in division can be given: —

Dividing numbers with like signs gives plus; with unlike, minus.

TO ILLUSTRATE:

1. Divide $ab^3 - cb^2 + 2b$ by b.

$$b\,\underline{)\,ab^3 - cb^2 + 2b}$$
$$ab^2 - cb + 2$$

---

2. $5a^5 - 2a^4bc + 3a^7 \div a^2 = ?$

$$a^2\,\underline{)\,5a^5 - 2a^4bc + 3a^7}$$
$$5a^3 - 2a^2bc + 3a^5$$

Notice that the exponents must be subtracted.

---

3. $4ab + 2ab^2 - 6a^2b \div 2ab$.

$$2ab\,\underline{)\,4ab + 2ab^2 - 6a^2b}$$
$$2 + b - 3a$$

---

4. Divide $8x^3y - 16x^2y^2 + 12xy^2$ by $2xy$.

$$2xy\,\underline{)\,8x^3y - 16x^2y^2 + 12xy^2}$$
$$4x^2 - 8xy + 6y$$

Notice that the coefficients must be divided.

---

5. Divide $x^2 - 2xy + y^2$ by $x - y$.

$$x - y\,)\,x^2 - 2xy + y^2\,(\,x - y$$
$$\underline{x^2 - xy}$$
$$-xy + y^2$$
$$\underline{-xy + y^2}$$

6. Divide $x^2 - y^2$ by $x + y$.

$$\begin{array}{r}
x + y\,)\,x^2 - y^2\,(\,x - y \\
\underline{x^2 + xy} \\
- xy - y^2 \\
\underline{- xy - y^2}
\end{array}$$

For the Pupil:

1. Give the law of signs in division.
2. Divide $25a^2x$ by $5ax$.
3. Divide $x^3 + 3x^2y + 3xy^2 + y^3$ by $x + y$.

FACTORING.

Factoring in algebra is the same as in arithmetic. Both the **inspection** and the **division** methods are used.

By Inspection.

1. Factor $24a^2$.
   Factors $= 2, 2, 2, 3, a, a$.

2. Factor $2a^2\,(x - y)$.
   Factors $= 2, a, a, x - y$.

NOTE.—Remember that all within a parenthesis is regarded as one factor.

3. Factor $a^2 + 2ab + b^2$.
   This is the square of the sum of two numbers. Factors $= a + b$ and $a + b$.

4. Factor $a^2 - 2\,ab + b^2$.
   Factors $= a - b$ and $a - b$.

5. Factor $a^2 - b^2$.
   Factors $= a - b$ and $a + b$.

By Division.

1. Factor $420a^5\,b^2$.

$$\begin{array}{l}
2\,)\,\underline{420} \\
\;\;2\,)\,\underline{210} \\
\;\;\;\;5\,)\,\underline{105} \\
\;\;\;\;\;\;3\,)\,\underline{21} \\
\;\;\;\;\;\;\;\;\;\;7
\end{array}$$

Factors $= 2, 2, 5, 3, 7, a, a, a, a, a, b, b$.

For the Pupil:

1. Factor $12a^2$.
2. Factor $48x^2y^3$.
3. Factor $x^2 - y^2$.

GREATEST COMMON DIVISOR AND LEAST COMMON MULTIPLE.

The method is the same as in arithmetic. Turn to pages 135 and 140 and review.

1. Find the G. C. D. of a^3, $4ax$, $3a^2y$.

$$\begin{aligned} a^3 &= a \times a \times a \\ 4ax &= 2 \times 2 \times a \times x \\ 3a^2y &= 3 \times a \times a \times y \\ \text{G. C. D.} &= a. \end{aligned}$$

---

2. Find the G. C. D. of $30c\,(x - y)$, $15mx - 15my$.

$$\begin{aligned} 30c\,(x - y) &= 3 \times 2 \times 5 \times c \times x - y \\ 15mx - 15my &= 3 \times 5 \times m \times x - y \\ \text{G. C. D.} &= 15\,(x - y) \text{ or, } 15x - 15y. \end{aligned}$$

---

3. Find the L. C. M. of a^3, $4ax$, $3a^2y$.

$$\begin{aligned} a^3 &= \underline{a \times a \times a} \\ 4ax &= \underline{2 \times 2} \times a \times \underline{x} \\ 3a^2y &= \underline{3} \times a \times a \times \underline{y} \\ \text{L. C. M.} &= a \times a \times a \times 2 \times 2 \times x \times y \times 3 \\ &= 12a^3xy. \end{aligned}$$

FRACTIONS.

$\frac{a}{x}$, $\frac{3ab}{2}$, $\frac{2}{bcd}$ are fractions such as we find in algebra.

$\frac{a}{x}$ is read "a divided by x" or "a over x".

$\frac{3ab}{2}$ is read "$3ab$ divided by 2" or "$3ab$ over 2".

Fractions in algebra are governed by the same principles as those in arithmetic :—

(*1*) *They may be reduced to lowest terms.*

Reduce $\frac{28a^2bx}{56ab^3x}$ to lowest terms.

Dividing both numerator and denominator by the G. C. D. $28abx$, we have $\frac{a}{2b^2}$.

(*2*) *They may be reduced to higher terms.*

Reduce $\frac{2b}{ax}$ to a fraction whose denominator is $8a^2bx$.

$8a^2bx \div ax = 8ab.$

$\frac{2b}{ax} = \frac{8ab \times 2b}{8ab \times ax} = \frac{16ab^2}{8a^2bx}$.

(*3*) *They may be reduced to a common denominator.*

Reduce $\frac{xy}{4}$ and $\frac{z}{2a}$ to a common denominator.

$$4 = 2 \times 2$$
$$2a = 2 \times a$$

L. C. D. of fractions $= 4a$.

$$\frac{xy}{4} = \frac{axy}{4a} + \frac{2z}{4a} = \frac{axy + 2z}{4a}$$
$$\frac{z}{2a} = \frac{2z}{4a}.$$

(*4*) *They may be added or subtracted.*

Add $\frac{a}{b}$ and $\frac{c}{x}$.

$$\frac{a}{b} + \frac{c}{x} = \frac{ax + bc}{bx}.$$

Notice that we simply reduced the fractions to L. C. D. and added the numerators.

From $\frac{a}{3}$ take $\frac{b}{4}$.

$$\frac{a}{3} - \frac{b}{4} = \frac{4a - 3b}{12}.$$

Notice that the L. C. D. is found and the numerators subtracted.

(*5*) ***They may be multiplied or divided.***

1. $\frac{a}{x} \times b = ?$

$$\frac{a}{x} \times b = \frac{ab}{x}.$$

2. $\frac{a}{x} \div b = ?$

$$\frac{a}{x} \div b = \frac{a}{x} \times \frac{1}{b} = \frac{a}{xb}.$$

3. $a \div \frac{x}{y} = ?$

$$a \div \frac{x}{y} = a \times \frac{y}{x} = \frac{ay}{x}.$$

EQUATIONS WITH ONE UNKNOWN NUMBER.

In arithmetic we learned that different forms may be given an equation without changing the equality. The same holds true with equations in algebra.

The equations may be:—

1. Turned about—See page 380.

$$5x - 10 = 6x - 14.$$

Turning the numbers about, we have—

$$6x - 14 = 5x - 10.$$

---

2. Transposed—

$$5x - 10 = 6x - 14.$$

Transposing so that similar terms are on the same side, we have—

$$6x - 5x = 14 - 10$$
$$x = 4.$$

---

3. Simplified—See page 380.

$$\frac{7x}{2} - \frac{2x - 3}{3} = 12.$$

Simplifying, by multiplying through by 6 to clear of fractions, we have—

$$21x - (4x - 6) = 72$$
$$21x - 4x + 6 = 72$$
$$21x - 4x = 66$$
$$17x = 66$$
$$x = 3\tfrac{15}{17}.$$

4. *Multiplied*—See page 380.

$$6x + 4 = 8x - 10.$$

Transposing, we have—

$$6x - 8x = -10 - 4$$
$$-2x = -14.$$

Multiplying through by -1, we have—

$$2x = 14$$
$$x = 7.$$

In actual work, the process of multiplying by -1 is not written. The signs are merely changed.

---

5. *Divided*—See page 381.

$$16x - 12 = 5x - 1$$

Transposing, $11x = 11$

Dividing through by 11, we have

$$x = 1.$$

PROBLEMS WITH ONE UNKNOWN NUMBER.

1. The sum of two numbers is 320 and the larger is 3 times the smaller. What are the two numbers?

Let x = smaller number
Then $3x$ = larger number
$4x$ = sum
$4x = 320$
$x = 80$ smaller number
$3x = 240$ larger number.

---

2. From four times a number I subtract $\frac{3}{5}$ of it plus 8, and have 15 left. What is the number?

Let x = number

Then $4x - \left(\frac{3x}{5} + 8\right) = 15$

Clearing of parenthesis, $4x - \frac{3x}{5} - 8 = 15.$

Clearing of fractions by multiplying through by 5, we have—

$$20x - 3x - 40 = 75$$
$$17x = 115$$
$$x = 6\tfrac{13}{17}$$

3. Bessie and Cecil have $480 in the savings bank. Bessie's share of it is 4 times Cecil's. How much does each have in the bank?

Let x = Cecil's share
Then $4x$ = Bessie's share

$$x + 4x = \$480$$
$$5x = \$480$$
$$x = \$96, \text{ or Cecil's}$$
$$4x = \$384, \text{ or Bessie's.}$$

---

4. The area of a rectangle is 98 sq. in. The longer side is twice the shorter. What is the length and width?

Let x = shorter side
Then $2x$ = longer side

$$2x \times x = 98 \text{ sq. in.}$$
$$2x^2 = 98 \text{ sq. in.}$$
$$x^2 = 49 \text{ sq. in.}$$
$$x = 7, \text{ the length in in. of shorter side.}$$
$$2x = 14, \text{ length in in. of longer side.}$$

---

5. Rose, Grace, and Lillie read 39 galleys of proof in 10 hours. Rose read one galley less than Grace, and Lillie one galley more. How many galleys did each read?

Let x = galleys Grace read.
Then $x - 1$ = galleys Rose read.
$x + 1$ = galleys Lillie read.

$$3x = 39 \text{ galleys.}$$
$$x = 13 \text{ galleys, what Grace read.}$$
$$x - 1 = 12 \text{ galleys, what Rose read.}$$
$$x + 1 = 14 \text{ galleys, what Lillie read.}$$

6. One number is five times another. Their difference is 64. What are the numbers?

Let x = the smaller.
Then $5x$ = the greater.

$4x$ = the difference.
$4x = 64$.
$x = 16$, or the smaller.
$5x = 80$, or the greater.

---

7. On being asked her age, a lady replied, that if from $\frac{1}{2}$ her age you took $\frac{1}{8}$ of her age, the remainder would be 15. What was her age?

Let x = her age.
Then $\frac{1}{2}x - \frac{1}{8}x = 15$ yr.
Clearing of fractions,
$4x - x = 120$ yr.
$3x = 120$ yr.
$x = 40$ yr., her age.

---

8. A book has a certain number of pages. Another has 27 more than 7 times as many. Both have 987 pages. How many pages in each book?

Let x = pages in smaller book.
Then $7x + 27$ = pages in larger.

$8x + 27 = 987$ pages.
$8x = 960$ pages.
$x = 120$ pages, No. in smaller book.
$7x + 27 = 867$ pages, No. in larger book.

PART TWELVE.

The Arithmetic Dictionary.

> *"Life, to be worthy of a rational being, must be always in progression; we must always purpose to do more or better than in time past."*
>
> —*Samuel Johnson.*

THE DULL PUPIL.

There is no subject of school study which so often brings down upon the pupil's head the term "blockhead," as Arithmetic. Often this is because the pupil does not grasp the subject readily. More often the cause is the inability to present the subject in a way that is clear to him. Do not lay all blame at the student's door. Remember that many of the world's most illustrious men did not reach high water mark in all of their school work.

Sir Walter Scott, loved all over the world for his writings, once had thundered in his ears by a Scotch schoolteacher, "Dunce you are, and dunce you will ever remain." Linnæus, the founder of modern botany, was another dunce at school. Goldsmith's schoolmistress said he was "impenetrably stupid." Henry Ward Beecher was classed with the dunces. Chatterton was sent home as an incorrigible dunce. Grant was a dull and awkward Ohio boy. Newton, profound mind in all scientific thought, was termed a dullard as a boy. Shakespeare's record at the Stratford Grammar School was a poor one. Edmund Burke, whose eloquence was to rouse all England, often wore a dunce cap.

And so, good teacher, the list might be continued indefinitely. Don't excuse it by saying these were exceptions, for those in your charge may be the geniuses of a new generation. So, though your patience be often tried, deal gently with them.

And to you, mothers of dull boys or girls, remember that through patience, and effort rightly directed, may come more than you had ever dreamed for ;— the earnest, untiring worker, the conservative business man who has everyone's confidence, the genius of letters, or the thoughtful scientist. The searchlight of biography reveals innumerable successes of children such as yours. — *Selected.*

THE ARITHMETIC DICTIONARY AND GLOSSARY OF BUSINESS TERMS.

Above par. Greater than face value. At a premium.

Abstract numbers are those whose unit has no name, as, 20, 4, 6.

Acceptance of a draft or bill of exchange is the promise of the drawee to pay it according to its terms.

Account. A written statement of debits and credits.

Accountant. One trained to keep accounts.

Account current is a written statement showing the debits and credits of business done between two parties.

Account purchase is a detailed statement by a commission merchant to his principal, showing cost of goods bought, and expense incurred in their purchase.

Account sales is a written statement of a consignee or commission merchant, of goods sold for a consignor. It includes the sales, expenses, charges, and net proceeds.

Accrued. Due and unpaid interest.

Accurate interest, or, **exact interest** is interest found on a principal for the exact time designated.

Acute angle is any angle of less than 90°.

Addends are the numbers to be added.

Addition is the process of finding the sum of given numbers.

Ad valorem duty is a tax assessed on imported merchandise at a certain per cent on its import value.

An **agent** is a person with discretionary power authorized to do business for another called the principal.

Aliquot parts are equal or exact parts. We speak of 12½, 6¼, 25, 16⅔, etc., as the aliquot parts of 100.

Alligation is the way of forming a certain mixture of two or more articles of different values.

(It is not treated in this HELP as it is not in use.)

Altitude of a figure or plane is the perpendicular distance from the base to the angle or side most distant.

Amount. (*a*) In interest, the amount is the sum of the principal and interest. (*b*) In general, it is the sum of two or more numbers.

Ampere. The unit which measures the strength of an electric current.

Analysis. The process of reasoning from a whole to its parts.

Angular or **circular measure** is that used in measuring circles, direction, longitude, etc.

Annual interest is simple interest on both principal and each year's interest from the date each simple interest is due until settlement.

Annuity. A sum paid periodically, usually once a year.

Antecedent. The first term in a ratio.

Apothecaries' weight is that used in retailing medicine or in preparing prescriptions.

Appraiser. One who sets a value or price upon goods, usually not his own, and when the value is in dispute.

Approximation. A result so near the correct one that it is satisfactory for the given purpose.

Arabic notation is the notation in common use. The characters are, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Arc. Any part of the circumference of a circle.

Are. A square Decameter.

Area or **surface** is shown by the number of square units in it.

Arithmetic is the science of numbers and their use.

Arithmetical progression is any series of numbers increasing or decreasing by a common difference.

Assessed value is the value put upon a thing by the assessor.

Assessor. The person who estimates the value of taxable property.

Assets are all the property belonging to an individual or corporation.

Assignee. A person to whom an assignment is made.

Assignment is a conditional transfer of property for safe keeping.

Assignor. The person who makes an assignment.

At a discount. Below face value. Below par.

At a premium. Above face value. Above par.

Audit. To examine the books, vouchers, etc., of a business.

Auditor. One appointed to examine and settle accounts.

Average. The result of adding several quantities and dividing by the number of quantities.

Average of payments. See equation of payments.

Average term of credit is the time to elapse before payment of several amounts due at different times can be made at once without loss to either party.

Avoirdupois weight is that used in current business where troy or apothecaries' weights are not used.

Axiom. A self-evident truth.

Balance. The difference between two accounts. The amount still to be paid.

Balancing books. Making a balance sheet from the ledger.

Balance sheet. A condensed statement showing the condition of business.

Bale. A package of goods, or produce.

Bank. An institution authorized by law to loan money on notes and other written obligations, or to issue notes or bills to circulate as money, and to receive deposits.

Bank book. The pass book of a depositor.

Bank discount is interest reckoned for the term of discount on the value at maturity.

Banker. One who does a banking business.

Bankrupt. (*a*) Unable to pay debts due. (*b*) A person unable to pay his debts.

Bank stock. Shares in the capital stock of a bank.

Base. (*a*) The line or surface of a figure on which it is supposed to rest. (*b*) In percentage it is the number of which a per cent is considered.

Below par. Less than face value. At a discount.

Bill. A written statement of goods sold, or services rendered. It usually gives the names of the parties concerned, the name and cost of each item, and all dates of the transactions.

Bill head. A printed form on which bills are written.

Bill of entry. A bill of goods entered at the custom house.

Bill of exchange is the same as a *draft.* It is a written order directing one person to pay a certain sum of money to another.

Bill of lading. (*a*) A written statement of goods shipped with terms of delivery. (*b*) A receipt from a transportation company for goods to be carried.

Bill of sale. A written contract for the sale of goods.

Bill of sight. A temporary form of entering goods at the custom house for examination.

Bills payable. Moneys to be paid out.

Bills receivable. Moneys due.

Bisect. To divide in two equal parts.

Bond. An interest bearing obligation of a public or private corporation guaranteeing the holder a specified sum of money on a specified date, and insuring a specified interest payable at stated intervals.

Bonded warehouse. A government building where goods, on which duty is to be paid, may be stored, by giving a bond for the payment of the duty.

Brace. A pair of double brackets, as, $\{ \}$.

Brackets. A pair of lines like these [ ]. They mean the same as a parenthesis and are used outside of the parenthesis.

Breakage. An allowance for broken bottles of imported liquids.

Broker. One who buys and sells goods for a principal, and in the principal's name, usually.

Brokerage. The fee or pay given a broker.

Bull. The name given to a person on the stock exchange who tries to raise the price of stocks that he may sell at a gain.

Bullion. Uncoined gold or silver.

Calendar or civil year. From midnight of December 31 to midnight of December 31, three hundred sixty-five days later.

Cancel. To strike out common factors in the terms of a fraction.

Cancellation. The striking out of common factors in both dividend and divisor.

Capacity. In arithmetic it means the number of units of volume which a thing will hold.

Capital. Money or property invested in a business enterprise.

Capitation tax. See poll tax.

Cash book. A book for the entry of money expended and money received.

Cashier. One having charge of paying out and receiving money.

Centigrade. A thermometer whose boiling point is 100° and freezing point zero. The space between is divided into 100 equal parts, called degrees.

Certificates of stock. Written statements from corporation officers stating that the holder owns the number of shares of stock specified therein.

Certified check. A check having indorsed upon it the word, "Certified," name of bank, date, and signature of the cashier of the bank. This makes the bank responsible for its payment.

Chain. A set of 100 connected links, each 7.92 inches long, used by land surveyors. It is 66 ft. long.

Check. An order on a bank or place of deposit asking it to pay the sum named in the order to the person in whose favor it is drawn, or to his order, or bearer, on demand.

Check book. A book of printed blank checks.

Check clerk. A clerk who examines the accounts of other clerks.

Chord. A straight line from one point in the circumference of a circle to another point in the circumference but not passing through the center of the circle.

Circle. A plane figure bounded by a curved line, every point of which is equidistant from a point within, called the center.

Circular measure. See ANGULAR MEASURE.

Circulating decimal. One in which certain figures continue to be repeated indefinitely.

Circumference. The boundary of a circle.

Circumscribed. Used to designate a figure drawn to exactly include some other.

Clearing house. An organized system by which the bankers of a city at some central establishment collect and interchange their bills, checks, and other obligations. Clearing houses are found in all important cities, and greatly reduce the labor and cash balances required in settlements.

Closing an account. Balancing the debtor and creditor sides by placing the difference on the smaller side, naming it "Balance" or "Profit and Loss" and drawing lines just below it.

Coefficient. An algebraic term used to designate a factor placed to the left of the literal part of an expression, thus in $12a$, 12 is the coefficient of a.

Collateral. Pledges for security.

Collector. One whose duty it is to collect moneys.

Commerce. The business of exchanging one commodity for another.

Commercial discount. A reduction from the list price or amount of a bill of goods or from a debt.

Commercial paper. Any paper in which a money consideration is named, as, notes, drafts, bills of exchange, etc.

Commission. Compensation allowed an agent for services.

Common denominator. One common to two or more fractions.

Common divisor of two or more numbers. A number that exactly divides each of them without a remainder.

Common factor. One found in all the numbers considered, as, 2 is a common factor of 6, 8, and 14.

Common fractions are fractions expressed by placing one of the numbers of the fraction above a line and the other below it.

Common multiple of two or more numbers. A number that exactly contains each of them.

Complex fraction. One whose numerator or denominator contains a simple fraction.

Composite number. One that contains other factors than itself and unity.

Compound interest. Interest on the principal and on all interest from the time it is due.

Compound proportion. A proportion where one or both ratios are compound.

Compound quantities or **numbers.** Quantities expressed in two or more units of the same kind of measure, as, 2 hours and 6 minutes.

Compound repetend. One that contains more than one figure, as, 286286, etc.

Concrete number. One whose unit is named, as, 3 feet.

Cone. A solid whose generation may be illustrated by rotating a right triangle on one of its sides for an axis.

Consequent. The second term of a ratio.

Consign. To send goods to a factor or agent for sale.

Consignee. The one to whom goods are sent.

Consignment. Goods sent.

Consignor. The person or firm sending goods.

Continued product. The result of two or more multiplications.

Contract. An agreement between two or more persons or parties.

Contributions. Joint payment of money to an undertaking.

Copartner. A person engaged in a partnership.

Cord. Wood or stone occupying 128 cubic feet of space.

Cord foot. A pile 1 ft. long, 4 ft. wide, and 4 ft. high.

Corporation. An organization of persons incorporated legally, and thereby authorized to do business as one person.

Cost. The price paid for goods delivered.

Counting room. A merchant's business office.

Couplet. Both terms of a ratio considered together.

Coupon. A small interest note attached to the margin of a bond.

Coupon bond. One payable to bearer, and so transferable without indorsement.

Creditor. One to whom money is due.

Cube. (*a*) A solid whose edges are equal and whose six faces are equal squares. (*b*) The third power of a number. (*c*) To raise to the third power.

Cube root. One of the three equal factors of a number.

Cubic measure. That used in measuring solids as to their contents.

Currency. The paper money and coin used as the circulating medium of a country.

Customer. A regular buyer of goods at a place.

Custom house. A government office for the collection of duties, and for entering and clearing ships.

Customs. Taxes on goods imported or exported.

Cylinder. A solid generated by rotating a parallelogram on one of its sides for an axis.

Days of grace. An allowance of three days after a note becomes due in which to pay it.

Debt. That which is owed to another.

Debtor. The person who owes money to another.

Decagon. A figure having ten equal sides.

Decimal. Pertaining to ten.

Decimal fraction. One whose denominator is ten or some power of ten. Usually fractions written with the decimal point.

Decimal point. A period placed after the units' figure to mark its place.

Decimal system. Any system of writing numbers in which the scale is ten. That is, where ten units of one order make one of the next order.

Denominate number. One whose unit is named, as, 4 hours, 34¢.

Denominator. The namer. Usually the number of a fraction which shows the number of parts into which the unit is divided.

Diameter. (*a*) The shortest distance through the center of a body. (*b*) A line through the center of a circle terminating in the circumference.

Difference. The number which added to the smaller produces the greater.

Digit. One of the symbols, 1, 2, 3, 0, 9, etc.

Direct tax. One levied directly on persons or property.

Discount. A reduction from the usual or normal rate or price.

Dividend. (*a*) The number to be divided. (*b*) A profit to be distributed among stockholders.

Division. The process of finding how many times one number contains another of the same kind.

Divisor. The number that shows into how many parts the dividend is to be divided, or one of the parts.

Domestic exchange. Exchange between different sections of the same state or country.

Dower. The widow's life estate in her husband's real property.

Draft. A written order of one person or firm to another to pay the sum specified therein to the party named therein; a bill of exchange.

Drawer. See Maker.

Dry measure. That sometimes used in measuring grain, fruit, vegetables, etc.

Due bill. A written acknowledgment of debt.

Duties. Indirect taxes levied by the government on imported goods.

Dyne. The force required to add one centimeter in one second to the velocity of one gram of matter.

East longitude. Longitude east of the prime meridian, or, from 0° east to 180°.

Endorsement. See Indorsement.

Endorser. See Indorser.

Equal. Having the same value.

Equation. The statement of equality between two ratios.

Equation of payments. The way of finding the date on which two or more debts, due at different times, may be paid without loss to debtor or creditor. Average of payments.

Equilateral. Having all sides equal.

Equilateral triangle. One whose sides are equal.

Erg. The work done in overcoming a dyne of force in a distance of one centimeter.

Even numbers. Those exactly divisible by 2.

Evolution. The process of finding one of the equal factors of a number.

Exchange. The system of paying money in distant places without transmitting the actual money.

Exchange broker. One engaged in negotiating foreign and domestic exchange.

Excise. A tax on articles produced at home.

Exponent. A small figure to the right and above a number, to show how many times it is to be used as a factor.

Extremes. The first and last terms of a proportion, or series.

Face. The sum named in the paper or note; face value.

Face value. Par value. Value named in the paper.

Factor. (*a*) An agent who buys and sells in his own name. (*b*) An exact divisor of a number.

Factoring. Resolving composite numbers into their factors.

Fahrenheit. The name of a thermometer whose boiling point is 212° and freezing point 32°. The space between is divided into 180 equal parts called degrees.

Federal currency. The current coins of the U. S.

Figures. (*a*) Symbols used to represent numbers. (*b*) Diagrams to represent geometrical forms.

Fire insurance. Insurance against loss by fire.

Fiscal. Pertaining to money.

F. o. b. Free on board.

Foot pound. The work done in overcoming a pound of force in a distance of one foot.

Foreign exchange. Exchange dealing with drafts or bills of exchange drawn in one country and payable in another.

Fraction. Means broken, and is a part of a unit or number of parts of a unit.

Fractional number. A collection of fractional units, as, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{3}{18}$.

Fractional unit. One of the equal parts into which a number is divided, as, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{20}$.

Freight. (*a*) Goods being moved from one place to another. (*b*) Charge for moving or carrying goods.

Frustum of a cone. The part of a cone remaining when the top has been removed by cutting across the cone parallel to the base.

Funded. Turned into a permanent interest bearing loan.

Funds. The supply of money or capital.

Gain. The selling price minus the cost. See PROFIT.

Gauging. Measuring the capacity of casks, etc.

Government bonds. Bonds issued by the U. S. government.

Gram. (*a*) A measure of weight. (*b*) In mechanics it is the force needed to add 980 centimeters to the velocity of one gram of matter in one second.

Gram centimeter. The work done in overcoming a gram of force in a distance of one centimeter.

Greatest common divisor of two or more numbers. The greatest number that will exactly divide each of them.

Greatest common measure of two or more numbers. The greatest number that will exactly divide each of them.

Gross weight. Weight before any deductions are made.

Hexagon. A plane figure having six equal sides.

Honor. To accept and pay when due.

Horse power. The power which will do 550 foot pounds of work in a second.

Hypotenuse. The longest side of a right triangle.

Import. To bring in from other countries.

Importer. One who buys goods from abroad.

Improper fraction. One whose numerator is not less than the denominator.

Indemnify. To reimburse; to pay in case of loss.

Index. A small figure written above and to the left of the radical sign showing the root to be found.

Indirect tax. One levied on merchandise before it reaches the consumer, as licenses, duties, etc.

Indorsement. (*a*) The signature; (*b*) the partial payment written on the back of commercial paper.

Indorser. (*a*) One who guarantees payment. (*b*) The one who signs on the back of a paper, thereby guaranteeing its payment.

Insolvency. Inability to pay debts when due.

Instalment. A part payment.

Insurance. A written guarantee of indemnity in case of loss or damage.

Insurance premium. The sum paid for the insurance.

Insured. The one secured against loss.

Insurer. The one who takes the risk. The underwriter.

Integer. A whole number.

Integral number. A whole number, as, 4, 1.

Interest. That which is paid for the use of money. It is usually money.

Internal revenue. Taxes collected in the form of internal duties.

International date line. The imaginary line agreed upon by civilized nations as the place where one day ends and another begins.

Investment. The laying out of money in the purchase of goods or property.

Invoice. A list sent to a purchaser or consignee, of articles bought or to be bought, with prices.

Involution. The process of raising a number to a certain power.

Isosceles triangle. One having two sides that are equal.

Joint and several note. One in which the makers are each liable for the full amount of the note as well as being jointly liable.

Joint note. One in which the makers are jointly liable for the amount.

Joint stock. Stock held jointly.

Latitude of a place. Distance north or south of the equator.

Leakage. An allowance for loss of imported liquids in transportation.

Leap year. One of 366 days. It occurs every 4 years and every year exactly divisible by 400.

Leats common denominator. The least denominator that can be common to the fractions considered.

Least common multiple of two or more numbers. The least number that will exactly contain each of them.

Ledger. A book where summaries of accounts are kept.

Legal rate. The rate by law.

Letter of credit. A letter addressed to some one at a distance, asking him to pay the sum named in it to the person specified, or to pay it as such person requests. The person addressed is reimbursed by drawing on the party writing the letter, or debiting it on the account between the parties.

Liabilities. The unsettled debts and obligations of an organization.

Like numbers. Numbers of the same unit value.

Linear measure. That used in measuring lines or distance.

Liquid measure. That used in measuring liquids ordinarily.

List price. The price named in the manufacturer's or dealer's list.

Liter. The unit of capacity in the metric system;—a cubic decimeter.

Loan. Money to be returned after temporary use.

Logarithm. The exponent of the power to which a number, called the base, must be raised to produce a number.

Long division. Division in which the processes are written.

Longitude of a place. Its distance east or west of the prime or standard meridian. The meridian of Greenwich, England, is generally considered the prime meridian.

Loss. The cost price minus the selling price.

Magnitude is anything. It answers the question "How much."

Maker, drawer, or **promisor,** of a note. The one who signs it.

Mantissa. Something left over.

Market value. The value that can be received in the open market.

Maturity of a note. The date on which it legally becomes due.

Mean proportional. A term used as the antecedent in one ratio and as the consequent in the other ratio of a proportion.

Means. The second and third terms of a proportion.

Measure. (*a*) A unit or standard of comparison. (*b*) To determine quantity by use of a unit.

Mensuration. The measuring of lines, surfaces, and solids.

Merchandise. Whatever is bought and sold in trade.

Meridian. A semi-circumference, from pole to pole.

Meter. The standard unit of length in the metric system.

Micron. The one millionth part of a meter.

Mint. A place where money is coined.

Minuend. The number from which another is taken.

Mixed number. The sum of an integer and a fraction, as, $4\frac{1}{2}$.

Monetary. Pertaining to money.

Money. Coin. Currency lawful in place of coin, as, bank notes.

Mortgage. (*a*) To convey property as security for a debt, the conveyance to be void on payment of the debt. (*b*) The paper containing the mortgage contract.

Multiple. The result of multiplying integers by integers.

Multiplicand. The number to be multiplied.

Multiplication. A short process of finding the sum when a number is taken a certain number of times as an addend.

Multiplier. The number which shows how many times the multiplicand is taken.

National banks. Those established under provisions or acts of Congress.

Negotiable. Capable of being transferred by indorsement.

Net. Without further discount. It may mean after all expenses are taken out.

Net price. The price after discounts or expenses have been taken out.

Net proceeds. What remains from a sale when all expenses and discounts are paid.

Non negotiable. Not transferable by indorsement.

Notation. The writing of numbers.

Note. A written or printed paper promising payment of an acknowledged debt at a certain time.

Number. (*a*) A collection of units of the same kind. (*b*) To name by numbers.

Numeration. The naming of numbers.

Numerator. The numberer. The part of a fraction which shows the number of parts considered.

Obligation. Liability. Debt.

Odd numbers. Those not exactly divisible by two.

Ohm. The unit of resistance in measuring electricity.

Open account. An unsettled account.

Order of units. A name designating the number in a group, as, *tens, hundreds, etc.*

Outstanding debts. Debts unpaid.

Panic. A financial crisis.

Par. Face. Face value.

Parallel lines. Lines that will not meet however far extended.

Parallelogram. A quadrilateral whose opposite sides are equal.

Parallelopiped. A prism whose bases are parallelograms.

Parenthesis. A pair of curved lines inclosing two or more quantities, ( ).

Partial payment. A payment made to discharge part of an obligation.

Partial product. The result of multiplying by part of the multiplier.

Partitive proportion. The separation of a number into parts bearing a certain relation to each other.

Partnership. A business association of two or more persons.

Par value. Face value.

Pay. To give an equivalent.

Payee. (*a*) The one to whose order a paper is payable. (*b*) One to whom money is paid.

Payer. One who pays.

Pendulum. A body suspended from a fixed point and moving freely about that point as a center.

Per capita. By the head.

Per cent. By the hundred.

Percentage. (*a*) The arithmetic of per cent. (*b*) The product of the base by the number representing the per cent.

Percentage of a number. One or more hundredths of a number.

Perimeter. The distance around a plane figure.

Period. (*a*) A group of three figures. (*b*) A dot.

Personal property. Movable property, as cattle, furniture, etc.

Policy. The written contract between the insurer and person in whose favor insurance is given.

Poll Tax. One levied on every male citizen over 21 yr. and not exempt by law. A capitation tax.

Port of entry. A port where a custom house is established.

Pound. (*a*) The force required to add 32.16 feet in one second to the velocity of one pound of matter. (*b*) Sixteen ounces. (*c*) Twelve ounces troy or apothecaries'.

Poundal. The force required to add one foot in one second to the velocity of one pound of matter.

Power. The product of two or more equal factors.

Preferred stocks. Those on which a dividend preference is given.

Premium. (*a*) Excess of market value over par value. (*b*) Money paid for insurance.

Present value of a debt, due in the future. A sum which, if put at interest, would just amount to the debt when it becomes due.

Present worth. The present value of a debt due at some future date. Present value.

Price. Current value. Sum demanded.

Prime factor. A prime number that will exactly divide the given number.

Prime meridian. The principal or base meridian.

Prime number. One only divisible by itself and unity without a remainder.

Principal. (*a*) Money drawing interest. (*b*) The one for whom an agent, factor, or broker works.

Principle. A truth derived from the properties and relations of numbers or quantity.

Prism. A solid whose lateral faces are parallelograms, and whose bases are similar, equal and parallel plane figures.

Problem. A question to be solved.

Product. The result of multiplication.

Profit. The excess of selling price over cost.

Profit and loss. A class of percentage problems in which loss or profit or both are the elements considered.

Proof. Evidence of accuracy.

Proper fraction. One whose numerator is less than its denominator.

Property insurance. Insurance against loss to property.

Property tax. One levied on real or personal property.

Proportion. The statement of equality between two ratios.

Pro rata. A proportional distribution.

Protest. A formal declaration of non-payment of commercial paper.

Protractor. An instrument for making circular measurements.

Pure decimal. One having no integral unit attached and no common fraction, as, .24, .147.

Pyramid. A solid whose base is a polygon, and two or more of whose faces are triangular.

Pythagorean theorem. The one first proved by Pythagoras, that the square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides.

Quadrilateral. A figure having four sides.

Quantity. Answer to the query "How much."

Quotient. The result of division.

Radius. (*a*) Distance from the center of a circle to its circumference. (*b*) A line from the center of a circle to its circumference.

Range. A row of townships extending north or south from the base line.

Rate. In percentage, the ratio of the base expressed in hundredths, as, 6% is the rate.

Rate per cent. The number of hundredths in the rate, as 6 is the rate per cent, when we speak of $\frac{6}{100}$ of a thing.

Ratio. The relative magnitude of two numbers or quantities of the same kind.

Real estate. See REAL PROPERTY.

Real property. That which is fixed and immovable, as houses, lands, etc.

Rèaumur thermometer. One that has the boiling point 80° above the freezing point, which is zero. The space between is divided into 80 equal parts called degrees.

Receipt. (*a*) A written acknowledgment of payment. (*b*) The receiving of a thing.

Reciprocal of a number. One divided by that number.

Rectangle. A quadrilateral all of whose angles are right angles.

Reduction. The process of changing the unit in which a quantity is expressed without changing the value of the quantity.

Reduction ascending. Changing units of one order to those of a higher order, as, inches to feet.

Reduction descending. Changing units of one order to those of a lower order.

Reduction of denominate numbers. Changing them from one denomination to another, as, changing yards to rods.

Refund. To repay. To pay back.

Registered bond. One payable to order, interest on which is only paid to the person whose name is registered on the books of the corporation.

Reinsurance. Where an insurance company lets part of its risk to another company.

Remainder. Same as difference.

Repetend. A figure or set of figures that continue to repeat indefinitely.

Resources or **assests** of an organization. Its property, and debts due it.

Retailer. One who sells goods in small quantities.

Revenue. Income. Return.

Rhomboid. A parallelogram whose adjoining sides are not equal and which contains no right angle.

Rhombus. An oblique angled equilateral parallelogram. A rhomb.

Risk. The total liability of the insurer in any event.

Roman notation. The notation using Roman letters for number symbols.

Root. One of the equal factors of a number.

Scale. (*a*) An instrument of measurement. (*b*) A system of fixed proportion, as the scale of 1 in. to the foot, etc.

Scalene triangle. One having no two sides equal.

Secant. A straight line that intersects a curve or figure.

Section. A portion. Separate part.

Sector. That part of a circle bounded by two radii and the arc that subtends them.

Segment. That portion of a circle included between a chord and its arc.

Selling price. The gross amount received for goods.

Series. Any succession of terms whose values may be found by a fixed law.

Share. (*a*) A portion. (*b*) One of the equal parts into which capital stock is divided.

Share of stock. One of the equal parts into which the capital of a company is divided.

Short division. Division where the operation is performed mentally.

Sight draft. A draft payable as soon as accepted.

Silent partner. See SPECIAL PARTNER.

Similar fractions. Those having a common denominator.

Similar numbers. Numbers representing units of the same name, as, $4, $6; 3 men, 4 men.

Similar triangles. Triangles of the same shape.

Simple fractions. Those whose numerators and denominators are integral numbers.

Simple interest. Interest on the principal only.

Simple repetend. One with but one figure, which repeats, as, 3333, etc.

Sinking fund. A fund set apart for the redemption of debt.

Solar day. The time between two successive passages of the sun over the same meridian.

Solid. A magnitude having length, breadth, and thickness.

Solution. The process by which a result is obtained.

Solvency. Ability to pay all claims.

Special partner or **silent partner.** One who is not known actively in the business.

Specific duty. A tax assessed on imported goods as so much per pound, yard, ton, etc.

Specific gravity of a substance. The ratio of a definite volume of it to an equal volume of the standard, usually water.

Sphere. A solid bounded by a curved surface, all points of which are equally distant from the center.

Square. (*a*) An equilateral rectangle. (*b*) To raise a number to the second power. (*c*) The second power of a number.

Square measure. That used in measuring surface.

Square root of a number. One of its two equal factors.

Standard time. A system of uniform time.

Stere. A cubic meter.

Stock. Capital invested in business.

Stock broker. One whose business is to deal in stocks, bonds, etc.

Stock exchange. An unincorporated association of brokers for the purpose of dealing in stocks, bonds, etc.

Submultiple of a number. An exact divisor of it.

Subtraction. The process of finding the difference between two numbers.

Subtrahend. The number taken from the minuend.

Sum. The result of addition.

Surd. An indicated root whose value cannot be exactly expressed in figures.

Surety. Security. Bondsman.

Surface. A magnitude having length and breadth only. See AREA.

Surveyor's measure. Land measures used by surveyors.

Tare. An allowance for the weight of boxes, barrels, etc., in which goods are imported.

Tariff. A list of duties imposed on goods passing from one country to another.

Tax. A contribution levied on persons, property, incomes, or business, for public purposes.

Teller. The officer in a bank who receives and pays money.

Tender. To offer or present for acceptance.

Term. Any number or quantity with the plus or minus sign.

Term of discount. The time from date of discount to the date when the debt becomes legally due.

Theorem. A statement of a principle to be demonstrated.

Thermometer. An instrument for measuring temperature.

Time. The common idea of continuous existence.

Time draft. One maturing at a future specified time.

Time to run. The years, months, and days before commercial paper is due.

Tonnage. (*a*) A tax levied on vessels, exclusive of cargoes, for the privilege of entering a port. (*b*) The weight of goods carried in a ship.

Trade discount. A discount allowed from the face of a bill or list price.

Transact. To perform business.

Trapezoid. A quadrilateral with two equal sides.

Treasury. A place where public money is kept.

Treasury notes. Notes issued by the government, and received in payment for all dues, except duties on imports.

Triangle. A plane figure of three sides.

Troy weight. That used for weighing precious metals.

True discount. The difference between a debt at maturity and its present value.

Trustee. One who is entrusted with another's property.

Ultimo or **ult.** Last month.

Underwriter. An insurer.

Unit. (*a*) Any standard of measurement. (*b*) A single thing.

United States money. The legal money of the United States.

Unit of a fraction. One, or the thing that is divided into parts.

Units. (*a*) More than one unit. (*b*) The standards by which comparisons are made.

Unity. One.

Usage of trade. Custom.

Usury. Exorbitant interest.

Valid. Having legal force. Binding.

Value. Estimated worth. Amount which can be had in exchange for a thing.

Verify. To establish the truth of a statement by trial. Prove.

Vinculum. A horizontal line placed just below, or just above, two or more quantities, to show that they are to be considered as one, as, $4 + \underline{2 \times 3} - 4 = 6$.

Void. Having no legal or binding force.

Volt. The unit of electromotive force.

Volume of a solid. The number of cubical units it contains.

Wages. Hire. Reward. Salary.

Watt. The power which will do ten million ergs of work in a second.

West longitude. Longitude west of the prime meridian from 0° to 180°.

X, y, and **z.** These are used to designate unknown quantities. Their use is algebraic.

Yard. Three feet.

Yard stick. A stick exactly a yard in length.

Year. Three hundred sixty-five days.

Yield. Produce. Render.

Zero. A cipher, 0. Naught. Nothing.

GENERAL INDEX.

PAGE

Above par 364
Abstract number 477
Abstract ratio 384
Acceptance of a bill 340
Accepted, bill of exchange 340
Account, cash 220
    purchase 221
    sales 221
Accountant's proof of addition 68
Accumulations of compound interest 333
Accuracy in addition 59
Acute angle 244
    triangle 442
Addends 59
Addition 58
    accountant's proof of 68
    accuracy in 59
    algebra, in 460
    be correct in 58
    decimals, of 199
    defined 59
    denominate quantities, of 248
    dollars and cents, of 182
    fractions, of 161
    getting fractions ready for 159
    large numbers, of 65
    mixed numbers, of 163
    problems in, of fractions 163
    sign of 55
    simple decimals, of 187
Ad valorem duties 354
Agent 303
Algebra 455
    addition in 460
    division in 466
    equations in 471
    expressions in 459
    fractions in 469
    multiplication in 464
    principles of 465
    subtraction in 462
Altitude of a triangle 443
A. M. — forenoon 43
Amount in interest 310
And, use of 31, 195
Angle 243
    acute 244
    obtuse 244
    right 244

PAGE

Angles and lines 437
Angular measure 243
Annexing 0's to a decimal 198
Annual interest 328
Annuities 361
    certain 361
    contingent 361
    final value of 361
    in arrears 361
    in reversion 361
    perpetual 361
    policies of 361
    present value of 361
    problems in 362
    table of 362
Antecedent, the 383
Apothecaries' measure 239
    weight 240
Apothem of a polygon 447
Approximate roots 406
April, derivation of 241
Aptness of pupils 12
Arc 243
Area, of a circle 449
    illustrated 50
    parallelogram, of a 441
    rectangle, of a 440
    states, of 86
    trapezoid, of a 441
    triangles, of 442
Arithmetic, mental 420
    practical 420
    written 436
Ascending reduction 226
Assessment of shares of stock 365
Assessor 349
    procedure of 351
Assured, the 356
Assurer, the 356
At a discount, draft 343
    stock 364
At a premium, draft 343
    stock 364
At par, draft 343
August, derivation of 242
Austrian methods 79, 209
Average 398
    of payments 400
    problems in 399, 401
    time 400

PAGE
Avoirdupois weight 239
Axioms 386

Bain on definitions 130
Bank Bills 181
Bank Discount 334
Bank Draft Forms 341
Bank Drafts 340
Banker's method, of adding 67
  finding interest 314
Bankrupt 395
Barn, estimate of hay in 281
Base, to find, when rate and percentage are given 291
Base boards, allowance for in plastering 275
Base Line 230
Bear, a 376
Below par, stock 364
Beneficiary, in Insurance 376
Bills 218
  receipted 219
Bills of Exchange, copies of 344
  sending 339
Binomial 459
Blank Indorsement 320
Board foot 262
Bond 365
  coupon 365
  discount on 367
  premium on 367
  problems in 372
  quotations 372
  registered 366
Bonds and Stocks 363
  difference between 366
Borrowing in subtraction 77
Braces 58
Brackets 58
Breakage 354
Brick and stone 268
Broken line, a 438
Broker, a 303
  stock 364
Brokerage, stock and bond 374
Bull, a 376
Bunch of shingles, a 270
Bushel, a 49
  Winchester 237
Business month 241
Buyer three 376
Buying long 375
Buying problems 308

Cæsar, month named after 242
Calendar, the 48
  Gregorian 241
  Julian 241
Canada, express orders to 347

PAGE
Cancellation 144
  examples in 144
  in proportion 392
  problems in 146
Cancelling like factors 144
Capacity measures 237
Capacity of tanks, finding the 280
Capitation tax 349
Carat, a 257
Carlisle table 368
Carpeting 275
Carrying, in addition 63
  in multiplication 94
  later method 64
Cash, stock sold 376
Cash account 220
Casting out the nines 68
Cent, derivation of 185
Central time 253
Certain annuities 361
Certificate of stock 364
Chain, Gunter's 227
Changes in equations 379
Changing fractions to higher terms. 157
Changing, improper fractions to whole or mixed numbers 155
  mixed numbers to improper fractions 156
Character of early work 13
Check plan in division of decimals .. 207
Child tells stories 22, 23
Chord, a 448
Circle, a 243, 448
  arc of 448
  area of 449
Circles 243, 448
Circular measures 243
  table of 244
Circulates 214
Circulating decimals 213
  how indicated 214
Circumference, a 243, 448
  to find 448
Circus story 14
City lots 235
Civil engineers, measure used by ... 227
Civil service method of adding 66
Classes of percentage problems 289
Clearing of fractions in division 171
Coefficient 458
Coinage, history of our 184
Coins of U. S 181
  values of foreign 246
Collector, a 303
  of taxes 349
Colonial money, kinds used 184
Colored blocks 23
Colored objects 17
Colored pages, use of 18

PAGE
Combinations, teaching the......... 18
  in subtraction.................... 74
Comma, use of........................ 57
Commercial, discount............... 306
  draft............................ 342
Commission......................... 302
Commission merchant................ 302
Committee of Twelve, report of..... 224
Common, denominator in decimal fractions................. 197, 198
  Divisor, Greatest................ 135
  factors.......................... 138
  fractions........................ 149
    reduced to decimals............ 213
    to write as per cent........... 288
  interest method.................. 310
  Multiple, Least.................. 140
  signs............................. 55
  stock............................ 371
  way of finding time in interest.......................... 314
  year............................. 241
Comparison, in primary arithmetic. 40
  of check and ordinary plans in division of decimals......... 208
  of solids........................ 454
  of surfaces...................... 451
  of weights.................. 240, 431
Compass............................. 44
Complete statements in giving answers............................. 18
Complex decimals................... 216
  fractions........................ 171
Composite factor................... 132
  number........................... 131
Compound, denominate quantities. 226
  interest......................... 330
    table of....................... 333
  partnership...................... 395
Computer's Method in Subtraction. 79
Concrete ratio..................... 384
Cones.............................. 452
Congress gallon.................... 289
Consequent, the.................... 382
Contingent annuities............... 361
Copies of bills of exchange........ 344
Cord, a............................ 277
Cord foot.......................... 277
Corn, estimate of bu. in a crib.... 282
Corner, a.......................... 376
  in land survey................... 232
  stones, in land survey........... 233
Corporation, a..................... 364
  assessment on shares............. 365
  directors of..................... 365
  gross earnings of................ 365
  net earnings of.................. 365
  scrip dividend in................ 371
  shareholders in.................. 365
Correction lines................... 232
Cost marks......................... 217
  methods.......................... 258
Counting............................ 14
  by ones........................... 16
  in addition....................... 62
  to 50............................. 16
Country schools, practical arithmetic in.......................... 294
Couplet, in ratio.................. 383
Coupon bond........................ 365
Coupons, interest on............... 365
Creamery book, page from........... 259
Credit, letters of.................. 345
  term of.......................... 400
Creditor, definition............... 218
Crib, estimate of corn in.......... 282
Cubes, geometrically............... 416
  of numbers....................... 412
Cubic measure table................ 236
  meter............................ 427
  unit, a.......................... 236
Curved line, a..................... 438
Custom house, a.................... 354
Customs............................ 353
Cylinder, the...................... 453

Date, equated...................... 400
  Line, International.............. 265
  of discount...................... 335
  of maturity...................... 335
David Page......................... 420
Day................................. 43
  where it begins.................. 254
Days of grace...................... 321
  of week........................... 49
Debtor, defined.................... 218
Decagon, a regular................. 447
December, derivation of............ 243
Decimal point, the............. 57, 216
Decimals........................... 185
  addition of...................... 199
  annexing 0's to right of......... 198
  check plan in division of........ 207
  circulating...................... 218
  complex.......................... 216
  defined.......................... 216
  dividing by 10 or powers of 10... 211
  division of...................... 204
  division of, by Austrian plan.... 209
  division of, by multiples of 10... 211
  exercises in..................... 196
  formally......................... 190
  making divisor a whole number in.............................. 206
  mixed............................ 216
  multiplication of................ 200
  names of places.................. 198
  orders of........................ 198

PAGE
Decimals—*Continued.*
  places equal in dividend and divisor ... 206
  problems in ... 214
  reading ... 216
  recurring ... 214
  reduced to, common denominator ... 197
    lowest terms ... 197
    common fractions ... 213
  reducing mixed numbers to ... 214
  remainder in division of ... 210
  repetends in ... 214
  short cuts in division of ... 212
  short summary of ... 216
  simple work in ... 186
  square root of ... 408
  subtraction of ... 199
  whole numbers with ... 194
  writing ... 216
Denominate fractions ... 251
  numbers ... 225
  quantities, compound ... 226
    addition of ... 248
    multiplication of ... 250
    simple ... 226
Derivation of, months ... 241
  names of U. S. coins ... 185
Descending reduction ... 226
Developing idea of percentage ... 285
Devices, in addition ... 61
Diagram of gov't. land survey ... 231
Diameter, of circle ... 243
Difference defined ... 76
Different forms of equal fractions .. 156
Dime, derivation of ... 185
Directors of corporation ... 365
Direct Taxes ... 348
  problems in ... 351
Discount, bank ... 334
  date of ... 335
  on bonds ... 367
  term of ... 335
  trade ... 306
Disks, teaching fractions with ... 151
Dishonored bill ... 340
Disraeli, Benjamin ... 53
Dissimilar terms ... 458
Dividend and divisor having equal number of decimal places ... 206
Dividend, the ... 105
Dividends, insurance ... 357
  partial ... 111
  scrip ... 371
  stock ... 365
Dividing dollars and cents ... 184
  equations ... 381, 472
Division ... 103
  by measurement ... 24, 104

PAGE
  by partition ... 24, 104
  clearing of fractions in ... 171
  drills in ... 107
  exact definitions in ... 105
  facts about ... 105
  first difficulty in ... 113
  illustrative problems in ... 106
  in algebra ... 472, 473
  laws of signs ... 465, 467
  long ... 111
  problems in ... 118
  proofs in ... 118
  ratio idea in ... 104
  second difficulty in ... 113
  short ... 109
  sign of ... 56
  teaching primary idea of ... 24
  three difficulties in, the ... 114
  three uses of, the ... 104
  ways of expressing ... 106
  ways of, in fractions ... 170
  why troublesome ... 103
  written problems in ... 120
Division of decimals ... 204
  Austrian plan, by the ... 209
  Check plan in ... 207
  multiples of ten, by ... 211
  ordinary plan ... 205
  remainder in the ... 210
  short cuts in ... 212
  ten or powers of ten, by ... 211
Division of fractions ... 169
  oral problems in ... 119
Divisor and dividend having equal number of decimals ... 206
Divisor, the ... 105
  ending in one or more 0's ... 116
  exact ... 132
  no decimal point in ... 205
Dollar, derivation of name ... 185
  how separated from cents ... 182
Dollars and cents, adding ... 182
  dividing ... 184
  multiplying ... 183
  subtracting ... 183
  writing ... 182
Domestic exchange ... 343
  postal rates ... 348
Domino method, the ... 19
  devices with ... 19
Double colon in proportion ... 388
  corners in land survey ... 232
Drafts ... 340
  at a discount ... 343
  at a premium ... 343
  at par ... 343
  forms of ... 341
Drawee of draft ... 340
Drawer of draft ... 340

PAGE
Drawing, in primary number work. 26
Drills, addition 62
  division 107
  multiplication 90
  subtraction 74
Duties 353
  ad valorem 354
  specific 354
Duty of kindergarten teacher 14

Eagle, derivation of name 185
Early fractions, how learned 149
Earnings, gross 365
Eastern time 253
Endowment policies 357
English linear table 227
  money table 247
  square measure table 230
  system of long measure 226
Equal fractions in different forms 156
Equality, sign of 55
Equal number of decimal places in dividend and divisor 206
Equals, added to equals 386
  divided by equals 387
  multiplied by equals 387
  subtracted from equals 387
Equated date 400
Equation of payments 400
Equations 22, 379
  changes in 380
  divided 381, 472
  dividing 381
  members of 379
  multiplied 380, 472
  simple forms of 381
  simplified 380, 471
  terms of 379
  transposed 380, 471
  truth of 379
  turned about 380, 471
  with one unknown quantity 471
Equiangular triangle 442
Equilateral triangles 441
Estimates on farm or ranch 279
  practical 262
Even number 132
Exact, divisor 132
  method of finding time in interest 314
  time table 316
Examples in cancellation 144
Exchange 339
Exchange, domestic 343
  foreign 343
  the stock 375
Exercises in decimals 196
Explanation of stock quotations 369
Exponent 405

PAGE
Expressions in algebra 459
Express orders 347
Extremes of a proportion 388

Face of note 319
Factor, a 302
Factoring 129, 131
  by division 134
  by inspection 133
  large numbers 134
  small numbers 133
Factors, common 133
  composite 132
  of a number 131
  prime 132
  prime to each other 133
Facts about, brick and stone 268
  carpeting 275
  cord wood 277
  division 105
  lumber 262
  painting and plastering 274
  shingles and laths 270
  wall paper 273
Farm, estimates on the 279
Fathom, a 257
February, derivation of name 242
Federal money 246
Fetzga, on rules 180
Figures, primary, how written 18
Final value of annuities 361
Finding the, rate of tax 351
  missing part in a ratio 384
First difficulty in division 112
Five-cent piece as a metric measure 482
Fool problems 178
Foot 42
Foreign, coins, values of 346
  exchange 343
  money orders 345
  money problems 347
  postal rates 348
Formal decimal fractions 190
Forms of drafts 341
For quick percentage results 298
Forty-five facts of addition 60
Fractions 148
  addition of 161
  and integers, multiplication of 166
    by fractions 167
  changing to higher terms 157
    to lower terms 158
  clearing of, in division 171
  complex 171
  decimal 185, 216
  defined 154
  denominate 251
  denominator of 154
  division of 169

PAGE

Fractions—*Continued.*
  early, how learned.............. 149
  easiest and best way to divide in 170
  how to teach small.............. 151
  improper.............. 154
  in algebra.............. 469
  kinds of.............. 154
  multiplication of.............. 166
  numerator of.............. 154
  problems in.............. 173
  proper.............. 154
  reciprocal of.............. 170
  review of principles of.............. 177
  subtraction of.............. 164
  teaching, things to remember in 150
  teaching with disks.............. 151
  terms of.............. 154
  value of a.............. 177
  ways to divide in.............. 169
Fraternal insurance.............. 356
Fundamental operations.............. 53
  problems in.............. 124

Gain.............. 300
Gallon, congress.............. 289
Gallons, capacity of tanks in.............. 280
Garfield, James A.............. 283
General partners.............. 395
Geometrical demonstration of square root.............. 408
Geometrically, cubes.............. 416
Getting fractions ready for adding or subtracting.............. 159
Gladstone, Wm. E.............. 129
Goods sold by the, hundred.............. 258
  thousand.............. 258
  ton.............. 259
Government land survey.............. 230
  bonds.............. 181
Grace, days of.............. 321
Gram, the.............. 421
Greatest Common Divisor..........185, 469
Greenwich, meridian of.............. 252
Gregorian calendar.............. 241
Gross earnings.............. 365
Grouping objects.............. 19
Guide meridians.............. 232
Gunter's chain.............. 227

Half dozen.............. 28
  foot.............. 28
  sections in land survey.............. 233
Hand, a.............. 257
Handy metric tables.............. 432
Haste in primary work.............. 13
Hay, estimates of, in barn or stack.. 281
Heptagon, regular.............. 447
Hexagon, regular.............. 447
Higher terms, changing fractions to 157

PAGE

Higher units of the metric system.. 421
History of our coinage.............. 184
Holder of a note.............. 320
  of a draft.............. 340
Holland, J. G.............. 435
Horizontal line, a.............. 438
Hour, an.............. 43
How to, teach small fractions.............. 151
  tell prime factors of a number... 134
  use exact-time table.............. 316
Hull's, John, mint.............. 184
Hundred, goods sold by the.............. 258
Hundredths, writing.............. 192
Hypotenuse, the.............. 445

Illustrative bills.............. 218
Imperfect squares.............. 406
Improper fractions.............. 154
  changing to a whole or mixed number.............. 155
  changing to mixed numbers.... 156
Inches.............. 42
Income tax.............. 349
Indian money.............. 184
Indicated ratio.............. 383
Indirect taxes.............. 358
  customs.............. 358
  duties.............. 358
  excises.............. 358
  internal revenue.............. 358
  problems in.............. 354
  uses of.............. 353
  when to teach.............. 354
Indorsee.............. 320
Indorsement in blank.............. 320
  in full.............. 320
  short.............. 320
Indorser.............. 320
Industrial insurance.............. 357
Insolvent.............. 395
Inspection method.............. 198
  work by.............. 196
Insurance.............. 356
  dividends.............. 357
  endowment.............. 357
  fraternal.............. 356
  industrial.............. 358
  life.............. 356
  life policies.............. 357
  old line.............. 356
  on personal property.............. 357
  policy.............. 356
  premium.............. 357
  problems in.............. 358
  property.............. 356
  risk.............. 356
  tables.............. 358
Insured, the.............. 356
Insurer, the.............. 356

PAGE
Integers and fractions, multiplication of 166
Interest 309
    annual 328
    -bearing note 337
    common method of finding 314
    compound 330
    coupons 365
    merchant's rule 324
    more than partial payment 327
    problems in 317
    6% method 314
    60-day method 313
    United States rule 324
    ways of reckoning time in 314
International Date Line 255
In the kindergarten 18
Introductory talk 12
Inventing problems 31
Isosceles triangles 442

January, derivation of 242
Joint and several notes 323
Joint notes 323
Julian calendar 241
July, derivation of 242
June, derivation of 242
Jupiter 242

Keys in price marks 217
Kindergarten, the 18
    story 14
Kinds of, fractions 154
    partnership 395
Knot, a 257
Known numbers 456

Land survey 231
    double corners in 232
    government 230
    sections 233
    standard corners in 232
Language training in number work 22
Large numbers, factoring 134
Laths and shingles 269
Leakage 354
Leap year 241
Least Common Multiple 140
Legal, interest table 334
    weight of bushel 238
Length of second's pendulum 214
Letters of credit 345
Liability, defined 395
Library, The Success 404
Life insurance 356
    policies 357
Like factors, cancelling 144
Limit of express orders 347
Line, a 437

PAGE
Linear measure 226
    table of 227
Lines, and angles 437
    parallel 438
    perpendicular 438
Link, a 227
Liquid measure 47
    table of 239
List prices 306
Liter 421, 428
Literal part 457
Little people, measures for 40
    tests for 51
Locating land 234
Location of International Date Line 255
Locke, on rules 180
Long division 111
Longitude and time 252
    problems in 255
Long measure 42, 226
    English system 226
    our system 226
Long ton 240
Lord Baltimore's coinage 185
Loss and profit 300
Lots, city 235
Lower terms, fractions to 158
    decimals to 197
    metric measures 421
Lumber, facts about 263
    problems in 264
    table of measures 267

Macaulay 408
Maker of a note 319
Making change 45, 79
Making divisor a whole number in decimals 206
Map, time 254
March, derivation of 242
Margin, a 376
Market value of stock 364
Marks, price 217
Material for number stories 26
Maturity, date of 335
May, derivation of 242
McClellan and Dewey 148
Means of a proportion 388
Measurement in primary arithmetic 40
Measures, angular 243
    apothecaries' 239
    capacity, metric 428
    capacity, of 237
    circular 243
    civil engineer's 227
    cubic 236
    dry 237
    English square 230
    length, metric 422

PAGE
Measures, angular—*Continued.*
  liquid 287
  little people, for 40
  long 226
  lumber 267
  paper 248
  particular 256
  surface 229
  surface, metric 425
  time, of 241
  value, of 245
  volume, metric 427
  volume, of 286
  weight 289
  weight, metric 430
Measuring public lands 282
Medicines, weighing of 240
Members of an equation 379
Memorizing tables 289
Mensuration
Mental arithmetic 420
Merchant, commission 302
Merchant's rule in interest 324
Meridians 230
Meter, the 421
Metric measures 421
  capacity, of 428
  higher 421
  length, of 422
  lower 421
  problems in 425, 426, 431
  reductions in 422
  surface, of 425
  volume, of 427
  weight, of 430
Metric tables, handy 432
Mill, derivation of 185
  use of, principal 350
  writing 182
Minim, a 289
Mint of John Hull 184
Mints, where 185
Minuend defined 76
Minus 55
Minutes 43
Miscellaneous problems in fractions 173
Mixed decimals 216
Mixed numbers 154
  adding 163
  reduced to decimals 214
  subtracting 165
Money 45
  Indian 184
Money orders, express 347
  foreign 345
  United States 347
Monomial 459
Months 49
  how named 242

PAGE
Mortality tables 366
Mother and teacher 404
Mountain time 253
Multiple, a 140
Multiples of ten, in division of decimals 211
  multiplying by 204
Multiplicand, defined 89, 98
Multiplication 89
  algebra, in 464
  decimals, of 200
  denominate quantities, of 251
  drills in 90
  fractions, of 166
  oral problems in 99
  proofs of 97
  sign laws 464
  sign of 56
  written problems, in 100
Multiplier defined 89
  greater than 10 95
Multiplying by multiples of ten 204
  by 10, 100, etc 203
  dollars and cents 183
  equations 380
  fractions by fractions 167
  when multiplier is negative 465
  when it is positive 464

Names of, decimal places 193
  months, how given 242
  parts of quantities 457
Neatness in addition 73
Negative numbers 459
Negotiable note 319
Net earnings of a corporation 365
  prices 304
  proceeds 303
  weight 354
Newspapers and kindergartens 13
No decimal in divisor 205
Nonagon, regular 447
Non interest bearing note problems 396
Notes 319
  bearing no interest 396
  demand 323
  face of 319
  forms 322
  holder of 320
  negotiable 319
  joint 323
  joint and several 323
  partial payments on 323
  promissory 319
  time 323
November, derivation of name 243
Numbers, above ten 32
  above twenty 35
  composite 131

PAGE

Numbers, above ten—*Continued.*
- cubes of 412
- denominate 225
- equal to same number 387
- even 182
- exponent of 405
- factoring 181
- factors of 131
- for little people 16
- from 1 to 10 14
- from 1 to 100 17
- how compared 382
- known 456
- mixed 154
- negative 459
- odd 182
- percentage of a, number to find. 289
- positive 459
- prime 131
- table 248
- to find a, when rate and percentage are given 291
- unknown 456

Number stories 22
- in multiplication 25
- material for 26

Numerator of fractions 154
Numerical ratio 383

Obtuse angle 244, 439
- triangle 442

Octagon, regular 447
October, derivation of name 243
Odd number 182
Old line insurance 356
One unknown quantity 472
- problems in 471

Oral problems, in division 119
- multiplication 99
- subtraction 75

Order of writing numbers 80
Orders, foreign money 345
- post-office 347

Orders of decimal fractions 193
Ordinary plan of dividing decimals. 205
Originality, don't stifle 127
- in solutions 127

"Or order" 319
Our coinage, history of 184
Our system of long measure 226

Pacific time 253
Page, David 420, 436
Page, from creamery book 259
- from time book 260

Painting and plastering 274
Papering 278
Paper, measure table 248
- money 181

PAGE

Parallel lines 488
Parallelogram, a 439
- area of 441

Parallels 230
Parenthesis 57, 494
- preceded by the minus sign 462
  - plus sign 462

Parker, Francis W 224
Partial dividends 111
- payments 323
- payments less than interest due. 327
- products 95

Particular measures 256
Partition 104
Partitive proportion 398
- problems in 398

Partnership 395
- kinds of 395
- problems in 396
- terms used in 395

Part one number is of another, to find 172
Parts of 10, teaching the 18
Par value 364
Pathfinder, the 255
Payee, the 319
- of draft 340

Payments, average of 400
- equation of 400
- partial 323

Peck 49
Pendulum, length of second's 214
Penn, William 179
Pentagon, regular 447
Percent, on or off 306
- table of values 287
- to write a common fraction as .. 288
- to write a decimal as 287

Percentage 285
- developing the idea 285
- of a number 289
- problems in 289
- rate in 289
- results quickly 298
- various problems in 294

Perch, a 268
Perfect squares 406
Perimeter of a polygon 447
Perpendicular lines 488
Perpetual annuities 301
Persian, from the 419
Personal property insurance 357
Philadelphia, mint at 185
Phillips, Wendell 147
Picturing problems 26
Pint 47
Placing the multiplier, in algebra .. 464
Plane surfaces 439
Plan to teach decimals 190

PAGE
Plastering and painting 274
Plus sign, the 55
P. M., afternoon 43
Point, decimal 216
Policy, annuity 361
  of insurance 356
Poll tax 349
Polygons, regular 447
Polynomials 459
Positive numbers 459
Postal rates 348
Post-office orders 347
Practical, arithmetic in country
    schools 224
  cost methods 258
  estimates 262
  teaching 54
Preferred stock 372
Premium, drafts at a 343
  in insurance 357
  on bonds 367
  table of insurance 358
Present, value of annuities 361
  worth and true discount 338
Price marks 217
  keys of 217
Prices, list 306
Primary, idea of division 24
  teachers 12
Prime factor 132, 134
  numbers 131
  number table 135
Principal 308
  in interest 309
  meridian 230
Principles in fractions, review of ... 177
Problems in, addition 64, 65
  addition of fractions 163
  annuities 362
  area of triangles 442
  average 398
  average of payments 400
  bonds 372
  carpeting 276
  decimals 214
  direct taxes 351
  division 118
  foreign money 247
  fractions 178
  fundamental operations, the 124
  indirect taxes 354
  insurance ....,.... 358
  laths and shingles 271
  longitude and time 255
  lumber 264
  metric system 425, 426, 431
  partitive proportion 393
  partnership 396
  percentage 289
  profit and loss 300
  proportion 389
  ratio 385
  stocks 367
  with one unknown quantity 472
  wood measure 278
Procedure of assessor 351
Proceeds 335
Product, definition of 89, 93
  in decimal fractions 201
  of means equals product of ex-
    tremes 388
Profit and loss 300
Promissory notes 319
Proofs, in addition 67
  division 118
  multiplication 97
  subtraction 81
Proper fractions 154
Property, insurance 356
  tax 349
Proportion 387
  cancellation in 392
  double colon in 388
  extremes of a 388
  means of a 388
  partitive 393
  problems in 389
  product of means equals product
    of extremes 389
  terms used in 388
Protractor, the 245
Proving additions 67
Psychology of Number 148
Pupil, telling the 103
Purchase, account 221
Purpose of kindergartens 13
Pyramids 452

Quadrilateral, a ....
Quantities, addition of denominate 248
  compound denominate 296
  multiplication of denominate ... 250
  names of parts of 226
  simple denominate 226
  subtraction of denominate 249
Quart, a 47
Quarter sections, in land survey 233
Quick percentage results 298
Quotations, bond 373
  of stock, explanations of 369
Quotient, the 105
  writing the 115

Radical sign, the 405
Radius 243, 448
Ranch, estimates on the 279
Rapid adding 66

PAGE

- Rate, and base given, to find percentage 293
  - and percentage given, to find base 291
  - in percentage 289
  - of tax, finding the 351
  - per cent 289
- Ratio 382
  - abstract 384
  - concrete 384
  - couplet in 383
  - finding the missing part in 384
  - idea in division 104
  - indicated 383
  - numerical 383
  - problems in 385
  - sign of a 383
  - terms of a 383
  - writing a 384
- Reading, and writing numbers 30
  - decimals 216
  - numbers 31
    - from 10 to 100 35
  - simple decimals 187
- Reason for pointing off in multiplication of decimals 201
- Receipted bills 219
- Reciprocal of a fraction 170
- Rectangle, a 439
  - area of 440
- Recurring decimals 214
- Reducing, common fractions to decimals 213
  - decimal fractions to common denominator 197, 198
  - fractions to higher terms 157
    - to lower terms 157
  - mixed numbers to decimals 214
- Reduction, ascending 226
  - descending 226
  - in metric system 423
  - of decimal fractions to lowest terms 197
- Registered bonds 366
- Regular, stock sold 316
- Regular polygons 447
- Reinsurance 358
- Remainder, definition of 76
  - in division of decimals 210
  - what to do with 110
- Repeater, in price marks 217
- Repetend 214
- Report of Committee of Twelve 224
- Results in percentage quickly 298
- Review of principles in fractions 177
- Rhomboid, a 440
- Rhombus, a 440
- Right angle 243
  - triangle 442, 445

PAGE

- Risk in insurance 356
- Roman notation, interesting facts about 37, 38
- Romans, in names of months 242
- Root, cube 412
  - square 405
- Rules, teaching of 180

- Sales, account 221
- Scalene triangles 442
- Schools, practical arithmetic in country 224
- Scrip dividend 371
- Second difficulty in division 113
- Second's pendulum, length of 214
- Sections, in land survey 233
- Section sixteen, for schools 233
- Seller three 376
- Selling problems 304
- Selling short 375
- Sending bills of exchange 339
- September, derivation of 242
- Shareholders, of corporation 365
- Share of stock 364
- Shingles and laths 269
- Short, cuts in division of decimals 212
  - division 109
  - indorsement 320
- Short summary of decimals 216
- Sides of a triangle 27
- Sign, a 244
  - addition, of 55
  - division, of 56
  - equality, of 55
  - minus 55
  - multiplication 56
  - %, of 286
  - plus 55
  - radical, the 405
  - ratio, of a 383
  - subtraction 55
- Signs, common 55
- Silent partner 395
- Similar solids 454
  - surfaces 451
  - terms 458
- Simple decimals 186
  - forms of equations 381
  - partnership 395
- Simplifying equations 380
- Single express order, limit of 347
- Six per cent method 312
- Sixty-day method 313
- Slant height of pyramid or cone 452
- Small fractions, how to teach 151
- Small numbers, factoring 133
- Smith, David Eugene 54
- Solar year 257

PAGE
Solids 452
  similar 454
Solutions, originality in 127
Some uses for taxes 349
Span, a 256
Special, partner 395
  receipts 326
Specific duties 354
Spencer's Education 130
Sphere, the 453
Square, a 257
  measure 50, 230
  on the hypotenuse 446
  root 405
  geometrically 408
  of decimals 408
  units 229
  uses of 411
Squares, imperfect 406
  numbers, of 405
  perfect 406
Stack, estimates of hay in 281
Standard, corners in land survey 232
  of U. S. money 182
Stere, the 427
Sterling money 246
Stock 364
  above par 364
  at a premium 364
  below par 364
  broker 364
  certificate of 364
  common 371
  dividend 365
  exchange 375
  market value of 364
  par value of 364
  preferred 372
  problems in 367
  quotations 369, 371
  share of 364
  sold cash 376
  sold regular 376
  watered 375
Stocks and bonds 363
  difference between 366
Stone, a 256
Stone and brick 268
Straight line, a 438
Subtraction, algebra in 462
  combinations in 73
  decimals, of 199
  defined 76
  denominate quantities, of 249
  dollar and cents, of 183
  fractions, of 164
  mixed numbers 165
  proofs of 81
  sign of 55

PAGE
Subtrahend defined 76
Success Library, The 404
Sum 59
Summary of decimals 216
Surface measures 229
  metric 425
Surfaces, plane 439
  similar 451
Surplus 364
Survey, diagram of township 231
Surveyor's linear table 227
System, metric 421

Table of, annuity values 362
  apothecaries' measures 239
  avoirdupois weight 239
  circular measure 244
  compound interest 333
  cubic measure 236
  English linear measure 227
  English money 247
  English square measure 230
  exact time 316
  foreign coins 246
  handy metric 432
  insurance premiums 358
  legal interest rates 334
  liquid measures 239
  lumber measures 267
  memorizing of 239
  paper measure 248
  percentage values 287
  prime numbers 135
  showing 9 mill tax 350
  Sterling money 247
  surveyor's linear measure 227
  time measures 241
  Troy weight 240
Tanks, finding capacity of 280
Tare 354
Tax, collector 349
  table 350
Taxes, capitation 349
  direct 348
  finding rate of 351
  income 349
  indirect 353
    customs 353
    duties 353
    excises 353
    internal revenue 353
  why needed 353
  poll 349
  problems, in direct 351
    indirect 354
  property 349
  some uses for 349
  use of the mill in 350
Teacher and mother 404

PAGE

- Teaching, fractions with discs 151
  - fractions, things to remember in 150
  - practical 54
- Telling the pupil 103
- Ten, or powers of ten 203, 211
- Term of credit 400
- Terms, algebraic 458
  - dissimilar 458
  - fraction, of a 154
  - ratio, of a 383
  - discount, of 335
  - equation, of 379
  - of stock exchange 375
  - similar 458
  - used in percentage 288
  - in proportion 388
- Tests for little people 51
- Third, difficulty in division 114
  - side of triangles, finding the 445
- Thousand, goods sold by the 258
- Thousandths, illustrating 192
- Three, classes of interest problems 317
  - difficulties in division 112
  - uses of division, the 104
- Tier, of townships 230
- Time, average of 400
  - belts, the 253
  - books 260
  - common way of finding, in interest 314
  - draft 342
  - exact method of finding, in interest 314
  - measures of 241
  - notes 323
  - telling 48
  - ways of reckoning, in interest 314
- Ton, the long 240
  - goods sold by 259
- Townships 233
- Tracts of land, locating, by government survey 234
- Trade discount 306
- Trapezium, a 440
- Trapezoid, a 440
  - area of 440
- Treasurer 349
- Treasury notes 181
- Trial quotient figure, to find 113
- Triangles 441
  - area of 442
  - finding the third side of 445
- Trinomial 459
- Troy weight 240
- True discount 338
- Truth of equations 379
- Twelve, report of the Committee of 234
- Twenty, numbers above 35

PAGE

- Underwriter 356
- United States, money 181
  - coins of 181
    - derivation of names 185
    - standard of 182
- United States rule interest 324
- Unknown, numbers 456
  - quantity, x for the 385
- Use of, *and* 31
  - comma 57
  - mill, the 350
- Uses for square roots 411
- Uses of, taxes, some 349
  - indirect taxes 353

- Valuable facts about notes 320
- Value, measures of 245
  - of a fraction 177
  - foreign coins 246
- Various problems in percentage 294
- Vertex 438
- Vertical line, a 438
- Vinculum 58
- Volume, measures of 236
  - solids, of 452

- Wainscoting, allowance for 275
- Wall paper 273
- Watered stock 307
- Week, days of 49
- Weight, measures of 239
  - as fixed by law 238
  - avoirdupois 239
  - comparison of 240
  - metric 430
  - Troy 240
- When to study longitude and time 252
- When to teach indirect taxes 354
- Where the day begins 254
- Why division is troublesome 103
- "Without recourse," on notes 321
- Wood measure 277
- Words, "or order" 319
- Work by inspection 198
- Writing, decimals 216
  - dollars and cents 182
  - hundredths 192
  - mills 182
  - quotient, the 115
  - ratio, a 384

- X for the unknown quantity 385

- Yard 42
- Year, common 241
  - months of 49
  - solar 257

MAY 1 4 1924